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Research Article

A T-S Fuzzy Model-Based Adaptive Exponential Synchronization Method for Uncertain Delayed Chaotic Systems: An LMI Approach

Choon Ki Ahn

Department of Automotive Engineering, Seoul National University of Science and Technology, 172 Gongneung 2-dong, Nowon-gu, Seoul 139-743, Republic of Korea

Correspondence should be addressed to Choon Ki Ahn, hironaka@snut.ac.kr

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This paper proposes a new fuzzy adaptive exponential synchronization controller for uncertain time-delayed chaotic systems based on Takagi-Sugeno (T-S) fuzzy model. This synchronization controller is designed based on Lyapunov-Krasovskii stability theory, linear matrix inequality (LMI), and Jesen's inequality. An analytic expression of the controller with its adaptive laws of parameters is shown. The proposed controller can be obtained by solving the LMI problem. A numerical example for time-delayed Lorenz system is presented to demonstrate the validity of the proposed method.

1. Introduction

Chaos synchronization is an important subject both theoretically and practically, for applications requiring oscillations out of chaos, machine and building structural stability analysis, chaos generators design and so on. Chaos synchronization, first described by Fujisaka and Yamada [1] in 1983, did not received great attention until 1990 [2]. From then on, chaos synchronization has been developed extensively due to its various applications [3]. During the last decade, several techniques for handling chaos synchronization have been developed, such as variable structure control [4], OGY method [5], observer-based control [6], active control [7], backstepping design technique [8], $\mathscr{H}_\infty$ approach [9], and passivity based method [10].

Time delay inevitably appears in many physical systems such as aircraft, chemical, and biological systems. Unlike ordinary differential equations, time delayed systems are infinite dimensional in nature and time-delay is, in many cases, a source of instability. The stability issue and the performance of time delayed systems are, therefore, both of theoretical and practical importance. Since Mackey and Glass [11] first found chaos in time delayed system, there has been increasing interest in time delayed chaotic systems [12, 13]. The synchronization problem for time delayed chaotic systems is also investigated by several researchers [14–20].

In recent years, fuzzy logic methodology has been proven effective in dealing with complex nonlinear systems containing certainties that are otherwise difficult to model. Among various kinds of fuzzy methods, Takagi-Sugeno (T-S) fuzzy model provides a successful method to describe certain complex nonlinear systems using some local linear subsystems [21, 22]. In [23], a fuzzy feedback control method was proposed for chaotic synchronization and chaotic model following control. The authors in [24, 25] proposed fuzzy observer-based chaotic synchronization and secure communication. In [26, 27], fuzzy adaptive synchronization methods for chaotic systems with unknown parameters were proposed. In spite of these advances in T-S fuzzy model-based chaos control and synchronization, most works were restricted to chaotic systems without time-delay. Due to finite signal transmission times, switching speeds and memory effects, time delayed systems are ubiquitous in nature, technology, and society [28, 29]. Time delayed chaotic systems are also interesting because the dimension of their chaotic dynamics can be increased by increasing the delay time sufficiently [30]. For this reason, the time delayed chaotic system has been suggested as a good candidate for secure communication [31]. The dimension of solution space of time delayed chaotic systems is infinite and so more than one positive Lyapunov exponents could be produced just by some low-dimension delayed chaotic systems. Therefore, communication system with a higher security level can be designed by means of time delayed chaotic systems. In addition, the time delayed system can be considered as a special case of spatiotemporal system [32]. From the above point of view, we can see that the study of fuzzy synchronization of time delayed chaotic systems is of high practical importance. To the best of our knowledge, however, for the fuzzy synchronization problem of time delayed chaotic systems, there is no result in the literature so far, which still remains open and challenging. This situation motivates our present investigation.

Motivated by the above discussions, the aim of this paper is to investigate the fuzzy adaptive exponential synchronization problem for time delayed chaotic systems with unknown parameters. T-S fuzzy model is adopted for the modeling of time delayed chaotic drive and response systems. Based on this fuzzy model, a new fuzzy synchronization controller is designed and an analytic expression of the controller with its adaptive laws of parameters is shown. By the proposed scheme, the closed-loop error system is adaptively exponentially synchronized. By virtue of Lyapunov-Krasovskii stability theory, linear matrix inequality (LMI) approach, and Jesen’s inequality, an existence criterion for the proposed controller is represented in terms of the LMI, that can be readily checked by using some standard numerical packages [33].

This paper is organized as follows. In Section 2, we formulate the problem. In Section 3, a fuzzy adaptive exponential synchronization controller is proposed for time delayed chaotic systems with unknown parameters. In Section 4, an application example for time delayed Lorenz system is given, and finally, conclusions are presented in Section 5.

2. Problem Formulation

Consider a class of uncertain time delayed chaotic systems described by the following.

$$\begin{aligned}&\text{Fuzzy Rule } i: \\ &\text{IF } \omega_1 \text{ is } \vartheta_{i1} \text{ and} \cdots \omega_s \text{ is } \vartheta_{is} \text{ THEN} \\ &\dot{x}(t) = A_i x(t) + \overline{A}_i x(t-\tau) + \eta_i(t) + \sum_{k=1}^{p} \Phi_k(x(t))\theta_k + \sum_{l=1}^{q} \Psi_l(x(t-\tau))\phi_l,\end{aligned} \tag{2.1}$$

where $x(t) \in R^n$ is the state vector, $\tau > 0$ is the time-delay of the chaotic system (2.1), $A_i \in R^{n\times n}$ and $\overline{A}_i \in R^{n\times n}$ are known constant matrices, $\eta_i(t) \in R^n$ denotes a bias term which is generated by the fuzzy modeling procedure, $\Phi_k(x(t))$ $(k = 1,\ldots,p) : R^n \to R^{n\times\lambda}$ and $\Psi_l(x(t))$ $(l = 1,\ldots,q) : R^n \to R^{n\times\mu}$ are activation function matrices, $\theta_k \in R^\lambda$ $(k = 1,\ldots,p)$ and $\phi_l \in R^\mu$ $(l = 1,\ldots,q)$ represent the uncertain constant parameter vectors, ω_j $(j = 1,\ldots,s)$ is the premise variable, ϑ_{ij} $(i = 1,\ldots,r,\ \ j = 1,\ldots,s)$ is the fuzzy set that is characterized by membership function, r is the number of the IF-THEN rules, and s is the number of the premise variables.

Using a standard fuzzy inference method (using a singleton fuzzifier, product fuzzy inference, and weighted average defuzzifier), the system (2.1) is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\omega)\left[A_i x(t) + \overline{A}_i x(t-\tau) + \eta_i(t) + \sum_{k=1}^{p} \Phi_k(x(t))\theta_k + \sum_{l=1}^{q} \Psi_l(x(t-\tau))\phi_l\right], \tag{2.2}$$

where $\omega = [\omega_1,\ldots,\omega_s]$, $h_i(\omega) = \varpi_i(\omega)/\sum_{i=1}^{r}\varpi_j(\omega)$, $\varpi_i : R^s \to [0,1]$ $(i = 1,\ldots,r)$ is the membership function of the system with respect to the fuzzy rule i. h_i can be regarded as the normalized weight of each IF-THEN rule and it satisfies

$$h_i(\omega) \geq 0, \qquad \sum_{i=1}^{r} h_i(\omega) = 1. \tag{2.3}$$

The system (2.2) is considered as a drive system. The synchronization problem of system (2.2) is considered by using the drive-response configuration. According to the drive-response concept, the controlled fuzzy response system is described by the following rules.

$$\begin{aligned}&\text{Fuzzy Rule } i: \\ &\text{IF } \omega_1 \text{ is } \vartheta_{i1} \text{ and} \cdots \omega_s \text{ is } \vartheta_{is} \text{ THEN} \\ &\dot{\hat{x}}(t) = A_i \hat{x}(t) + \overline{A}_i \hat{x}(t-\tau) + \eta_i(t) + u(t),\end{aligned} \tag{2.4}$$

where $\hat{x}(t) \in R^n$ is the state vector of the response system and $u(t) \in R^n$ is the control input. The fuzzy response system can be inferred as

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(\omega)\left[A_i \hat{x}(t) + \overline{A}_i \hat{x}(t-\tau) + \eta_i(t) + u(t)\right]. \tag{2.5}$$

Define the synchronization error $e(t) = \hat{x}(t) - x(t)$. Then we obtain the synchronization error system

$$\dot{e}(t) = \sum_{i=1}^{r} h_i(\omega)\left[A_i e(t) + \overline{A}_i e(t-\tau) - \sum_{k=1}^{p}\Phi_k(x(t))\theta_k - \sum_{l=1}^{q}\Psi_l(x(t-\tau))\phi_l + u(t)\right]. \tag{2.6}$$

Throughout this paper, we define that $\hat{\theta}_k(t)$ $(k = 1,\ldots,p)$ and $\hat{\phi}_l(t)$ $(l = 1,\ldots,q)$ are the estimate values of θ_k and ϕ_l, respectively.

Definition 2.1 (Adaptive exponential synchronization). With nonzero initial conditions, the error system (2.6) is adaptively exponentially synchronized if the synchronization error $e(t)$ satisfies

$$\|e(t)\| < M\exp(-Nt), \tag{2.7}$$

where M and N are positive constants, under the control $u(t)$ with the adaptive laws $\hat{\theta}_k(t)$ and $\hat{\phi}_l(t)$ $(k = 1,\ldots,p,\ l = 1,\ldots,q)$.

The purpose of this paper is to design the controller $u(t)$ with the adaptive laws $\hat{\theta}_k(t)$ and $\hat{\phi}_l(t)$ $(k = 1,\ldots,p,\ l = 1,\ldots,q)$ guaranteeing the adaptive exponential synchronization for time delayed chaotic systems with unknown parameters.

3. An LMI-Based Fuzzy Adaptive Exponential Synchronization

In this section, we present the LMI problem for achieving the fuzzy adaptive exponential synchronization of time delayed chaotic systems with unknown parameters.

Theorem 3.1. *If there exist $P = P^T > 0$, $Q = Q^T > 0$, $R = R^T > 0$, $S = S^T > 0$, $W = W^T > 0$, and M_j such that*

$$\begin{bmatrix} A_i^T P + PA_i + M_j + M_j^T + \kappa P + \dfrac{\exp(\kappa\tau)-1}{\kappa}Q + R + S & P\overline{A}_i & W \\ \overline{A}_i^T P & -\exp(-\kappa\tau)R & -W \\ W & -W & \kappa W - \dfrac{1}{\tau}Q \end{bmatrix} < 0 \tag{3.1}$$

for $i, j = 1,2,\ldots,r$, where $\kappa > 0$ is an enough small real number properly selected, then the fuzzy adaptive exponential synchronization is achieved under the control

$$u(t) = \sum_{j=1}^{r} h_j(\omega)K_j(\hat{x}(t) - x(t)) - \sum_{k=1}^{p}\Phi_k(x(t))\hat{\theta}_k(t) - \sum_{l=1}^{q}\Psi_l(x(t-\tau))\hat{\phi}_l(t), \tag{3.2}$$

and the adaptive laws

$$\dot{\hat{\theta}}_k(t) = \Gamma \Phi_k^T(x(t)) P(\hat{x}(t) - x(t)) \exp(\kappa t), \quad (k = 1, \dots, p),$$
$$\dot{\hat{\phi}}_l(t) = \Upsilon \Psi_l^T(x(t-\tau)) P(\hat{x}(t) - x(t)) \exp(\kappa t), \quad (l = 1, \dots, q), \tag{3.3}$$

where Γ *and* Υ *are positive definite matrices for design.*

Proof. The fuzzy adaptive exponential synchronization controller can be constructed via the parallel distributed compensation. The controller is described by the following rules.

$$\begin{aligned} &\text{Fuzzy Rule } j: \\ &\text{IF } \omega_1 \text{ is } \vartheta_{j1} \text{ and} \cdots \omega_s \text{ is } \vartheta_{js} \text{ THEN} \\ &u(t) = K_j e(t) - \sum_{k=1}^{p} \Phi_k(x(t)) \hat{\theta}_k(t) - \sum_{l=1}^{q} \Psi_l(x(t-\tau)) \hat{\phi}_l(t), \end{aligned} \tag{3.4}$$

where $K_j \in R^{n \times m}$ is the gain matrix of the controller for the fuzzy rule j. The fuzzy controller can be inferred as

$$u(t) = \sum_{j=1}^{r} h_j(\omega) K_j e(t) - \sum_{k=1}^{p} \Phi_k(x(t)) \hat{\theta}_k(t) - \sum_{l=1}^{q} \Psi_l(x(t-\tau)) \hat{\phi}_l(t). \tag{3.5}$$

The closed-loop error system with the control input (3.5) can be written as

$$\dot{e}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\omega) h_j(\omega) \left[(A_i + K_j) e(t) + \overline{A}_i e(t-\tau) - \sum_{k=1}^{p} \Phi_k(x(t)) \tilde{\theta}_k(t) - \sum_{l=1}^{q} \Psi_l(x(t-\tau)) \tilde{\phi}_l(t) \right], \tag{3.6}$$

where $\tilde{\theta}_k(t) = \hat{\theta}_k(t) - \theta_k$ and $\tilde{\phi}_l(t) = \hat{\phi}_l(t) - \phi_l$. Consider the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V(t) = {} & \exp(\kappa t) e^T(t) P e(t) + \int_{-\tau}^{0} \exp(-\kappa \beta) \int_{t+\beta}^{t} \exp(\kappa \alpha) e^T(\alpha) Q e(\alpha) d\alpha \, d\beta \\ & + \int_{-\tau}^{0} \exp(\kappa(t+\sigma)) e^T(t+\sigma) \operatorname{Re}(t+\sigma) d\sigma \\ & + \exp(\kappa t) \left[\int_{-\tau}^{0} e(t+\sigma) d\sigma \right]^T W \left[\int_{-\tau}^{0} e(t+\sigma) d\sigma \right] + \sum_{k=1}^{p} \tilde{\theta}_k^T(t) \Gamma^{-1} \tilde{\theta}_k(t) \\ & + \sum_{l=1}^{q} \tilde{\phi}_l^T(t) \Upsilon^{-1} \tilde{\phi}_l(t). \end{aligned} \tag{3.7}$$

The time derivative of $V(t)$ along the trajectory of (3.6) is

$$\begin{aligned}
\dot{V}(t) &= \exp(\kappa t)\dot{e}(t)^T P e(t) + \exp(\kappa t) e^T(t) P \dot{e}(t) + \kappa \exp(\kappa t) e^T(t) P e(t) + \frac{\exp(\kappa\tau)-1}{\kappa} \\
&\quad \times \exp(\kappa t) e^T(t) Q e(t) - \exp(\kappa t)\int_{t-\tau}^{t} e^T(\sigma) Q e(\sigma) d\sigma + \exp(\kappa t) e(t)^T \operatorname{Re}(t) \\
&\quad - \exp(\kappa(t-\tau)) e^T(t-\tau) \operatorname{Re}(t-\tau) + \kappa \exp(\kappa t)\left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]^T W \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right] \\
&\quad + \exp(\kappa t)[e(t)-e(t-\tau)]^T W \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right] + \exp(\kappa t)\left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]^T \\
&\quad \times W[e(t)-e(t-\tau)] + 2\sum_{k=1}^{p}\tilde{\theta}_k^T(t)\Gamma^{-1}\dot{\hat{\theta}}_k(t) + 2\sum_{l=1}^{q}\tilde{\phi}_l^T(t)\Upsilon^{-1}\dot{\hat{\phi}}_l(t) \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\omega) h_j(\omega) \\
&\quad \times \Bigg\{ \exp(\kappa t) e^T(t)\left[A_i^T P + P A_i + P K_j + K_j^T P + \kappa P\right] e(t) \\
&\qquad + \exp(\kappa t) e^T(t) P \overline{A}_i e(t-\tau) + \exp(\kappa t) e^T(t-\tau) \overline{A}_i^T P e(t) \\
&\qquad - 2\exp(\kappa t)\sum_{k=1}^{p}\tilde{\theta}_k^T(t)\Phi_k^T(x(t)) P e(t) - 2\exp(\kappa t)\sum_{l=1}^{q}\tilde{\phi}_l^T(t)\Psi_l^T(x(t-\tau)) P e(t) \Bigg\} \\
&\qquad + \frac{\exp(\kappa\tau)-1}{\kappa}\exp(\kappa t) e^T(t) Q e(t) \\
&\quad - \exp(\kappa t)\int_{t-\tau}^{t} e^T(\sigma) Q e(\sigma) d\sigma + \exp(\kappa t) e(t)^T \operatorname{Re}(t) - \exp(\kappa(t-\tau)) e^T(t-\tau) \operatorname{Re}(t-\tau) \\
&\quad + \kappa \exp(\kappa t)\left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]^T W \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right] + \exp(\kappa t)[e(t)-e(t-\tau)]^T W \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right] \\
&\quad + \exp(\kappa t)\left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]^T W[e(t)-e(t-\tau)] + 2\sum_{k=1}^{p}\tilde{\theta}_k^T(t)\Gamma^{-1}\dot{\hat{\theta}}_k(t) + 2\sum_{l=1}^{q}\tilde{\phi}_l^T(t)\Upsilon^{-1}\dot{\hat{\phi}}_l(t).
\end{aligned} \tag{3.8}$$

Using the Jesen's inequality [34], we have

$$-\exp(\kappa t)\int_{t-\tau}^{t} e(\sigma)^T Q e(\sigma) d\sigma \le -\frac{\exp(\kappa t)}{\tau}\left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]^T Q \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]. \tag{3.9}$$

Finally, using (3.9), the time derivative of $V(t)$ can be obtained as

$$\begin{aligned}
\dot{V}(t) \leq{}& \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\omega)h_j(\omega)\exp(\kappa t) \\
&\times \Bigg\{ e^T(t)\left[A_i^T P + PA_i + PK_j + K_j^T P + \kappa P\right]e(t) \\
&+ \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]^T \left[\kappa W - \frac{1}{\tau}Q\right]\left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right] + e^T(t)P\overline{A}_i e(t-\tau) + e^T(t-\tau)\overline{A}_i^T Pe(t) \\
&+ \frac{\exp(\kappa\tau)-1}{\kappa}e^T(t)Qe(t) + e(t)^T \operatorname{Re}(t) - \exp(-\kappa\tau)e^T(t-\tau)\operatorname{Re}(t-\tau) \\
&+ [e(t)-e(t-\tau)]^T W\left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right] + \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]^T W[e(t)-e(t-\tau)] \Bigg\} \\
&+ 2\sum_{k=1}^{p}\tilde{\theta}_k^T(t)\Gamma^{-1}\left[\dot{\hat{\theta}}_k(t) - \Gamma\Phi_k^T(x(t))Pe(t)\exp(\kappa t)\right] \\
&+ 2\sum_{l=1}^{q}\tilde{\phi}_l^T(t)\Upsilon^{-1}\left[\dot{\hat{\phi}}_l(t) - \Upsilon\Psi_l^T(x(t-\tau)) \times Pe(t)\exp(\kappa t)\right] \\
={}& \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\omega)h_j(\omega)\exp(\kappa t) \\
&\times \left\{ \begin{bmatrix} e(t) \\ e(t-\tau) \\ \int_{t-\tau}^{t} e(\sigma)d\sigma \end{bmatrix}^T \begin{bmatrix} (1,1) & P\overline{A}_i & W \\ \overline{A}_i^T P & -\exp(-\kappa\tau)R & -W \\ W & -W & \kappa W - \frac{1}{\tau}Q \end{bmatrix} \times \begin{bmatrix} e(t) \\ e(t-\tau) \\ \int_{t-\tau}^{t} e(\sigma)d\sigma \end{bmatrix} - e^T(t)Se(t) \right\} \\
&+ 2\sum_{k=1}^{p}\tilde{\theta}_k^T(t)\Gamma^{-1}\left[\dot{\hat{\theta}}_k(t) - \Gamma\Phi_k^T(x(t))Pe(t)\exp(\kappa t)\right] \\
&+ 2\sum_{l=1}^{q}\tilde{\phi}_l^T(t)\Upsilon^{-1}\left[\dot{\hat{\phi}}_l(t) - \Upsilon\Psi_l^T(x(t-\tau))Pe(t)\exp(\kappa t)\right],
\end{aligned} \tag{3.10}$$

where

$$(1,1) = A_i^T P + PA_i + PK_j + K_j^T P + \kappa P + \frac{\exp(\kappa\tau)-1}{\kappa}Q + R + S. \tag{3.11}$$

If the adaptive laws (3.3) are used and the following matrix inequality is satisfied:

$$\begin{bmatrix} (1,1) & P\overline{A}_i & W \\ \overline{A}_i^T P & -\exp(-\kappa\tau)R & -W \\ W & -W & \kappa W - \dfrac{1}{\tau}Q \end{bmatrix} < 0, \tag{3.12}$$

for $i, j = 1, 2, \ldots, r$, then we have

$$\begin{aligned} \dot{V}(t) &< -\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\omega)h_j(\omega)\exp(\kappa t)e^T(t)Se(t) \\ &= -\exp(\kappa t)e^T(t)Se(t). \end{aligned} \tag{3.13}$$

That is, $\dot{V}(t) < 0$ for all $e(t) \neq 0$. Thus, it implies that $V(t) < V(0)$ for any $t \geq 0$. In addition, from (3.7), one has

$$\begin{aligned} &V(t) < V(0) \\ &\quad = e^T(0)Pe(0) + \int_{-\tau}^{0} \exp(-\kappa\beta) \int_{\beta}^{0} \exp(\kappa\alpha)e^T(\alpha)Qe(\alpha)d\alpha\, d\beta \\ &\qquad + \int_{-\tau}^{0} \exp(\kappa(\sigma))e^T(\sigma)\operatorname{Re}(\sigma)d\sigma + \left[\int_{-\tau}^{0} e(\sigma)d\sigma\right]^T W \left[\int_{-\tau}^{0} e(\sigma)d\sigma\right] + \sum_{k=1}^{p}\widetilde{\theta}_k^T(0)\Gamma^{-1}\widetilde{\theta}_k(0) \\ &\qquad + \sum_{l=1}^{q}\widetilde{\phi}_l^T(0)\Upsilon^{-1}\widetilde{\phi}_l(0). \end{aligned} \tag{3.14}$$

Also, we have

$$V(t) \geq \lambda_{\min}(P)\exp(\kappa t)\|e(t)\|^2, \tag{3.15}$$

where $\lambda_{\min}(P)$ is the minimum eigenvalue of the matrix P. It follows immediately from (3.14) and (3.15) that

$$\begin{aligned}
\|e(t)\| &< \frac{1}{\sqrt{\lambda_{\min}(P)\exp(\kappa t)}} \\
&\quad \times \Bigg\{ e^T(0)Pe(0) + \int_{-\tau}^{0} \exp(-\kappa\beta) \int_{\beta}^{0} \exp(\kappa\alpha) e^T(\alpha) Q e(\alpha) d\alpha\, d\beta \\
&\qquad + \int_{-\tau}^{0} \exp(\kappa(\sigma)) e^T(\sigma)\,\mathrm{Re}(\sigma) d\sigma + \left[\int_{-\tau}^{0} e(\sigma)d\sigma\right]^T W \left[\int_{-\tau}^{0} e(\sigma)d\sigma\right] \\
&\qquad + \sum_{l=1}^{q} \widetilde{\phi}_l^T(0)\Upsilon^{-1}\widetilde{\phi}_l(0) + \sum_{k=1}^{p} \widetilde{\theta}_k^T(0)\Gamma^{-1}\widetilde{\theta}_k(0) \Bigg\}^{1/2} \\
&= \frac{1}{\sqrt{\lambda_{\min}(P)}} \\
&\quad \times \Bigg\{ e^T(0)Pe(0) + \int_{-\tau}^{0} \exp(-\kappa\beta) \int_{\beta}^{0} \exp(\kappa\alpha) e^T(\alpha) Q e(\alpha) d\alpha\, d\beta \\
&\qquad + \int_{-\tau}^{0} \exp(\kappa(\sigma)) e^T(\sigma)\,\mathrm{Re}(\sigma) d\sigma + \left[\int_{-\tau}^{0} e(\sigma)d\sigma\right]^T W \left[\int_{-\tau}^{0} e(\sigma)d\sigma\right] \\
&\qquad + \sum_{l=1}^{q} \widetilde{\phi}_l^T(0)\Upsilon^{-1}\widetilde{\phi}_l(0) + \sum_{k=1}^{p} \widetilde{\theta}_k^T(0)\Gamma^{-1}\widetilde{\theta}_k(0) \Bigg\}^{1/2} \exp\left(-\frac{\kappa}{2}t\right).
\end{aligned} \tag{3.16}$$

If we let

$$\begin{aligned}
M &= \frac{1}{\sqrt{\lambda_{\min}(P)}} \\
&\quad \times \Bigg\{ e^T(0)Pe(0) + \int_{-\tau}^{0} \exp(-\kappa\beta) \int_{\beta}^{0} \exp(\kappa\alpha) e^T(\alpha) Q e(\alpha) d\alpha\, d\beta + \int_{-\tau}^{0} \exp(\kappa(\sigma)) \\
&\qquad \times e^T(\sigma)\,\mathrm{Re}(\sigma) d\sigma + \left[\int_{-\tau}^{0} e(\sigma)d\sigma\right]^T W \left[\int_{-\tau}^{0} e(\sigma)d\sigma\right] \\
&\qquad + \sum_{l=1}^{q} \widetilde{\phi}_l^T(0)\Upsilon^{-1}\widetilde{\phi}_l(0) + \sum_{k=1}^{p} \widetilde{\theta}_k^T(0)\Gamma^{-1} \times \widetilde{\theta}_k(0) \Bigg\}^{1/2} > 0, \\
N &= \frac{\kappa}{2} > 0,
\end{aligned} \tag{3.17}$$

we obtain (2.7). If we let $M_j = PK_j$, (3.12) is equivalently changed into the LMI (3.1), then the gain matrix of the control input $u(t)$ is given by $K_j = P^{-1}M_j$. This completes the proof. □

Remark 3.2. Various efficient convex optimization algorithms can be used to check whether the LMI (3.1) is feasible. In this paper, in order to solve the LMI, we utilize MATLAB LMI Control Toolbox [35], which implements state-of- the-art interior-point algorithms.

4. Numerical Example

Consider the following time delayed Lorenz system [36]:

$$\begin{aligned}
\dot{x}_1(t) &= -10x_1(t) + 10x_2\left(t - \frac{1}{6}\right), \\
\dot{x}_2(t) &= 28x_1(t) - x_2(t) - x_1(t)x_3(t), \\
\dot{x}_3(t) &= x_1(t)x_2(t) - \chi x_3\left(t - \frac{1}{6}\right).
\end{aligned} \tag{4.1}$$

The parameter χ is assumed unknown. By defining two fuzzy sets, we can obtain the following fuzzy drive system that exactly represents the nonlinear equation of the time delayed Lorenz system under the assumption that $x_1(t) \in [-d, d]$ with $d = 20$:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(\omega)\left[A_i x(t) + \overline{A}_i x\left(t - \frac{1}{6}\right) + \eta_i + \Psi_1\left(x\left(t - \frac{1}{6}\right)\right)\phi_1\right], \tag{4.2}$$

where

$$\begin{aligned}
&A_1 = \begin{bmatrix} -10 & 0 & 0 \\ 28 & -1 & -d \\ 0 & d & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -10 & 0 & 0 \\ 28 & -1 & d \\ 0 & -d & 0 \end{bmatrix}, \qquad \phi_1 = \chi, \\
&\overline{A}_1 = \overline{A}_2 = \begin{bmatrix} 0 & 10 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \eta_1 = \eta_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \Psi_1\left(x\left(t - \frac{1}{6}\right)\right) = \begin{bmatrix} 0 \\ 0 \\ -x_3\left(t - \frac{1}{6}\right) \end{bmatrix}.
\end{aligned} \tag{4.3}$$

The membership functions are

$$h_1(\omega) = \frac{1}{2}\left(1 + \frac{x_1(t)}{d}\right), \qquad h_2(\omega) = \frac{1}{2}\left(1 - \frac{x_1(t)}{d}\right). \tag{4.4}$$

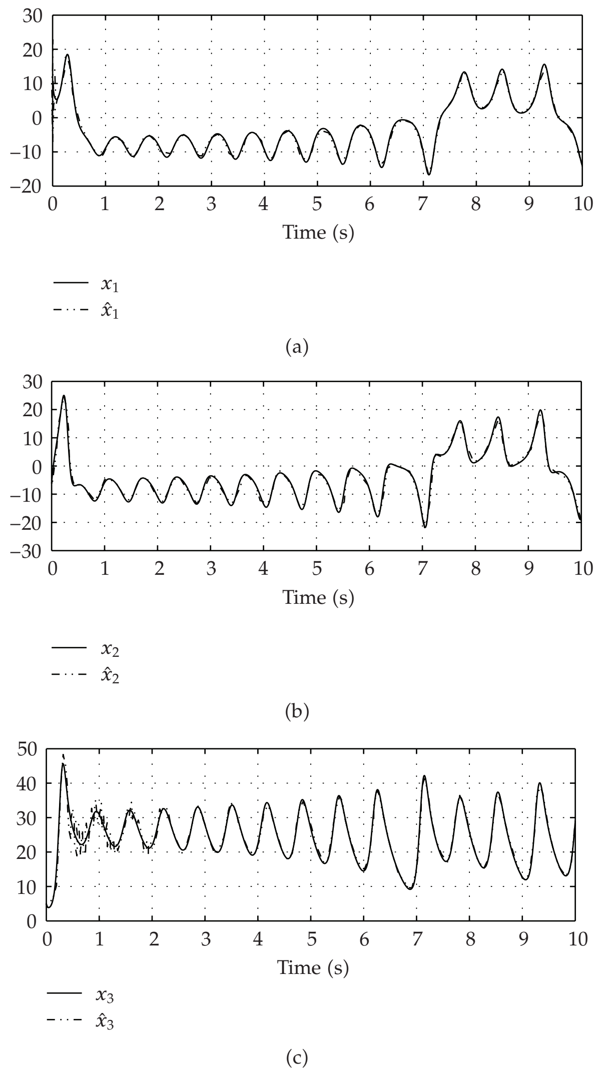
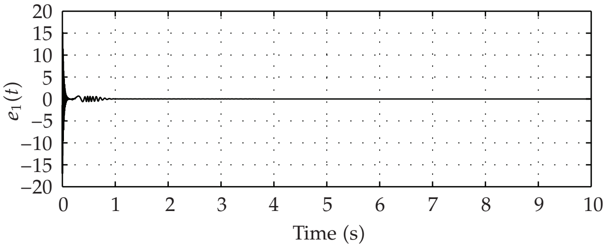


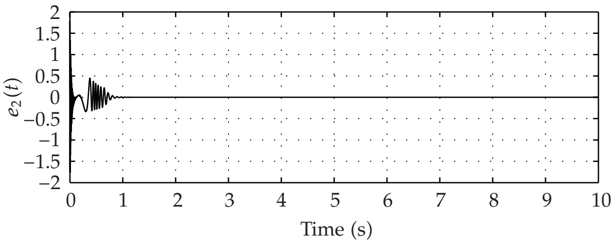
Figure 1: State trajectories.

For the numerical simulation, we use parameters $\kappa = 0.05$, $\phi_1 = 8/3$, and $\Upsilon = 10$. Applying Theorem 3.1 to the fuzzy system (4.2) yields

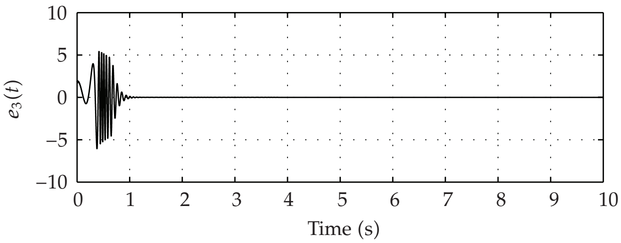
$$P = \begin{bmatrix} 0.0109 & 0.0009 & 0.0000 \\ 0.0009 & 1.0117 & 0.0000 \\ 0.0000 & 0.0000 & 1.0117 \end{bmatrix}, \qquad M_1 = \begin{bmatrix} -1.4994 & -112.5918 & -8.6076 \\ 84.2734 & -0.5964 & -0.3152 \\ 8.6076 & 0.3152 & -1.6045 \end{bmatrix},$$
$$M_2 = \begin{bmatrix} -1.4994 & -42.5072 & -0.4721 \\ 14.1889 & -0.5964 & 0.2439 \\ 0.4721 & -0.2439 & -1.6045 \end{bmatrix}. \tag{4.5}$$



(a)



(b)



(c)

Figure 2: Synchronization errors.

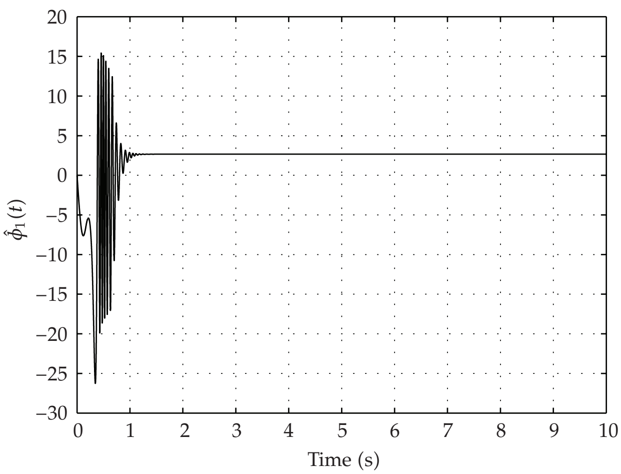


Figure 3: The estimate value $\widehat{\phi}_1(t)$ of parameter ϕ_1.

Figure 1 shows state trajectories when the initial conditions are given by $(x_1(0), x_2(0), x_3(0)) = (10, -6.2, 5.1)$, $(\hat{x}_1(0), \hat{x}_2(0), \hat{x}_3(0)) = (8.6, -4.41, 7)$, and $\hat{\phi}_1(0) = 0$. From Figure 1, it can be seen that drive and response systems are indeed achieving chaos synchronization. Figure 2 plots the time responses of synchronization errors. The estimate $\hat{\phi}_1(t)$ of the uncertain parameter ϕ_1 is illustrated at Figure 3, which shows that the estimate $\hat{\phi}_1(t)$ approaches rapidly to target value 8/3. Simulation results reveal that the response system controlled using the proposed synchronization method performs well. The effectiveness and accuracy of the proposed method is demonstrated.

5. Conclusion

In this paper, a new fuzzy adaptive exponential synchronization scheme, which consists of time delayed fuzzy drive and response systems, is proposed for time delayed chaotic systems with unknown parameters. Based on Lyapunov-Krasovskii stability theory and LMI formulation, the proposed scheme can guarantee the adaptive exponential synchronization. The synchronization problem for the time delayed Lorenz system is given to illustrate the effectiveness of the proposed scheme. Finally, the proposed synchronization method has the advantage that it can be effectively used to adaptive exponential control and synchronization of other uncertain time delayed nonlinear systems described by a T-S fuzzy model.

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