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Von Neumann–Jordan Constant for Lebesgue–Bochner Spaces*

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The von Neumann–Jordan (NJ-) constant for Lebesgue–Bochner spaces $L_p(X)$ is determined under some conditions on a Banach space X. In particular the NJ-constant for $L_r(c_p)$ as well as c_p (the space of p-Schatten class operators) is determined. For a general Banach space X we estimate the NJ-constant of $L_p(X)$, which may be regarded as a sharpened result of a previous one concerning the uniform non-squareness for $L_p(X)$. Similar estimates are given for Banach sequence spaces $l_p(X_i)$ (l_p-sum of Banach spaces X_i), which gives a condition by NJ-constants of X_i's under which $l_p(X_i)$ is uniformly non-square. A bi-product concerning 'Clarkson's inequality' for $L_p(X)$ and $l_p(X_i)$ is also given.

Keywords: Von Neumann–Jordan constant; Lebesgue–Bochner space; l_p-sum of Banach spaces; Uniform non-squareness; Clarkson's inequality; Interpolation

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1 INTRODUCTION AND PRELIMINARIES

Let X be a Banach space. The *von Neumann–Jordan* (NJ-) *constant* for X (Clarkson [4]), we denote it by $C_{\mathrm{NJ}}(X)$, is the smallest constant C for which

$$\frac{1}{C} \leqq \frac{\| x+y \|^2 + \| x-y \|^2}{2\,(\| x \|^2 + \| y \|^2)} \leqq C \tag{1}$$

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holds for all x and y in X with $(x,y)\neq(0,0)$. A classical result of Jordan and von Neumann [8] implies that $1\leqq C_{\mathrm{NJ}}(X)\leqq 2$ for any Banach space X; and X is a Hilbert space if and only if $C_{\mathrm{NJ}}(X)=1$. Clarkson [4] showed that $C_{\mathrm{NJ}}(L_p)=2^{2/\min\{p,p'\}-1}$, $1/p+1/p'=1$. Recently Kato and Miyazaki [10,9] determined the NJ-constant for $L_p(L_q)$ (L_q-valued L_p-space), Sobolev spaces $W_p^k(\Omega)$ [10], and for $C_c(K)$ (the space of continuous functions with compact support on a locally compact Hausdorff space K; [9]). On the other hand, the authors [11,19] gave a sequence of new results about the NJ-constant. In particular they showed that: (i) X is super-reflexive if and only if X admits an equivalent norm with NJ-constant less than 2 [11]; this was refined as (ii) X is uniformly non-square if and only if $C_{\mathrm{NJ}}(X)<2$ [19].

In this note we first state Clarkson's procedure to obtain the NJ-constant of L_p [4] in a generalized setting, and then we determine the NJ-constant for Lebesgue–Bochner spaces $L_p(X)$ under some conditions on a Banach space X. As corollaries the NJ-constant for $L_r(c_p)$ as well as c_p (the space of p-Schatten class operators) is determined, and the results in [4,9,10] stated above are also obtained. Next, we estimate C_{NJ} $(L_p(X))$ for a general Banach space X, which is best possible in several cases. Previous results on uniform non-squareness (Smith and Turett [17]) and super-reflexivity (Pisier [15]) for $L_p(X)$ are obtained as immediate consequences. Similar estimates are also given for Banach sequence spaces $l_p(X_i)$ (l_p-sum of Banach spaces X_i), which implies in particular that $l_p(X_i)$ is uniformly non-square if and only if $\sup C_{\mathrm{NJ}}$ $(X_i)<2$. As a bi-product it is derived that 'Clarkson's inequality' holds in $L_p(X)$, resp. in $l_p(X_i)$ if and only if it holds in X, resp. in each X_i (for the former, see Kato and Takahashi [12]).

Let X be a Banach space and let $1\leqq p\leqq\infty$. Let $L_p(X)$ be the Lebesgue–Bochner space on an arbitrary measure space (S,μ), that is, the space of all (equivalence classes of) X-valued μ-measurable functions f on S such that $\|f\|_{L_p(X)}:=\{\int_S\|f(\cdot)\|_X^p\,\mathrm{d}\mu\}^{1/p}$ (resp. $\operatorname{ess\,sup}_S\|f(\cdot)\|_X$) for $1\leqq p<\infty$ (resp. $p=\infty$) is finite. For $X=\boldsymbol{K}$ (reals or complexes) $L_p(\boldsymbol{K})$ is denoted by L_p as usual. The Banach sequence space $l_p(X_i)$ is the l_p-sum of Banach spaces X_i's, that is, the space of all sequences $x=\{x_i\}$ with $x_i\in X_i$ and $\|x\|_p:=\left\{\sum_{i=1}^{\infty}\|x_i\|^p\right\}^{1/p}<\infty$ (cf. e.g., [16]).

A Banach space X is called *$(2,\varepsilon)$-convex*, $\varepsilon>0$, provided $\min\{\|x+y\|,\|x-y\|\}\leqq 2(1-\varepsilon)$ whenever $\|x\|\leqq 1$, $\|y\|\leqq 1$ (cf. [20,5]).

X is called *uniformly non-square* if it is $(2, \varepsilon)$-convex for some $\varepsilon > 0$ ([6]; cf. [1]). X is said to be *super-reflexive* ([7]; cf. [1,20]) if any Banach space which is finitely representable in X is reflexive (a Banach space Y is said to be finitely representable in X when any finite-dimensional subspace of Y can be found in X, with an approximation as good as one wants). It is well known that uniformly convex spaces are uniformly non-square, and uniformly non-square spaces are super-reflexive; super-reflexive spaces are just those uniformly convexifiable (cf. [1,7,20]).

Let

$$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Let $l_r^2(X)$, $1 \leqq r \leqq \infty$, denote the X-valued l_r^2-space. In the following, $p', q', r', \ldots$ denote the conjugate numbers of $p, q, r, \ldots$

2 VON NEUMANN – JORDAN CONSTANT FOR $L_p(X)$

We start with the following lemma.

LEMMA 1 *Let* $1 \leqq t \leqq 2$.

(i) $C_{\mathrm{NJ}}(X) = 2^{2/t-1}$ *if and only if*

$$\|A : l_t^2(X) \to l_2^2(X)\| = 2^{1/t}; \tag{2}$$

and hence $C_{\mathrm{NJ}}(X') = C_{\mathrm{NJ}}(X)$ (X' *is the dual space of* X).

(ii) *If* X *contains a nearly isometric copy of* l_t^2 *or* $l_{t'}^2$ (*in particular if* l_t *or* $l_{t'}$ *is finitely representable in* X), *then* $C_{\mathrm{NJ}}(X) \geqq 2^{2/t-1}$.

Proof (i) is readily seen by noting that the first and second inequalities in (1) are equivalent; put $x + y = u$, $x - y = v$.

(ii) Assume that for any $\lambda > 1$ there exists a two-dimensional subspace X_0 of X and an isomorphism T from l_t^2 onto X_0 such that

$$\lambda^{-1}\|x\| \leqq \|Tx\| \leqq \lambda\|x\| \quad \text{for all } x \in l_t^2.$$

Then for any x, y in l_t^2

$$\frac{\|x+y\|^2+\|x-y\|^2}{2(\|x\|^2+\|y\|^2)} \leqq \lambda^4 C_{\mathrm{NJ}}(X),$$

whence $C_{\mathrm{NJ}}(l_t^2) \leqq \lambda^4 C_{\mathrm{NJ}}(X)$. Letting $\lambda \to 1$, we have the conclusion because $C_{\mathrm{NJ}}(l_t^2) = C_{\mathrm{NJ}}(l_{t'}^2) = 2^{2/t-1}$ ([4]; see also [10]).

Clarkson's procedure to determine the NJ-constant for L_p [4] is stated in a generalized setting as follows.

PROPOSITION 2 *Let* $1 \leqq t \leqq 2$ *and let* $1/t + 1/t' = 1$. *Assume that the* (t, t') *Clarkson inequality*

$$(\|x+y\|^{t'} + \|x-y\|^{t'})^{1/t'} \leqq 2^{1/t'}(\|x\|^t + \|y\|^t)^{1/t} \tag{3}$$

holds in X, and X contains a nearly isometric copy of l_t^2 *or* $l_{t'}^2$. *Then* $C_{\mathrm{NJ}}(X) = 2^{2/t-1}$.

Proof By (3) we have

$$\begin{aligned}
&\|A : l_2^2(X) \to l_2^2(X)\| \\
&\leqq \|I : l_2^2(X) \to l_t^2(X)\| \; \|A : l_t^2(X) \to l_{t'}^2(X)\| \; \|I : l_{t'}^2(X) \to l_2^2(X)\| \\
&\leqq 2^{1/t-1/2} 2^{1/t'} 2^{1/2-1/t'} = 2^{1/t},
\end{aligned} \tag{4}$$

where I's are identity operators. This implies $C_{\mathrm{NJ}}(X) \leqq 2^{2/t-1}$. The opposite inequality follows from Lemma 1 (ii).

Remark 3 In any Banach space *some* (t, t') Clarkson inequality, $1 \leqq t \leqq 2$, holds. Indeed, as is easily seen, $(1, \infty)$ Clarkson inequality is valid in any Banach space; and if $1 \leqq s < t \leqq 2$, (t, t') Clarkson inequality implies (s, s') inequality [18]. For some examples of Banach spaces in which (t, t') Clarkson inequality holds with $t > 1$ we refer the reader to [14].

By Proposition 2 we immediately obtain the NJ-constant for c_p as well as some previous results.

COROLLARY 4 (i) *Let* $1 \leqq p \leqq \infty$. *Let* $t = \min\{p, p'\}$. *Then for* $X = L_p$ (Clarkson [4]), $W_p^k(\Omega)$ (*Kato and Miyazaki* [10]), *and* c_p, $C_{\mathrm{NJ}}(X) = 2^{2/t-1}$.

(ii) $C_{\mathrm{NJ}}(C_c(K)) = 2$ (*Kato and Miyazaki* [9]).

Indeed, in c_p, the (t, t') Clarkson inequality holds and l_p is isometrically imbedded into c_p (McCarthy [13]; cf. [14]).

Lemma 5 (Takahashi and Kato [18; Theorem 2.3]) *Let* $1 \leqq p \leqq \infty$ *and let* $1 \leqq t \leqq 2$. *Assume that the* (t, t') *Clarkson inequality* (3) *holds in* X. *Then* (s, s') *Clarkson inequality holds in* $L_p(X)$, *where* $s = \min\{t, p, p'\}$.

Theorem 6 *Let* $1 \leqq p \leqq \infty$ *and* $1 \leqq t \leqq 2$. *Assume that the* (t, t') *Clarkson inequality* (3) *holds in* X.

(i) *If* $1 \leqq p \leqq t$ *or* $t' \leqq p \leqq \infty$, *then* $C_{\mathrm{NJ}}(L_p(X)) = 2^{2/r-1}$, *where* $r = \min\{p, p'\}$.
(ii) *If* $t \leqq p \leqq t'$, *and if* X *contains a nearly isometric copy of* l_t^2 *or* $l_{t'}^2$, *then* $C_{\mathrm{NJ}}(L_p(X)) = 2^{2/t-1}$.

Proof (i) By Lemma 5 (r, r') Clarkson inequality holds in $L_p(X)$. Since l_p^2 is isometrically imbedded into $L_p(X)$, we have $C_{\mathrm{NJ}}(L_p(X)) = 2^{2/r-1}$ by Proposition 2.

(ii) In this case (t, t') Clarkson inequality holds in $L_p(X)$ by Lemma 5. Since X, and *a fortiori* $L_p(X)$, is supposed to contain a nearly isometric copy of l_t^2 or $l_{t'}^2$, we have the conclusion.

By Theorem 6 we obtain the following.

Corollary 7 *Let* $1 \leqq p, q \leqq \infty$. *Let* $t = \min\{p, q, p', q'\}$. *Then*

(i) $C_{\mathrm{NJ}}(L_p(c_q)) = 2^{2/t-1}$,
(ii) $C_{\mathrm{NJ}}(L_p(L_q)) = 2^{2/t-1}$ (*Kato and Miyazaki* [10]).

Next we estimate the NJ-constant of $L_p(X)$ with a general X (and also that of $l_p(X_i)$).

Lemma 8 *Let* $1 \leqq p \leqq 2$ *and let* $1/p + 1/p' = 1$. *Then for any Banach space* X

(i) $$\|A : l_p^2(L_p(X)) \to l_{p'}^2(L_p(X))\| = \|A : l_p^2(X) \to l_{p'}^2(X)\|,$$
(ii) $$\|A : l_p^2(l_p(X_i)) \to l_{p'}^2(l_p(X_i))\| = \sup_i \|A : l_p^2(X_i) \to l_{p'}^2(X_i)\|.$$

Proof (i) Let us see the inequality '$\leqq$' (the converse inequality is trivial). For any f and g in $L_p(X)$ we have

$$\|f+g\|^{p'}_{L_p(X)} + \|f-g\|^{p'}_{L_p(X)}$$

$$= \left\{\int \|f(t)+g(t)\|^p \,\mathrm{d}\mu(t)\right\}^{p'/p} + \left\{\int \|f(t)-g(t)\|^p \,\mathrm{d}\mu(t)\right\}^{p'/p}$$

$$\leqq \left\{\int (\|f(t)+g(t)\|^{p'} + \|f(t)-g(t)\|^{p'})^{p/p'} \,\mathrm{d}\mu(t)\right\}^{p'/p}$$

(by Minkowski's inequality for $p/p' \leqq 1$)

$$\leqq \|A : l^2_p(X) \to l^2_{p'}(X)\|^{p'} \left\{\int (\|f(t)\|^p + \|g(t)\|^p) \,\mathrm{d}\mu(t)\right\}^{p'/p}$$

$$= \|A : l^2_p(X) \to l^2_{p'}(X)\|^{p'} \left(\|f\|^p_{L_p(X)} + \|g\|^p_{L_p(X)}\right)^{p'/p},$$

which gives the conclusion. The proof of (ii) goes in the same way.

THEOREM 9 *Let* $1 \leqq p \leqq \infty$, *and let* $t = \min\{p, p'\}$. *Then*

$$\max\{C_{\mathrm{NJ}}(L_p), C_{\mathrm{NJ}}(X)\} \leqq C_{\mathrm{NJ}}(L_p(X)) \leqq C_{\mathrm{NJ}}(L_p) \cdot C_{\mathrm{NJ}}(X)^{2/t'}, \quad (5)$$

where $1/p + 1/p' = 1/t + 1/t' = 1$.

Here one should note that $C_{\mathrm{NJ}}(L_p) = 2^{2/t-1}$, and hence the third term in (5) is not bigger than 2.

Proof The left-hand inequality of (5) is trivial since L_p and X are isometrically imbedded into $L_p(X)$. We prove the right-hand inequality of (5). Let $1 \leqq p \leqq 2$. Let $C_{\mathrm{NJ}}(X) = 2^{2/r-1}$, $1 \leqq r \leqq 2$. Then by Lemma 1

$$\|A : l^2_2(X) \to l^2_2(X)\| = 2^{1/r}. \quad (2)$$

On the other hand, we obviously have

$$\|A : l^2_1(X) \to l^2_\infty(X)\| = 1. \quad (6)$$

Put $\theta = 2/p'$ $(0 < \theta < 1)$. Then by interpolation (cf. [2], esp. Theorems 5.1.2, 4.2.1 and 4.1.2) with (6) and (2), we have

$$\|A : l^2_p(X) \to l^2_{p'}(X)\| \leqq 1^{1-\theta} 2^{\theta/r} = 2^{2/p'r},$$

from which it follows that

$$\|A : l_p^2(L_p(X)) \to l_{p'}^2(L_p(X))\| \leqq 2^{2/p'r}$$

by Lemma 8 (i). Therefore, in the same way as (4), we obtain

$$\|A : l_2^2(L_p(X)) \to l_2^2(L_p(X))\| \leqq 2^{1/p-1/p'+2/p'r}.$$

Put here $1/s = 1/p - 1/p' + 2/p'r$ (note that $1 \leqq s \leqq p \leqq 2$). Then we have $C_{\mathrm{NJ}}(L_p(X)) \leqq 2^{2/s-1} = 2^{2/p-1+2(2/r-1)/p'}$ by Lemma 1, which implies the right-hand inequality of (5). Let next $2 < p < \infty$ and let $C_{\mathrm{NJ}}(X) < 2$ (the right-hand inequality of (5) is trivial if $p = \infty$ or $C_{\mathrm{NJ}}(X) = 2$). Then X is reflexive by Theorem 6 in [11] (or Theorem 8 in [19]) and hence X' has the Radon–Nikodym property; therefore $L_p(X)' = L_{p'}(X')$. Consequently we obtain the conclusion by Lemma 1 and the preceding case.

Remark 10 Both inequalities of (5) in Theorem 9 are reduced to equality in the following cases; that is, we have:

(i) If $C_{\mathrm{NJ}}(X) = 1$, then $C_{\mathrm{NJ}}(L_p(X)) = C_{\mathrm{NJ}}(L_p)$.
(ii) If $C_{\mathrm{NJ}}(X) = 2$, then $C_{\mathrm{NJ}}(L_p(X)) = C_{\mathrm{NJ}}(X)$.
(iii) If $p = 2$, then $C_{\mathrm{NJ}}(L_2(X)) = C_{\mathrm{NJ}}(X)$ for all X.

Recall here the authors' results in [19,11] which state that X is uniformly non-square if and only if $C_{\mathrm{NJ}}(X) < 2$ [19]; and X is super-reflexive if and only if X admits an equivalent norm with NJ-constant less than 2 [11]. Now, Theorem 9 implies that for $1 < p < \infty$, $C_{\mathrm{NJ}}(L_p(X)) < 2$ if and only if $C_{\mathrm{NJ}}(X) < 2$. Therefore we immediately obtain the following well-known facts:

COROLLARY 11 *Let* $1 < p < \infty$.

(i) $L_p(X)$ *is uniformly non-square if and only if* X *is* (*Smith and Turett* [17]).
(ii) $L_p(X)$ *is super-reflexive if and only if* X *is* (*Pisier* [15]).

Similar estimates as (5) in Theorem 9 are valid for $l_p(X_i)$.

THEOREM 12 *Let* $1 \leqq p \leqq \infty$ *and let* $t = \min\{p, p'\}$. *Then*

$$\max\{C_{\mathrm{NJ}}(l_p), \sup_i C_{\mathrm{NJ}}(X_i)\} \leqq C_{\mathrm{NJ}}(l_p(X_i))$$
$$\leqq C_{\mathrm{NJ}}(l_p) \cdot \sup_i C_{\mathrm{NJ}}(X_i)^{2/t'}. \qquad (7)$$

The proof goes in the same way as that of Theorem 9 by using Lemma 8 (ii).

Remark 13 In inequalities (7), equality is simultaneously attained in the cases where (i) $\sup C_{\mathrm{NJ}}(X_i) = 1$ or 2, and (ii) $p = 2$.

Now, uniform non-squareness dose not lift to $l_p(X_i)$ from X_i's in general (see [16], esp. p. 152). Giesy [5; Corollary 18] gave the following condition under which this is the case: If X_i is $(2, \varepsilon_i)$-convex and if $\inf \varepsilon_i > 0$, then $l_p(X_i)$ is uniformly non-square. Our next result might provide a far simple condition which assures the uniform non-squareness of $l_p(X_i)$. By Theorem 12, combined with the authors' result in [19] stated above, we obtain:

COROLLARY 14 *Let* $1 < p < \infty$. *Then* $l_p(X_i)$ *is uniformly non-square if and only if* $\sup C_{\mathrm{NJ}}(X_i) < 2$.

Finally we see that Lemma 8 yields a bi-product concerning the (t, t') Clarkson inequality $(1 \leqq t \leqq 2)$

$$(\|x + y\|^{t'} + \|x - y\|^{t'})^{1/t'} \leqq 2^{1/t'}(\|x\|^t + \|y\|^t)^{1/t}. \qquad (3)$$

Since equality is always attained in (3) (put $y = 0$), the inequality (3) is represented as

$$\|A : l_t^2(X) \to l_{t'}^2(X)\| = 2^{1/t'}.$$

Therefore (3) holds in X if and only if it does in the dual space X' ([11, Theorem 3]). Lemma 8 and these observations lead us to the following theorem.

THEOREM 15 *Let* $1 \leqq p \leqq \infty$ *and* $t = \min\{p, p'\}$. *Then*:

(i) (t, t') *Clarkson inequality holds in* $L_p(X)$ *if and only if it holds in* X ([12, *Theorem* 4] *for the case* $1 \leqq p \leqq 2$; *cf. Lemma* 5).
(ii) (t, t') *Clarkson inequality holds in* $l_p(X_i)$ *if and only if it holds in each* X_i.

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