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Research Article

On the Distribution of the q-Euler Polynomials and the q-Genocchi Polynomials of Higher Order

Leechae Jang[1] and Taekyun Kim[2]

[1] *Department of Mathematics and Computer Science, KonKuk University, Chungju 380-701, South Korea*
[2] *Division of General Education-Mathematics, Kwangwoon University, Seoul 139-701, South Korea*

Correspondence should be addressed to Leechae Jang, leechae.jang@kku.ac.kr

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In 2007 and 2008, Kim constructed the q-extension of Euler and Genocchi polynomials of higher order and Choi-Anderson-Srivastava have studied the q-extension of Euler and Genocchi numbers of higher order, which is defined by Kim. The purpose of this paper is to give the distribution of extended higher-order q-Euler and q-Genocchi polynomials.

1. Introduction

The Euler numbers E_n and polynomials $E_n(x)$ are defined by the generating function in the complex number field as

$$\frac{2}{e^t+1}=\sum_{n=0}^{\infty}E_n\frac{t^n}{n!}\ (|t|<\pi),$$
$$\frac{2}{e^t+1}e^{xt}=\sum_{n=0}^{\infty}E_n(x)\frac{t^n}{n!}\ (|t|<\pi), \tag{1.1}$$

cf. [1–4]. The Bernoulli numbers B_n and polynomials $B_n(x)$ are defined by the generating function as

$$\frac{t}{e^t-1}=\sum_{n=0}^{\infty}B_n\frac{t^n}{n!},$$
$$\frac{t}{e^t-1}e^{xt}=\sum_{n=0}^{\infty}B_n(x)\frac{t^n}{n!}, \tag{1.2}$$

cf. [5–8]. The Genocchi numbers G_n and polynomials $G_n(x)$ are defined by the generating function as

$$\frac{2t}{e^t+1}=\sum_{n=0}^{\infty}G_n\frac{t^n}{n!},$$
$$\frac{2t}{e^t+1}e^{xt}=\sum_{n=0}^{\infty}G_n(x)\frac{t^n}{n!}, \tag{1.3}$$

cf. [9, 10]. It satisfies $G_0=0$, $G_1=1,\ldots$, and for $n\geq 1$,

$$G_n=2^n\left(B_n\left(\frac{1}{2}\right)-B_n\right). \tag{1.4}$$

Let p be a fixed odd prime number. Throughout this paper, $\mathbb{Z}_p, \mathbb{Q}_p$, and $\mathbb{C}_p$ will be, respectively, the ring of p-adic rational integers, the field of p-adic rational numbers and the p-adic completion of the algebraic closure of $\mathbb{Q}_p$. The p-adic absolute value in $\mathbb{C}_p$ is normalized so that $|p|_p=1/p$. When one talks of q-extension, q is variously considered as an indeterminate, a complex number $q\in\mathbb{C}$ or a p-adic number $q\in\mathbb{C}_p$. If $q\in\mathbb{C}$, one normally assumes $|q|<1$. If $q\in\mathbb{C}_p$, one normally assumes $|1-q|_p<1$. We use the notation

$$[x]_q=\frac{1-q^x}{1-q},\qquad [x]_{-q}=\frac{1-(-q)^x}{1+q}, \tag{1.5}$$

cf. [1–5, 9–23] for all $x\in\mathbb{Z}_p$. For a fixed odd positive integer d with $(p,d)=1$, set

$$X=X_d=\lim_{\overleftarrow{n}}\frac{\mathbb{Z}}{dp^n\mathbb{Z}},\quad X_1=\mathbb{Z}_p,$$
$$X^*=\bigcup_{\substack{0<a<dp\\(a,p)=1}}(a+dp\mathbb{Z}_p), \tag{1.6}$$
$$a+dp^n\mathbb{Z}_p=\{x\in X\mid x\equiv a\ (\operatorname{mod} dp^n)\},$$

where $a\in\mathbb{Z}$ lies in $0\leq a<dp^n$. For any $n\in\mathbb{N}$,

$$\mu_q(a+dp^n\mathbb{Z}_p)=\frac{q^a}{[dp^n]_q} \tag{1.7}$$

is known to be a distribution on X, cf. [1–5, 9–23].

We say that f is uniformly differentiable function at a point $a\in\mathbb{Z}_p$ and denote this property by $f\in UD(\mathbb{Z}_p)$, if the difference quotients

$$F_f(x,y)=\frac{f(x)-f(y)}{x-y} \tag{1.8}$$

have a limit $l=f'(a)$ as $(x,y)\to(a,a)$, cf. [4].

The p-adic q-integral of a function $f\in UD(\mathbb{Z}_p)$ was defined as

$$I_q(f)=\int_{\mathbb{Z}_p}f(x)d\mu_q(x)=\lim_{n\to\infty}\frac{1}{[p^n]_q}\sum_{x=0}^{p^n-1}f(x)q^x, \tag{1.9}$$

$$I_{-q}(f)=\int_{\mathbb{Z}_p}f(x)d\mu_{-q}(x)=\lim_{n\to\infty}\frac{1}{[p^n]_q}\sum_{x=0}^{p^n-1}f(x)(-q)^x, \tag{1.10}$$

cf. [14]. In (1.10), when $q \to 1$, we derive

$$I_{-1}(f_1) + I_{-1}(f) = 2f(0), \tag{1.11}$$

where $f_1(x) = f(x+1)$. If we take $f(x) = e^{tx}$, then we have $f_1(x) = e^{t(x+1)} = e^{tx}e^t$. From (1.11), we obtain

$$I_{-1}(e^{tx}) = \int_{\mathbb{Z}_p} e^{tx} d\mu_{-1}(x) = \frac{2}{e^t+1} = \sum_{n=0}^{\infty} E_n \frac{t^n}{n!}. \tag{1.12}$$

In view of (1.10), we can consider the q-Euler numbers as follows:

$$I_{-q}(e^{t[x]_q}) = \int_{\mathbb{Z}_p} e^{t[x]_q} d\mu_{-q}(x) = \sum_{n=0}^{\infty} E_{n,q} \frac{t^n}{n!}. \tag{1.13}$$

By (1.12) and (1.13), we obtain the followings.

Lemma 1.1. *For $n \in \mathbb{N}$,*

$$E_n = \frac{G_{n+1}}{n+1}. \tag{1.14}$$

Proof. We note that

$$\begin{gathered} tI_{-1}(e^{tx}) = \frac{2t}{e^t+1} = \sum_{n=0}^{\infty} G_n \frac{t^n}{n!} = \sum_{n=1}^{\infty} G_n \frac{t^n}{n!} = \sum_{n=0}^{\infty} \frac{G_{n+1}}{n+1} \frac{t^{n+1}}{n!}, \\ tI_{-1}(e^{tx}) = \sum_{n=0}^{\infty} \int_{\mathbb{Z}_p} x^n d\mu_{-1}(x) \frac{t^{n+1}}{n!}. \end{gathered} \tag{1.15}$$

From (1.15), we have

$$\frac{G_{n+1}}{n+1} = \int_{\mathbb{Z}_p} x^n d\mu_{-1}(x) = E_n. \tag{1.16}$$

The purpose of this paper is to give the distribution of extended higher order q-Euler and q-Genocchi polynomials. In [24], Choi-Anderson-Srivastava have studied the q-extension of the Apostol-Euler polynomials of order n, and the multiple Hurwitz zeta functions (see [24]). Actually, their results and definitions are not new (see [18, 20]) and the definition of the Apostol-Bernoulli numbers in their paper are exactly the same as the definition of the q-extension of Genocchi numbers. Finally, they conjecture that the following q-distribution relation holds:

$$([m]_q)^{k-1} \sum_{j=0}^{m-1} (-w)^j E^{(n)}_{k,q^m,w^m}\left(\frac{x+j}{m}\right) = E^{(n)}_{k,q,w}(x) \tag{1.17}$$

(see [24, Remark 6, page 735]). This seems to be nonsense as a conjecture. In this paper we give the corrected distribution relation related to the conjecture of Choi-Anderson-Srivastava in [24] (see Theorem 2.6). □

2. Weighted q-Genocchi number of higher order

In this section, we assume that $q \in \mathbb{C}_p$ with $|1-q|_p < 1$ or $q \in \mathbb{C}$ with $|q| < 1$. For $k \in \mathbb{N}$ and $w \in \mathbb{C}_p$ with $|1-w|_p < 1$, we define the weighted q-Euler numbers of order k as follows:

$$E_{n,q,w}^{(k)} = \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} q^{\sum_{j=1}^{k}(k-j)x_j} w^{x_1+\cdots+x_k} [x_1+\cdots+x_k]_q^n d\mu_{-q}(x_1)\cdots d\mu_{-q}(x_k). \tag{2.1}$$

We note that q-binomial coefficient is defined by

$$\binom{n}{k}_q = \frac{[n]_q[n-1]_q\cdots[n-k+1]_q}{[k]_q}, \tag{2.2}$$

cf. [20]. From (2.1), we obtain the following theorem.

Lemma 2.1. *For $k \in \mathbb{N}, n \in \mathbb{N} \cup \{0\}$ and $w \in \mathbb{C}_p$ with $|1-w|_p < 1$, one has*

$$E_{n,q,w}^{(k)} = [2]_q^k \sum_{m=0}^{\infty} \binom{m+k-1}{m}_q (-1)^m w^m q^m [m]_q^n. \tag{2.3}$$

Proof. From (2.1), we have

$$\begin{aligned}
E_{n,q,w}^{(k)} &= \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} q^{\sum_{j=1}^{k}(k-j)x_j} w^{x_1+\cdots+x_k} [x_1+\cdots+x_k]_q^n d\mu_{-q}(x_1)\cdots d\mu_{-q}(x_k) \\
&= \lim_{N\to\infty} \frac{1}{[p^N]_{-q}^k} \sum_{x_1,\ldots,x_k=0}^{p^N-1} q^{\sum_{j=1}^{k}(k-j)x_j} w^{x_1+\cdots+x_k} [x_1+\cdots+x_k]_q^n (-q)^{x_1+\cdots+x_k} \\
&= \frac{[2]_q^k}{2^k} \frac{1}{(1-q)^n} \lim_{N\to\infty} \sum_{x_1,\ldots,x_k=0}^{p^N-1} q^{\sum_{j=1}^{k}(k-j+1)x_j} (-1)^{x_1+\cdots+x_k} \\
&\quad \times w^{x_1+\cdots+x_k} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{l(x_1+\cdots+x_k)} \\
&= \frac{[2]_q^k}{2^k} \frac{1}{(1-q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l \frac{2^k}{\prod_{j=1}^{k}(1+q^{l+j}w)} \\
&= [2]_q^k \frac{1}{(1-q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l \sum_{m=0}^{\infty} \binom{m+k-1}{m}_q (-1)^m q^{lm} q^m w^m \\
&= [2]_q^k \sum_{m=0}^{\infty} \binom{m+k-1}{m}_q (-1)^m q^{lm} q^m w^m \frac{1}{(1-q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{lm} \\
&= [2]_q^k \sum_{m=0}^{\infty} \binom{m+k-1}{m}_q (-1)^m q^{lm} q^m w^m [m]_q.
\end{aligned} \tag{2.4}$$

Now we consider the following generating functions:

$$\begin{aligned} F_{q,w}^{(k)}(t) &= \sum_{n=0}^{\infty} E_{n,q,w}^{(k)} \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} [2]_q^k \sum_{m=0}^{\infty} \binom{m+k-1}{m}_q (-1)^m w^m q^m [m]_q^n \\ &= [2]_q^k \sum_{m=0}^{\infty} \binom{m+k-1}{m}_q (-1)^m w^m q^m e^{[m]_q t}. \end{aligned} \tag{2.5}$$

By (2.5), we can define the weighted q-Genocchi numbers of order k:

$$T_{q,w}^{(k)}(t) = t^k F_{q,w}^{(k)}(t) = \sum_{n=0}^{\infty} G_{n,q,w}^{(k)} \frac{t^n}{n!}. \tag{2.6}$$

From (2.1), (2.2), and (2.6), we note that

$$\begin{gathered} G_{0,q,w}^{(k)} = G_{1,q,w}^{(k)} = \cdots = G_{k-1,q,w}^{(k)} = 0, \\ t^k \sum_{n=0}^{\infty} E_{n,q,w}^{(k)} \frac{t^n}{n!} = \sum_{n=k}^{\infty} G_{n,q,w}^{(k)} \frac{t^n}{n!}. \end{gathered} \tag{2.7}$$

Thus, we obtain

$$\begin{aligned} \sum_{n=0}^{\infty} E_{n,q,w}^{(k)} \frac{t^n}{n!} &= \sum_{n=k}^{\infty} G_{n,q,w}^{(k)} \frac{t^{n-k}}{n!} \\ &= \sum_{n=k}^{\infty} G_{n+k,q,w}^{(k)} \frac{t^n}{(n+k)!} \\ &= \sum_{n=k}^{\infty} G_{n+k,q,w}^{(k)} \frac{1}{\binom{m+k-1}{m}} \frac{t^n}{n!}. \end{aligned} \tag{2.8}$$

From (2.8), we obtain the following recurrsion relation between q-Euler and q-Genocchi numbers of order k. □

Theorem 2.2. *For $k \in \mathbb{N}, n \in \mathbb{N} \cup \{0\}$ and $w \in \mathbb{C}_p$ with $|1-w|_p < 1$, one has*

$$\binom{m+k}{k} k! E_{n,q,w}^{(k)} = G_{n+k,q,w}^{(k)}. \tag{2.9}$$

For $k \in \mathbb{N}$, we also define the weighted q-Euler polynomials of order k as follows:

$$E_{n,q,w}^{(k)}(x) = \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} q^{\sum_{j=1}^{k}(k-j)x_j} w^{x_1+\cdots+x_k} \left[x + x_1 + \cdots + x_k\right]_q^n d\mu_{-q}(x_1) \cdots d\mu_{-q}(x_k). \tag{2.10}$$

From (2.9), we obtain the following theorem.

Theorem 2.3. *For $k \in \mathbb{N}$, $n \in \mathbb{N} \cup \{0\}$ and $w \in \mathbb{C}_p$ with $|1-w|_p < 1$, one has*

$$E_{n,q,w}^{(k)}(x) = [2]_q^k \sum_{m=0}^{\infty} \binom{m+k-1}{m}_q (-1)^m w^m q^m [x+m]_q^n. \tag{2.11}$$

Proof.

$$\begin{aligned}
E_{n,q,w}^{(k)}(x) &= \lim_{N\to\infty} \frac{1}{[p^N]_{-q}^k} \sum_{x_1,\ldots,x_k=0}^{p^N-1} q^{\sum_{j=1}^{k}(k-j)x_j} w^{x_1+\cdots+x_k} \left[x + x_1 + \cdots + x_k\right]_q^n (-q)^{x_1+\cdots+x_k} \\
&= \frac{[2]_q^k}{2^k} \frac{1}{(1-q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{lx} \lim_{N\to\infty} \sum_{x_1,\ldots,x_k=0}^{p^N-1} q^{\sum_{j=1}^{k}(k-j+l+1)x_j} (-1)^{x_1+\cdots+x_k} w^{x_1+\cdots+x_k} \\
&= \frac{[2]_q^k}{2^k} \frac{1}{(1-q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{lx} \frac{2^k}{\prod_{j=1}^{k}(1+q^{l+j}w)} \\
&= [2]_q^k \frac{1}{(1-q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{lx} \sum_{m=0}^{\infty} \binom{m+k-1}{m}_q (-1)^m q^{lm} q^m w^m \\
&= [2]_q^k \sum_{m=0}^{\infty} \binom{m+k-1}{m}_q (-1)^m q^{lm} q^m w^m [x+m]_q.
\end{aligned} \tag{2.12}$$

From (2.11), we consider the following generating functions:

$$\begin{aligned}
F_{q,w}^{(k)}(t,x) &= \sum_{n=0}^{\infty} E_{n,q,w}^{(k)}(x) \frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty} [2]_q^k \sum_{m=0}^{\infty} \binom{m+k-1}{m}_q (-1)^m w^m q^m [x+m]_q^n \\
&= [2]_q^k \sum_{m=0}^{\infty} \binom{m+k-1}{m}_q (-1)^m w^m q^m e^{[x+m]_q t}.
\end{aligned} \tag{2.13}$$

□

By (2.13), we can define the weighted q-Genocchi polynomials of order k as follows:

$$T_{q,w}^{(k)}(t,x) = t^k F_{q,w}^{(k)}(t,x) = \sum_{n=0}^{\infty} G_{n,q,w}^{(k)}(x) \frac{t^n}{n!}. \tag{2.14}$$

From (2.14), we note that

$$\begin{gathered}
G_{0,q,w}^{(k)}(x) = G_{1,q,w}^{(k)} = \cdots = G_{k-1,q,w}^{(k)}(x) = 0, \\
t^k \sum_{n=0}^{\infty} E_{n,q,w}^{(k)}(x) \frac{t^n}{n!} = \sum_{n=k}^{\infty} G_{n,q,w}^{(k)}(x) \frac{t^n}{n!}.
\end{gathered} \tag{2.15}$$

By comparing the coefficients on both sides, we see that

$$\begin{aligned}\sum_{n=0}^{\infty} E_{n,q,w}^{(k)}(x)\frac{t^n}{n!} &= \sum_{n=k}^{\infty} G_{n,q,w}^{(k)}(x)\frac{t^{n-k}}{n!} \\ &= \sum_{n=k}^{\infty} G_{n+k,q,w}^{(k)}(x)\frac{t^n}{(n+k)!} \\ &= \sum_{n=k}^{\infty} G_{n+k,q,w}^{(k)}(x)\frac{1}{\binom{m+k-1}{m}}\frac{t^n}{n!}.\end{aligned} \tag{2.16}$$

From (2.16), we obtain the following recursion relation between weighted q-Euler and weighted q-Genocchi polynomials of order k.

Theorem 2.4. *For $k \in \mathbb{N}, n \in \mathbb{N} \cup \{0\}$ and $w \in \mathbb{C}_p$ with $|1-w|_p < 1$, one has*

$$\binom{m+k}{k} k! E_{n,q,w}^{(k)}(x) = G_{n+k,q,w}^{(k)}(x). \tag{2.17}$$

Corollary 2.5. *For $k \in \mathbb{N}, n \in \mathbb{N} \cup \{0\}$ and $w \in \mathbb{C}_p$ with $|1-w|_p < 1$, one has*

$$\begin{aligned}G_{n+k,q,w}^{(k)}(x) &= k!\binom{n+k}{k}\frac{[2]_q^k}{(1-q)^n}\sum_{l=0}^{n}\binom{n}{l}(-1)^l q^{xl}\frac{1}{\prod_{j=1}^{k}(1+q^{l+j}w)} \\ &= k!\binom{n+k}{k}[2]_q^k\sum_{m=0}^{\infty}\binom{m+k-1}{m}_q(-1)^m w^m q^m [x+m]_q^n.\end{aligned} \tag{2.18}$$

Let $d \in \mathbb{N}$ with $d \equiv 1(\mathrm{mod}\, 2)$. Then we note that

$$\begin{aligned}E_{n,q,w}^{(k)}(x) &= \int_{\mathbb{Z}_p}\cdots\int_{\mathbb{Z}_p} q^{\sum_{j=1}^{k}(k-j)x_j} w^{x_1+\cdots+x_k}[x+x_1+\cdots+x_k]_q^n d\mu_{-q}(x_1)\cdots d\mu_{-q}(x_k) \\ &= \frac{[d]_q^m}{[d]_{-q}^k}\sum_{i_1,\ldots,i_k=0}^{d-1} q^{k\sum_{j=1}^{k}i_j-\sum_{j=2}^{k}(j-1)i_j}(-1)^{\sum_{j=1}^{k}i_j} w^{i_1+\cdots+i_k} \\ &\quad\times\int_{\mathbb{Z}_p}\cdots\int_{\mathbb{Z}_p}\left[\frac{x+\sum_{j=1}^{k}i_j}{d}+\sum_{j=1}^{k}x_j\right]_{q^d}^m (q^d)^{\sum_{j=1}^{k}(k-j)x_j}(w^d)^{x_1+\cdots+x_k} \\ &\quad\times d\mu_{-q^d}(x_1)\cdots d\mu_{-q^d}(x_k) \\ &= \frac{[d]_q^m}{[d]_{-q}^k}\sum_{i_1,\ldots,i_k=0}^{d-1} q^{k\sum_{j=1}^{k}i_j-\sum_{j=2}^{k}(j-1)i_j}(-1)^{\sum_{j=1}^{k}i_j} E_{m,q^d,w^d}^{(k)}\left(\frac{x+x_1+\cdots+x_k}{d}\right).\end{aligned} \tag{2.19}$$

Therefore, we obtain the following main results.

Theorem 2.6 (Distribution for higher order q-Euler polynomials). *For $d \in \mathbb{N}$ with $d \equiv 1 (\bmod 2), n \in \mathbb{N} \cup \{0\}$ and $w \in \mathbb{C}_p$ with $|1-w|_p < 1$, one has*

$$E_{n,q,w}^{(k)}(x) = \frac{[d]_q^m}{[d]_{-q}^k} \sum_{i_1,\ldots,i_k=0}^{d-1} q^{k\sum_{j=1}^{k} i_j - \sum_{j=2}^{k}(j-1)i_j} (-1)^{\sum_{j=1}^{k} i_j} E_{m,q^d,w^d}^{(k)}\left(\frac{x+x_1+\cdots+x_k}{d}\right). \tag{2.20}$$

For $k \in \mathbb{N}$, $w \in \mathbb{C}$ with $|w| < 1$, we easily see that

$$F_{q,w}^{(k)}(t,x) = [2]_q^k \sum_{m=0}^{\infty} \binom{m+k-1}{m}_q (-1)^m w^m q^m e^{[x+m]_q t} = \sum_{m=0}^{\infty} E_{m,q,w}^{(k)}(x) \frac{t^m}{m!}. \tag{2.21}$$

Thus we have

$$E_{n,q,w}^{(k)}(x) = \frac{d^n}{dt^n} F_{q,w}^{(k)}(t,x) = [2]_q^k \sum_{m=0}^{\infty} (-1)^m q^m w^m [x+m]_q^n \binom{m+k-1}{m}_q \tag{2.22}$$

Definition 2.7. For $s \in \mathbb{C}$, $k \in \mathbb{N}$ and $w \in \mathbb{C}$ with $|w| < 1$, one has

$$\zeta_{q,w,E}^{(k)}(s,x) = [2]_q^k \sum_{m=0}^{\infty} \frac{(-1)^m w^m q^m \binom{m+k-1}{m}_q}{[m+x]_q^s}. \tag{2.23}$$

Note that $\zeta_{q,w,E}^{(k)}(s,x)$ is analytic function in the whole complex s-plane. From (2.23), we derive the following.

Theorem 2.8. *For $n \in \mathbb{N} \cup \{0\}, k \in \mathbb{N}$ and $w \in \mathbb{C}_p$ with $|1-w|_p < 1$, one has*

$$\zeta_{q,w,E}^{(k)}(-n,x) = E_{n,q,w}^{(k)}(x). \tag{2.24}$$

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