MAGNETOHYDRODYNAMIC FLOW DUE TO NONCOAXIAL ROTATIONS OF A POROUS DISK AND A FOURTH-GRADE FLUID AT INFINITY

TASAWAR HAYAT AND YONGQI WANG

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The governing equations for the unsteady flow of a uniformly conducting incompressible fourth-grade fluid due to noncoaxial rotations of a porous disk and the fluid at infinity are constructed. The steady flow of the fourth-grade fluid subjected to a magnetic field with suction/blowing through the disk is studied. The nonlinear ordinary differential equations resulting from the balance of momentum and mass are discretised by a finite-difference method and numerically solved by means of an iteration method in which, by a coordinate transformation, the semi-infinite physical domain is converted to a finite calculation domain. In order to solve the fourth-order nonlinear differential equations, asymptotic boundary conditions at infinity are augmented. The manner in which various material parameters affect the structure of the boundary layer is delineated. It is found that the suction through the disk and the magnetic field tend to thin the boundary layer near the disk for both the Newtonian fluid and the fourth-grade fluid, while the blowing causes a thickening of the boundary layer with the exception of the fourth-grade fluid under strong blowing. With the increase of the higher-order viscosities, the boundary layer has the tendency of thickening.

1. Introduction

The formulation of shear stress for non-Newtonian fluids is a difficult problem, which has not progressed very far from a theoretical standpoint. However, there is no single model which clearly exhibits all the properties of non-Newtonian fluids. For a more fundamental understanding, several empirical descriptions have established rheological models. For example, in most of these models, a significant reduction of the drag past solid walls has been observed. Moreover, elastic properties of real fluids are also present. A discussion of the various differential, rate-type, and integral models can be found in Schowalter [20], Huilgol [9], and Rajagopal [16].

In recent years, the fluids of differential type [22] have received special attention under a wide range of geometrical, dynamical, and rheological conditions. Some experiments by Barnes et al. [2] confirmed that an increase in the flow rate is possible and that the

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phenomenon appears to be governed by the shear-dependent viscosity. In fact, in [23] Walters and Townsend showed that the mean flow rate is unaffected by second-order viscoelasticity.

Although the second-grade model for steady flows is used to predict the normal stress differences, it does not correspond to shear thinning or thickening if the shear viscosity is assumed to be constant. For this reason, some experiments may be well described by the fluids of grade three or four [3, 10]. The third-grade model exhibits shear-dependent viscosity. Related studies of third-grade fluid are in [1, 4, 14, 18]. The model represented in this paper is the fourth-grade fluid. A similar model has been used by Kaloni and Siddiqui [13] to discuss the steady flow of a fourth-grade fluid between two infinite parallel plates rotating with the same angular velocity about two noncoincident axes. By expanding the variables in ascending powers of a suitable parameter, Kaloni and Siddiqui found the correction to the second-grade fluid solution. After making some observations about the complex shear modulus, they focused the attention on determining the forces on one of the plates. Numerical calculations are carried out for a third-grade fluid in which the constitutive coefficients are selected by assuming the relationship between third-grade fluid and the rigid dumbbell molecular model. Finally, they noted some differences between the result of this variable viscosity model and the second-grade fluid.

In the past years, several simple flow problems of classical hydrodynamics have received new attention in the more general context of magnetohydrodynamics (MHD). The study of the motion of non-Newtonian fluids in the absence as well as in the presence of a magnetic field has applications in many areas. A few examples are the flow of nuclear fuel slurries, flow of liquid metals and alloys such as the flow of gallium at ordinary temperatures (30°C), flow of plasma, flow of mercury amalgams, handling of biological fluids, flow of blood—a Bingham fluid with some thixotropic behaviour, coating of paper, plastic extrusion, and lubrication with heavy oils and greases.

Another important field of application is the electromagnetic propulsion. Basically, an electromagnetic propulsion system consists of a power source, such as a nuclear reactor, a plasma, and a tube through which the plasma is accelerated by electromagnetic forces. The study of such systems, which is closely associated with magnetochemistry, requires a complete understanding of the equation of state and transport properties such as diffusion, shear stress-shear rate relationship, thermal conductivity, electrical conductivity, and radiation. Some of these properties will undoubtedly be influenced by the presence of an external magnetic field which sets the plasma in hydrodynamic motion.

The aim of the present paper is to venture further in the regime of fourth-grade fluids. The available literature, to the best of our knowledge, are papers by Kaloni and Siddiqui [13] and Hayat et al. [8]. Thus, it seems that equations of unsteady fourth-grade fluid in the more general context of MHD and rotation have remained untouched. The present analysis models the nonlinear partial differential equations of fourth-grade fluid due to noncoaxial rotations of a porous disk and a fluid at infinity under the influence of a magnetic field perpendicular to the direction of motion. Throughout this study, the magnetic Reynolds number is assumed to be sufficiently small. The presented differential equations are in the more generalised form. Thus, the modelled partial differential equations are a significant contribution to understand the behaviour of fourth-grade fluids both from the physical and mathematical standpoints. Finally, the steady magnetohydrodynamic flow of a fourth-grade fluid due to noncoaxial rotations of a porous disk and a fluid at infinity is numerically solved.

2. Governing equations

We introduce a Cartesian coordinate system with the z-axis normal to the porous disk and the plane of the disk is $z = 0$. The axes of rotation, of both the disk and the fluid at infinity, are assumed to be in the plane $x = 0$, with the distance l between the axes. The common angular velocity of the disk and the fluid at infinity is taken as Ω. The fluid is electrically conducting and assumed to be permeated by an imposed magnetic field B_0 having no components in the x and y directions. Following Erdogan [5], we assume the velocity field of the form

$$\begin{aligned} u &= -\Omega y + f(z,t), \\ v &= \Omega x + g(z,t), \end{aligned} \tag{2.1}$$

where u and v are the x- and y-components of the velocity, respectively.

If the fluid is assumed to be homogeneous and incompressible, the continuity equation is expressed by the divergence-free condition

$$\operatorname{div} \mathbf{V} = 0, \tag{2.2}$$

where $\mathbf{V}$ is the velocity. On substituting (2.1) into (2.2), we obtain $\partial w/\partial z = 0$. Following Kaloni [11], we take $w = -W_0$. Obviously, $W_0 > 0$ is the suction velocity and $W_0 < 0$ corresponds to the blowing velocity normal to the disk. The velocity field can be considered as the summation of a helical motion $(-\Omega y, \Omega x, 0)$ and a translational motion $(f(z,t), g(z,t), -W_0)$.

The equations of motion governing the MHD flow are

$$\frac{D\mathbf{V}}{Dt} = \frac{1}{\rho}\operatorname{div}\mathbf{T} + \frac{1}{\rho}\mathbf{J}\times\mathbf{B}, \tag{2.3}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2.4}$$

$$\nabla \times \mathbf{B} = \mu_m \mathbf{J}, \tag{2.5}$$

$$\nabla \times \mathbf{E} = 0, \tag{2.6}$$

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V}\times\mathbf{B}). \tag{2.7}$$

In the above equations, D/Dt is the material time derivative, ρ the mass density, $\mathbf{T}$ the Cauchy stress tensor, $\mathbf{J}$ the current density, ∇ the gradient operator, μ_m the magnetic permeability, $\mathbf{E}$ the electric field, $\mathbf{B}$ the total magnetic field so that $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, $\mathbf{b}$ the induced magnetic field, and σ the electrical conductivity of the fluid. In addition, $\nabla \cdot \mathbf{J} = 0$ is acquired by using (2.5).

We make the following assumptions:

(i) the quantities ρ, μ_m, and σ are all constants throughout the flow field;
(ii) the magnetic field $\mathbf{B}$ is perpendicular to the velocity field $\mathbf{V}$ and the induced magnetic field $\mathbf{b}$ is negligible compared with the imposed field so that the magnetic Reynolds number is small [21];
(iii) the electric field $\mathbf{E}$ is assumed to be zero.

In view of these assumptions, the electromagnetic body force involved in (2.3) takes the linearised form (see [19])

$$\frac{1}{\rho}\mathbf{J}\times\mathbf{B} = \frac{\sigma}{\rho}\left[(\mathbf{V}\times\mathbf{B}_0)\times\mathbf{B}_0\right] = -n^*\mathbf{V}, \tag{2.8}$$

where $n^* = \sigma B_0^2/\rho$ has the same dimension as Ω and it plays an important role in the hydromagnetic analysis.

For the problem under consideration, the Cauchy stress of an incompressible homogeneous fourth-grade fluid is related to the fluid motion in the following manner:

$$\begin{aligned}\mathbf{T} = {}& -p\mathbf{I} + (\mu - \beta_1\Omega^2)\mathbf{A}_1 + (\alpha_1 - \gamma_1\Omega^2)\mathbf{A}_2 + (\alpha_2 - 2\gamma_2\Omega^2)\mathbf{A}_1^2 \\ & + \beta_2(\mathbf{A}_1\mathbf{A}_2 + \mathbf{A}_2\mathbf{A}_1) + \beta_3(\operatorname{tr}\mathbf{A}_1^2)\mathbf{A}_1 + \gamma_3\mathbf{A}_2^2 + \gamma_4(\mathbf{A}_2\mathbf{A}_1^2 + \mathbf{A}_1^2\mathbf{A}_2) \\ & + \gamma_5(\operatorname{tr}\mathbf{A}_2)\mathbf{A}_2 + \gamma_6(\operatorname{tr}\mathbf{A}_2)\mathbf{A}_1^2,\end{aligned} \tag{2.9}$$

where $-p\mathbf{I}$ is the spherical part of the stress due to the constraint of incompressibility, μ, α_i $(i = 1,2)$, β_i $(i = 1,2,3)$, γ_i $(i = 1,\dots,6)$ are all material constants, and $\mathbf{A}_1$ and $\mathbf{A}_2$ are the kinematical tensors defined by

$$\begin{aligned}\mathbf{A}_1 &= (\operatorname{grad}\mathbf{V}) + (\operatorname{grad}\mathbf{V})^T, \\ \mathbf{A}_2 &= \frac{D\mathbf{A}_1}{Dt} + \mathbf{A}_1(\operatorname{grad}\mathbf{V}) + (\operatorname{grad}\mathbf{V})^T\mathbf{A}_1.\end{aligned} \tag{2.10}$$

It should be noted that when $\alpha_i = 0$ $(i = 1,2)$, $\beta_i = 0$ $(i = 1,2,3)$, $\gamma_i = 0$ $(i = 1,\dots,6)$, the model reduces to the classical linearly viscous model which describes the motions of a Newtonian fluid. When $\beta_i = 0$ $(i = 1,2,3)$, $\gamma_i = 0$ $(i = 1,\dots,6)$, it reduces to a second-grade model, and when $\mu \geq 0$, $\alpha_1 \geq 0$, $\beta_3 \geq 0$, $|\alpha_1 + \alpha_2| \leq \sqrt{24\mu\beta_3}$, $\beta_i = 0$ $(i = 1,2)$, and $\gamma_i = 0$ $(i = 1,\dots,6)$, it reduces to a third-grade model which is compatible with thermodynamics in the sense of the Clausius-Duhem inequality and the requirement that the Helmholtz free energy be a minimum when the fluid is at rest [6].

Now substituting (2.1) and (2.10) in (2.9) gives

$$\begin{aligned}T_{11} = {}& -p + (\alpha_2 - 2\gamma_2\Omega^2)\left(\frac{\partial f}{\partial z}\right)^2 + 2\beta_2\frac{\partial f}{\partial z}\left(\frac{\partial^2 f}{\partial t\partial z} - W_0\frac{\partial^2 f}{\partial z^2} + \Omega\frac{\partial g}{\partial z}\right) \\ & + \gamma_3\left(\frac{\partial^2 f}{\partial t\partial z} - W_0\frac{\partial^2 f}{\partial z^2} + \Omega\frac{\partial g}{\partial z}\right)^2 + 2\gamma_6\left(\frac{\partial f}{\partial z}\right)^2\left[\left(\frac{\partial f}{\partial z}\right)^2 + \left(\frac{\partial g}{\partial z}\right)^2\right],\end{aligned}$$

$$T_{22} = -p + (\alpha_2 - 2\gamma_2\Omega^2)\left(\frac{\partial g}{\partial z}\right)^2 + 2\beta_2\frac{\partial g}{\partial z}\left(\frac{\partial^2 g}{\partial t\partial z} - W_0\frac{\partial^2 g}{\partial z^2} - \Omega\frac{\partial f}{\partial z}\right)$$

$$+ \gamma_3\left(\frac{\partial^2 g}{\partial t\partial z} - W_0\frac{\partial^2 g}{\partial z^2} - \Omega\frac{\partial f}{\partial z}\right)^2 + 2\gamma_6\left(\frac{\partial g}{\partial z}\right)^2\left[\left(\frac{\partial f}{\partial z}\right)^2 + \left(\frac{\partial g}{\partial z}\right)^2\right],$$

$$T_{33} = -p + [2\alpha_1 + \alpha_2 - 2\Omega^2(\gamma_1 + \gamma_2)]\left[\left(\frac{\partial f}{\partial z}\right)^2 + \left(\frac{\partial g}{\partial z}\right)^2\right]$$

$$+ 2\beta_2\left[\frac{\partial f}{\partial z}\left(\frac{\partial^2 f}{\partial t\partial z} - W_0\frac{\partial^2 f}{\partial z^2} + \Omega\frac{\partial g}{\partial z}\right) + \frac{\partial g}{\partial z}\left(\frac{\partial^2 g}{\partial t\partial z} - W_0\frac{\partial^2 g}{\partial z^2} - \Omega\frac{\partial f}{\partial z}\right)\right]$$

$$+ \gamma_3\left[\left(\frac{\partial^2 f}{\partial t\partial z} - W_0\frac{\partial^2 f}{\partial z^2} + \Omega\frac{\partial g}{\partial z}\right)^2\right.$$

$$\left. + \left(\frac{\partial^2 g}{\partial t\partial z} - W_0\frac{\partial^2 g}{\partial z^2} - \Omega\frac{\partial f}{\partial z}\right)^2 + 4\left(\left(\frac{\partial f}{\partial z}\right)^2 + \left(\frac{\partial g}{\partial z}\right)^2\right)^2\right]$$

$$+ 2(2\gamma_4 + 2\gamma_5 + \gamma_6)\left[\left(\frac{\partial f}{\partial z}\right)^2 + \left(\frac{\partial g}{\partial z}\right)^2\right]^2,$$

$$T_{12} = (\alpha_2 - 2\gamma_2\Omega^2)\left(\frac{\partial f}{\partial z}\right)\left(\frac{\partial g}{\partial z}\right)$$

$$+ \beta_2\left[\frac{\partial f}{\partial z}\left(\frac{\partial^2 g}{\partial t\partial z} - W_0\frac{\partial^2 g}{\partial z^2} - \Omega\frac{\partial f}{\partial z}\right) + \frac{\partial g}{\partial z}\left(\frac{\partial^2 f}{\partial t\partial z} - W_0\frac{\partial^2 f}{\partial z^2} + \Omega\frac{\partial g}{\partial z}\right)\right]$$

$$+ \gamma_3\left[\left(\frac{\partial^2 f}{\partial t\partial z} - W_0\frac{\partial^2 f}{\partial z^2} + \Omega\frac{\partial g}{\partial z}\right)\left(\frac{\partial^2 g}{\partial t\partial z} - W_0\frac{\partial^2 g}{\partial z^2} - \Omega\frac{\partial f}{\partial z}\right)\right]$$

$$+ 2\gamma_6\left(\frac{\partial f}{\partial z}\right)\left(\frac{\partial g}{\partial z}\right)\left[\left(\frac{\partial f}{\partial z}\right)^2 + \left(\frac{\partial g}{\partial z}\right)^2\right],$$

$$T_{13} = (\mu - \beta_1\Omega^2)\frac{\partial f}{\partial z} + (\alpha_1 - \gamma_1\Omega^2)\left(\frac{\partial^2 f}{\partial t\partial z} - W_0\frac{\partial^2 f}{\partial z^2} + \Omega\frac{\partial g}{\partial z}\right)$$

$$+ 2(\beta_2 + \beta_3)\frac{\partial f}{\partial z}\left[\left(\frac{\partial f}{\partial z}\right)^2 + \left(\frac{\partial g}{\partial z}\right)^2\right]$$

$$+ 2(\gamma_3 + \gamma_5)\left(\frac{\partial^2 f}{\partial t\partial z} - W_0\frac{\partial^2 f}{\partial z^2} + \Omega\frac{\partial g}{\partial z}\right)\left[\left(\frac{\partial f}{\partial z}\right)^2 + \left(\frac{\partial g}{\partial z}\right)^2\right]$$

$$+ \gamma_4\left[\left(2\left(\frac{\partial f}{\partial z}\right)^2 + \left(\frac{\partial g}{\partial z}\right)^2\right)\left(\frac{\partial^2 f}{\partial t\partial z} - W_0\frac{\partial^2 f}{\partial z^2} + \Omega\frac{\partial g}{\partial z}\right)\right.$$

$$\left. + \left(\frac{\partial f}{\partial z}\right)\left(\frac{\partial g}{\partial z}\right)\left(\frac{\partial^2 g}{\partial t\partial z} - W_0\frac{\partial^2 g}{\partial z^2} - \Omega\frac{\partial f}{\partial z}\right)\right],$$

$$
\begin{aligned}
T_{23} = {} & (\mu-\beta_1\Omega^2)\frac{\partial g}{\partial z}+(\alpha_1-\gamma_1\Omega^2)\left(\frac{\partial^2 g}{\partial t\partial z}-W_0\frac{\partial^2 g}{\partial z^2}-\Omega\frac{\partial f}{\partial z}\right)\\
& +2(\beta_2+\beta_3)\frac{\partial g}{\partial z}\left[\left(\frac{\partial f}{\partial z}\right)^2+\left(\frac{\partial g}{\partial z}\right)^2\right]\\
& +2(\gamma_3+\gamma_5)\left(\frac{\partial^2 g}{\partial t\partial z}-W_0\frac{\partial^2 g}{\partial z^2}-\Omega\frac{\partial f}{\partial z}\right)\left[\left(\frac{\partial f}{\partial z}\right)^2+\left(\frac{\partial g}{\partial z}\right)^2\right]\\
& +\gamma_4\left[\left(\left(\frac{\partial f}{\partial z}\right)^2+2\left(\frac{\partial g}{\partial z}\right)^2\right)\left(\frac{\partial^2 g}{\partial t\partial z}-W_0\frac{\partial^2 g}{\partial z^2}-\Omega\frac{\partial f}{\partial z}\right)\right.\\
& \qquad \left.+\left(\frac{\partial f}{\partial z}\right)\left(\frac{\partial g}{\partial z}\right)\left(\frac{\partial^2 f}{\partial t\partial z}-W_0\frac{\partial^2 f}{\partial z^2}+\Omega\frac{\partial g}{\partial z}\right)\right],
\end{aligned}
\tag{2.11}
$$

where $T_{12}=T_{21}$, $T_{13}=T_{31}$, and $T_{23}=T_{32}$.

In view of (2.1), (2.3), and (2.8), the three scalar momentum equations become

$$
\begin{aligned}
\rho\left(\frac{\partial f}{\partial t}-W_0\frac{\partial f}{\partial z}-\Omega g-\Omega^2 x\right) &= \frac{\partial T_{11}}{\partial x}+\frac{\partial T_{12}}{\partial y}+\frac{\partial T_{13}}{\partial z}-\sigma B_0^2(f-\Omega y),\\
\rho\left(\frac{\partial g}{\partial t}-W_0\frac{\partial g}{\partial z}+\Omega f-\Omega^2 y\right) &= \frac{\partial T_{21}}{\partial x}+\frac{\partial T_{22}}{\partial y}+\frac{\partial T_{23}}{\partial z}-\sigma B_0^2(g+\Omega x),\\
0 &= \frac{\partial T_{31}}{\partial x}+\frac{\partial T_{32}}{\partial y}+\frac{\partial T_{33}}{\partial z}+\sigma B_0^2 W_0.
\end{aligned}
\tag{2.12}
$$

Inserting the stress components (2.11) into (2.12), we obtain

$$
\begin{aligned}
\rho\left(\frac{\partial f}{\partial t}-W_0\frac{\partial f}{\partial z}-\Omega g\right) = {} & -\frac{\partial \hat{p}}{\partial x}-\sigma B_0^2(f-\Omega y)+(\mu-\beta_1\Omega^2)\frac{\partial^2 f}{\partial z^2}\\
& +(\alpha_1-\gamma_1\Omega^2)\left(\frac{\partial^3 f}{\partial t\partial z^2}-W_0\frac{\partial^3 f}{\partial z^3}+\Omega\frac{\partial^2 g}{\partial z^2}\right)\\
& +2(\beta_2+\beta_3)\frac{\partial}{\partial z}\left[\frac{\partial f}{\partial z}\left(\left(\frac{\partial f}{\partial z}\right)^2+\left(\frac{\partial g}{\partial z}\right)^2\right)\right]\\
& +2(\gamma_3+\gamma_5)\frac{\partial}{\partial z}\left[\left(\frac{\partial^2 f}{\partial t\partial z}-W_0\frac{\partial^2 f}{\partial z^2}+\Omega\frac{\partial g}{\partial z}\right)\left(\left(\frac{\partial f}{\partial z}\right)^2+\left(\frac{\partial g}{\partial z}\right)^2\right)\right]\\
& +\gamma_4\frac{\partial}{\partial z}\left[\left(2\left(\frac{\partial f}{\partial z}\right)^2+\left(\frac{\partial g}{\partial z}\right)^2\right)\left(\frac{\partial^2 f}{\partial t\partial z}-W_0\frac{\partial^2 f}{\partial z^2}+\Omega\frac{\partial g}{\partial z}\right)\right.\\
& \qquad \left.+\left(\frac{\partial f}{\partial z}\right)\left(\frac{\partial g}{\partial z}\right)\left(\frac{\partial^2 g}{\partial t\partial z}-W_0\frac{\partial^2 g}{\partial z^2}-\Omega\frac{\partial f}{\partial z}\right)\right],
\end{aligned}
$$

$$\begin{aligned}
\rho\left(\frac{\partial g}{\partial t}-W_0\frac{\partial g}{\partial z}+\Omega f\right)&=-\frac{\partial\hat{p}}{\partial y}-\sigma B_0^2(g+\Omega x)+(\mu-\beta_1\Omega^2)\frac{\partial^2 g}{\partial z^2}\\
&\quad+(\alpha_1-\gamma_1\Omega^2)\left(\frac{\partial^3 g}{\partial t\partial z^2}-W_0\frac{\partial^3 g}{\partial z^3}-\Omega\frac{\partial^2 f}{\partial z^2}\right)\\
&\quad+2(\beta_2+\beta_3)\frac{\partial}{\partial z}\left[\frac{\partial g}{\partial z}\left(\left(\frac{\partial f}{\partial z}\right)^2+\left(\frac{\partial g}{\partial z}\right)^2\right)\right]\\
&\quad+2(\gamma_3+\gamma_5)\frac{\partial}{\partial z}\left[\left(\frac{\partial^2 g}{\partial t\partial z}-W_0\frac{\partial^2 g}{\partial z^2}-\Omega\frac{\partial f}{\partial z}\right)\left(\left(\frac{\partial f}{\partial z}\right)^2+\left(\frac{\partial g}{\partial z}\right)^2\right)\right]\\
&\quad+\gamma_4\frac{\partial}{\partial z}\left[\left(\left(\frac{\partial f}{\partial z}\right)^2+2\left(\frac{\partial g}{\partial z}\right)^2\right)\left(\frac{\partial^2 g}{\partial t\partial z}-W_0\frac{\partial^2 g}{\partial z^2}-\Omega\frac{\partial f}{\partial z}\right)\right.\\
&\qquad\qquad\left.+\left(\frac{\partial f}{\partial z}\right)\left(\frac{\partial g}{\partial z}\right)\left(\frac{\partial^2 f}{\partial t\partial z}-W_0\frac{\partial^2 f}{\partial z^2}+\Omega\frac{\partial g}{\partial z}\right)\right],\\
\sigma B_0^2 W_0&=-\frac{\partial\hat{p}}{\partial z},
\end{aligned}\tag{2.13}$$

where the modified pressure $\hat{p}$ is given by

$$\begin{aligned}
\hat{p}=p&-\frac{\rho}{2}\Omega^2(x^2+y^2)+\left[2\Omega^2(\gamma_1+\gamma_2)-(2\alpha_1+\alpha_2)\right]\left[\left(\frac{\partial f}{\partial z}\right)^2+\left(\frac{\partial g}{\partial z}\right)^2\right]\\
&-2\beta_2\left[\left(\frac{\partial f}{\partial z}\right)\left(\frac{\partial^2 f}{\partial t\partial z}-W_0\frac{\partial^2 f}{\partial z^2}+\Omega\frac{\partial g}{\partial z}\right)+\left(\frac{\partial g}{\partial z}\right)\left(\frac{\partial^2 g}{\partial t\partial z}-W_0\frac{\partial^2 g}{\partial z^2}-\Omega\frac{\partial f}{\partial z}\right)\right]\\
&-\gamma_3\left[4\left(\left(\frac{\partial f}{\partial z}\right)^2+\left(\frac{\partial g}{\partial z}\right)^2\right)^2+\left(\frac{\partial^2 g}{\partial t\partial z}-W_0\frac{\partial^2 g}{\partial z^2}-\Omega\frac{\partial f}{\partial z}\right)^2+\left(\frac{\partial^2 f}{\partial t\partial z}-W_0\frac{\partial^2 f}{\partial z^2}+\Omega\frac{\partial g}{\partial z}\right)^2\right].
\end{aligned}\tag{2.14}$$

On eliminating the pressure gradient from (2.13) by cross differentiation, we finally obtain

$$\begin{aligned}
&\rho\left(\frac{\partial^2 f}{\partial t\partial z}-W_0\frac{\partial^2 f}{\partial z^2}-\Omega\frac{\partial g}{\partial z}\right)\\
&\quad=-\sigma B_0^2\frac{\partial f}{\partial z}+(\mu-\beta_1\Omega^2)\frac{\partial^3 f}{\partial z^3}+(\alpha_1-\gamma_1\Omega^2)\left(\frac{\partial^4 f}{\partial t\partial z^3}-W_0\frac{\partial^4 f}{\partial z^4}+\Omega\frac{\partial^3 g}{\partial z^3}\right)\\
&\qquad+2(\beta_2+\beta_3)\frac{\partial^2}{\partial z^2}\left[\frac{\partial f}{\partial z}\left(\left(\frac{\partial f}{\partial z}\right)^2+\left(\frac{\partial g}{\partial z}\right)^2\right)\right]\\
&\qquad+2(\gamma_3+\gamma_5)\frac{\partial^2}{\partial z^2}\left[\left(\frac{\partial^2 f}{\partial t\partial z}-W_0\frac{\partial^2 f}{\partial z^2}+\Omega\frac{\partial g}{\partial z}\right)\left(\left(\frac{\partial f}{\partial z}\right)^2+\left(\frac{\partial g}{\partial z}\right)^2\right)\right]\\
&\qquad+\gamma_4\frac{\partial^2}{\partial z^2}\left[\left(2\left(\frac{\partial f}{\partial z}\right)^2+\left(\frac{\partial g}{\partial z}\right)^2\right)\left(\frac{\partial^2 f}{\partial t\partial z}-W_0\frac{\partial^2 f}{\partial z^2}+\Omega\frac{\partial g}{\partial z}\right)\right.\\
&\qquad\qquad\left.+\left(\frac{\partial f}{\partial z}\right)\left(\frac{\partial g}{\partial z}\right)\left(\frac{\partial^2 g}{\partial t\partial z}-W_0\frac{\partial^2 g}{\partial z^2}-\Omega\frac{\partial f}{\partial z}\right)\right],
\end{aligned}$$

$$
\begin{aligned}
&\rho\left(\frac{\partial^2 g}{\partial t\partial z}-W_0\frac{\partial^2 g}{\partial z^2}+\Omega\frac{\partial f}{\partial z}\right)\\
&\quad=-\sigma B_0^2\frac{\partial g}{\partial z}+(\mu-\beta_1\Omega^2)\frac{\partial^3 g}{\partial z^3}+(\alpha_1-\gamma_1\Omega^2)\left(\frac{\partial^4 g}{\partial t\partial z^3}-W_0\frac{\partial^4 g}{\partial z^4}-\Omega\frac{\partial^3 f}{\partial z^3}\right)\\
&\qquad+2(\beta_2+\beta_3)\frac{\partial^2}{\partial z^2}\left[\frac{\partial g}{\partial z}\left(\left(\frac{\partial f}{\partial z}\right)^2+\left(\frac{\partial g}{\partial z}\right)^2\right)\right]\\
&\qquad+2(\gamma_3+\gamma_5)\frac{\partial^2}{\partial z^2}\left[\left(\frac{\partial^2 g}{\partial t\partial z}-W_0\frac{\partial^2 g}{\partial z^2}-\Omega\frac{\partial f}{\partial z}\right)\left(\left(\frac{\partial f}{\partial z}\right)^2+\left(\frac{\partial g}{\partial z}\right)^2\right)\right]\\
&\qquad+\gamma_4\frac{\partial^2}{\partial z^2}\left[\left(\left(\frac{\partial f}{\partial z}\right)^2+2\left(\frac{\partial g}{\partial z}\right)^2\right)\left(\frac{\partial^2 g}{\partial t\partial z}-W_0\frac{\partial^2 g}{\partial z^2}-\Omega\frac{\partial f}{\partial z}\right)\right.\\
&\qquad\qquad\left.+\left(\frac{\partial f}{\partial z}\right)\left(\frac{\partial g}{\partial z}\right)\left(\frac{\partial^2 f}{\partial t\partial z}-W_0\frac{\partial^2 f}{\partial z^2}+\Omega\frac{\partial g}{\partial z}\right)\right].
\end{aligned}
\tag{2.15}
$$

For a steady state, the above equations reduce to the following dimensionless forms:

$$
\begin{aligned}
&\bar{B}^2\frac{d\bar{f}}{d\bar{z}}-\bar{W}_0\frac{d^2\bar{f}}{d\bar{z}^2}-(1-\bar{\beta}_1\bar{\Omega}^2)\frac{d^3\bar{f}}{d\bar{z}^3}\\
&\quad=\bar{\Omega}\frac{d\bar{g}}{d\bar{z}}+(\bar{\alpha}_1-\bar{\gamma}_1\bar{\Omega}^2)\left(-\bar{W}_0\frac{d^4\bar{f}}{d\bar{z}^4}+\bar{\Omega}\frac{d^3\bar{g}}{d\bar{z}^3}\right)\\
&\qquad+2(\bar{\beta}_2+\bar{\beta}_3)\frac{d^2}{d\bar{z}^2}\left[\frac{d\bar{f}}{d\bar{z}}\left(\left(\frac{d\bar{f}}{d\bar{z}}\right)^2+\left(\frac{d\bar{g}}{d\bar{z}}\right)^2\right)\right]\\
&\qquad+2(\bar{\gamma}_3+\bar{\gamma}_5)\frac{d^2}{d\bar{z}^2}\left[\left(-\bar{W}_0\frac{d^2\bar{f}}{d\bar{z}^2}+\bar{\Omega}\frac{d\bar{g}}{d\bar{z}}\right)\left(\left(\frac{d\bar{f}}{d\bar{z}}\right)^2+\left(\frac{d\bar{g}}{d\bar{z}}\right)^2\right)\right]\\
&\qquad+\bar{\gamma}_4\frac{d^2}{d\bar{z}^2}\left[\left(2\left(\frac{d\bar{f}}{d\bar{z}}\right)^2+\left(\frac{d\bar{g}}{d\bar{z}}\right)^2\right)\left(-\bar{W}_0\frac{d^2\bar{f}}{d\bar{z}^2}+\bar{\Omega}\frac{d\bar{g}}{d\bar{z}}\right)\right.\\
&\qquad\qquad\left.-\left(\frac{d\bar{f}}{d\bar{z}}\right)\left(\frac{d\bar{g}}{d\bar{z}}\right)\left(\bar{W}_0\frac{d^2\bar{g}}{d\bar{z}^2}+\bar{\Omega}\frac{d\bar{f}}{d\bar{z}}\right)\right],\\
&\bar{B}^2\frac{d\bar{g}}{d\bar{z}}-\bar{W}_0\frac{d^2\bar{g}}{d\bar{z}^2}-(1-\bar{\beta}_1\bar{\Omega}^2)\frac{d^3\bar{g}}{d\bar{z}^3}\\
&\quad=-\bar{\Omega}\frac{d\bar{f}}{d\bar{z}}-(\bar{\alpha}_1-\bar{\gamma}_1\bar{\Omega}^2)\left(\bar{W}_0\frac{d^4\bar{g}}{d\bar{z}^4}+\bar{\Omega}\frac{d^3\bar{f}}{d\bar{z}^3}\right)\\
&\qquad+2(\bar{\beta}_2+\bar{\beta}_3)\frac{d^2}{d\bar{z}^2}\left[\frac{d\bar{g}}{d\bar{z}}\left(\left(\frac{d\bar{f}}{d\bar{z}}\right)^2+\left(\frac{d\bar{g}}{d\bar{z}}\right)^2\right)\right]\\
&\qquad-2(\bar{\gamma}_3+\bar{\gamma}_5)\frac{d^2}{d\bar{z}^2}\left[\left(\bar{W}_0\frac{d^2\bar{g}}{d\bar{z}^2}+\bar{\Omega}\frac{d\bar{f}}{d\bar{z}}\right)\left(\left(\frac{d\bar{f}}{d\bar{z}}\right)^2+\left(\frac{d\bar{g}}{d\bar{z}}\right)^2\right)\right]\\
&\qquad-\bar{\gamma}_4\frac{d^2}{d\bar{z}^2}\left[\left(\left(\frac{d\bar{f}}{d\bar{z}}\right)^2+2\left(\frac{d\bar{g}}{d\bar{z}}\right)^2\right)\left(\bar{W}_0\frac{d^2\bar{g}}{d\bar{z}^2}+\bar{\Omega}\frac{d\bar{f}}{d\bar{z}}\right)\right.\\
&\qquad\qquad\left.+\left(\frac{d\bar{f}}{d\bar{z}}\right)\left(\frac{d\bar{g}}{d\bar{z}}\right)\left(\bar{W}_0\frac{d^2\bar{f}}{d\bar{z}^2}-\bar{\Omega}\frac{d\bar{g}}{d\bar{z}}\right)\right].
\end{aligned}
\tag{2.16}
$$

The left-hand sides of (2.16) are linear to $\bar{f}$ and $\bar{g}$, respectively. The emerging dimensionless parameters are defined as

$$\bar{z} = \frac{\rho U_0 z}{\mu}, \qquad \bar{f} = \frac{f}{U_0}, \qquad \bar{g} = \frac{g}{U_0},$$
$$\bar{W}_0 = \frac{W_0}{U_0}, \qquad \bar{B}^2 = \frac{\mu\sigma B_0^2}{\rho^2 U_0}, \qquad \bar{\Omega} = \frac{\Omega\mu}{U_0^2}, \qquad \bar{\alpha}_1 = \frac{\alpha_1 \rho U_0^2}{\mu^2}, \tag{2.17}$$
$$\bar{\beta}_i = \frac{\beta_i \rho^2 U_0^4}{\mu^3} \quad (i = 1,2,3), \qquad \bar{\gamma}_i = \frac{\gamma_i \rho^3 U_0^6}{\mu^4} \quad (i = 1,3,4,5).$$

For the problem under consideration, the boundary conditions for the velocity components u and v are

$$u = -\Omega y, \quad v = \Omega x, \quad \text{at } z = 0,$$
$$u = -\Omega(y - l), \quad v = \Omega x, \quad \text{as } z \longrightarrow \infty. \tag{2.18}$$

We note that (2.16) are higher-order equations and thus require additional boundary conditions [12, 15, 17]. As we are solving the problem in an unbounded domain, it is possible to augment the boundary conditions by enforcing additional asymptotic structures at infinity. Here, we augment the additional boundary conditions by requiring that

$$\frac{d^n u}{dz^n} = 0, \quad \frac{d^n v}{dz^n} = 0, \quad \text{as } z \longrightarrow \infty \ (n = 1,2). \tag{2.19}$$

The boundary conditions (2.18) and (2.19) in terms of $\bar{f}$ and $\bar{g}$ can be expressed as

$$\bar{f}(\bar{z}) = 0, \quad \bar{g}(\bar{z}) = 0, \quad \text{as } \bar{z} = 0,$$
$$\bar{f}(\bar{z}) = 1, \quad \bar{g}(\bar{z}) = 0, \quad \frac{d^n \bar{f}}{d\bar{z}^n} = 0, \quad \frac{d^n \bar{g}}{d\bar{z}^n} = 0, \quad \text{as } \bar{z} \longrightarrow \infty \ (n = 1,2), \tag{2.20}$$

where we choose $U_0 = \Omega l$ and $\bar{\Omega} = 1$. For simplicity, in the following, we will drop the bars of the dimensionless variables $\bar{f}$, $\bar{g}$, and $\bar{z}$.

3. Numerical method

The coordinate transformation $\eta = 1/(z+1)$ is applied for transforming the semi-infinite physical domain $z \in [0, \infty)$ to a finite calculation domain $\eta \in [0,1]$, that is,

$$z = \frac{1}{\eta} - 1, \qquad \frac{d}{dz} = -\eta^2 \frac{d}{d\eta}, \qquad \frac{d^2}{dz^2} = \eta^4 \frac{d^2}{d\eta^2} + 2\eta^3 \frac{d}{d\eta},$$
$$\frac{d^3}{dz^3} = -\eta^6 \frac{d^3}{d\eta^3} - 6\eta^5 \frac{d^2}{d\eta^2} - 6\eta^4 \frac{d}{d\eta}, \tag{3.1}$$
$$\frac{d^4}{dz^4} = \eta^8 \frac{d^4}{d\eta^4} + 12\eta^7 \frac{d^3}{d\eta^3} + 36\eta^6 \frac{d^2}{d\eta^2} + 24\eta^5 \frac{d}{d\eta}.$$

With these transformations, the differential equations (2.16) can be rewritten in the forms

$$L_f(f(\eta),\eta) = N_f(f(\eta),g(\eta),\eta), \tag{3.2a}$$
$$L_g(g(\eta),\eta) = N_g(f(\eta),g(\eta),\eta), \tag{3.2b}$$

in which L_f and L_g are linear functions in f and g, respectively, and N_f and N_g stand for the remaining terms and, in general, are nonlinear functions in f and g. Here, we avoid writing the explicit forms of (3.2) because of their complexity. The boundary conditions (2.20) in terms of η can be rewritten as

$$\begin{gathered} f(0)=1, \qquad g(0)=0, \\ \left.\frac{d^n f}{d\eta^n}\right|_{\eta=0}=0, \quad \left.\frac{d^n g}{d\eta^n}\right|_{\eta=0}=0, \quad (n=1,2), \\ f(1)=0, \qquad g(1)=0. \end{gathered} \tag{3.3}$$

Due to nonlinear terms in the governing equations (3.2), we cannot solve the boundary value problem directly by a direct finite-difference method. Here we solve them by means of the method of successive approximation.

We can now define an iterative procedure determining sequences of functions $f^{(0)}(\eta)$, $f^{(1)}(\eta), f^{(2)}(\eta),\dots$ and $g^{(0)}(\eta), g^{(1)}(\eta), g^{(2)}(\eta),\dots$ in the following manner: $f^{(0)}(\eta)$ and $g^{(0)}(\eta)$ are chosen arbitrarily, then $f^{(1)}(\eta), f^{(2)}(\eta),\dots$ and $g^{(1)}(\eta), g^{(2)}(\eta),\dots$ are calculated successively from the following iteration steps:

$$L_f(f^{(k+1)}(\eta),\eta) = N_f(f^{(k)}(\eta),g^{(k)}(\eta),\eta), \tag{3.4a}$$
$$L_g(g^{(k+1)}(\eta),\eta) = N_g(f^{(k+1)}(\eta),g^{(k)}(\eta),\eta), \tag{3.4b}$$

where $k = 0,1,2,\dots$. Note that in the right-hand side of (3.4b) the value of f in the new iteration step $(k+1)$, which is just computed from (3.4a), has been used instead of that in the old iteration step (k) as usual. Test computations have shown that such a variance can provide much better stability. The effectiveness of the method is often influenced considerably by the form of the arrangement (3.2) of the given differential equations and by the choice of the starting functions $f^{(0)}(\eta)$ and $g^{(0)}(\eta)$; the method is generally more effective the closer $f^{(0)}(\eta)$ and $g^{(0)}(\eta)$ are to the solutions $f(\eta)$ and $g(\eta)$, respectively.

In our calculations, to achieve a better convergence, we use the so-called "method of successive under-relaxation." According to the iterations

$$\begin{aligned} L_f(\tilde{f}^{(k+1)}(\eta),\eta) &= N_f(f^{(k)}(\eta),g^{(k)}(\eta),\eta), \\ L_g(\tilde{g}^{(k+1)}(\eta),\eta) &= N_g(f^{(k+1)}(\eta),g^{(k)}(\eta),\eta), \end{aligned} \tag{3.5}$$

where $k = 0, 1, 2, \ldots$, we can obtain $\tilde{f}^{(k+1)}$ and $\tilde{g}^{(k+1)}$, then $f^{(k+1)}$ and $g^{(k+1)}$ are defined by the formulas

$$\begin{aligned} f^{(k+1)} &= f^{(k)} + \tau\left(\tilde{f}^{(k+1)} - u^{(k)}\right), \\ f^{(k+1)} &= f^{(k)} + \tau\left(\tilde{f}^{(k+1)} - u^{(k)}\right), \end{aligned} \tag{3.6}$$

where $0 < \tau \leq 1$, $\tau \in (0, 1]$, is a real under-relaxation parameter. We should choose τ so small that convergent iteration is reached. For $\tau = 1$, the successive under-relaxation method (3.5) with (3.6) is in correspondence with the simple iteration method (3.4).

Because (3.2a) and (3.2b) are differential equations, we can discretise (3.2) for M uniformly distributed discrete points in $\boldsymbol{\eta} = (\eta_1, \eta_2, \ldots, \eta_M) \in (0, 1)$ with a space grid size of $\Delta\eta = 1/(M + 1)$. Due to the special form of the boundary conditions (3.3), that is, four pair conditions at $\eta = \eta_0 = 0$, but only one pair at $\eta = \eta_{M+1} = 1$, we cannot use central differences to approximate the high-order derivatives emerging in (3.2) and (3.3) (if the order is greater than 2). For the third-order and fourth-order derivatives, the following finite-difference schemes are used; they are not central differences, but inclined to the side toward the boundary $\eta_0 = 0$; however, they are still of second-order accuracy:

$$\begin{aligned} \left.\frac{d^3 f}{d\eta^3}\right|_{\eta=\eta_i} &= \frac{3f_{i+1} - 10f_i + 12f_{i-1} - 6f_{i-2} + f_{i-3}}{2(\Delta\eta)^3} + \mathbb{O}(\Delta\eta^2), \\ \left.\frac{d^4 f}{d\eta^4}\right|_{\eta=\eta_i} &= \frac{2f_{i+1} - 9f_i + 16f_{i-1} - 14f_{i-2} + 6f_{i-3} - f_{i-4}}{(\Delta\eta)^4} + \mathbb{O}(\Delta\eta^2), \end{aligned} \tag{3.7}$$

where f_i is the numerical value of f at the point $\eta = \eta_i$.

By means of the finite-difference method, we actually obtain two linear equation systems from (3.5):

$$\mathbf{A}_f\tilde{\mathbf{f}}^{(k+1)} = \mathbf{b}_f, \qquad \mathbf{A}_g\tilde{\mathbf{g}}^{(k+1)} = \mathbf{b}_g, \tag{3.8}$$

where $\mathbf{A}_f$, $\mathbf{A}_g$ are $M \times M$ matrices and $\tilde{\mathbf{f}}^{(k+1)}$, $\mathbf{b}_f$ and $\tilde{\mathbf{g}}^{(k+1)}$, $\mathbf{b}_g$ are vectors

$$\begin{gathered} \tilde{\mathbf{f}}^{(k+1)} = \left[\tilde{f}_1^{(k+1)}, \tilde{f}_2^{(k+1)}, \ldots, \tilde{f}_M^{(k+1)}\right]^T, \\ \mathbf{b}_f = \left[N_f\left(f^{(k)}, g^{(k)}, \eta\right)|_{\eta=\eta_1}, N_f\left(f^{(k)}, g^{(k)}, \eta\right)|_{\eta=\eta_2}, \ldots, N_f\left(f^{(k)}, g^{(k)}, \eta\right)|_{\eta=\eta_M}\right]^T, \\ \tilde{\mathbf{g}}^{(k+1)} = \left[\tilde{g}_1^{(k+1)}, \tilde{g}_2^{(k+1)}, \ldots, \tilde{g}_M^{(k+1)}\right]^T, \\ \mathbf{b}_g = \left[N_g\left(f^{(k+1)}, g^{(k)}, \eta\right)|_{\eta=\eta_1}, N_g\left(f^{(k+1)}, g^{(k)}, \eta\right)|_{\eta=\eta_2}, \ldots, N_f\left(f^{(k+1)}, g^{(k)}, \eta\right)|_{\eta=\eta_M}\right]^T, \end{gathered} \tag{3.9}$$

evaluated at the discrete points $0 < \eta_1 < \eta_2 < \cdots < \eta_n < 1$.

In doing so, for each iterative step, two algebraic equation systems of the form (3.8) with bandwidth of six elements emerge, which can be solved, for example, by Gaussian elimination. The iteration should be carried out until the relative differences of the computed $f_i^{(k)}$, $f_i^{(k+1)}$ as well as $g_i^{(k)}$, $g_i^{(k+1)}$ $(i = 1, 2, \ldots, M)$ between two consecutive iterative steps (k) and $(k+1)$ are smaller than a given error chosen to be 10^{-10}.

4. Numerical results and discussions

We compute and compare the profiles of the dimensionless translational velocities in η-coordinate for two kinds of fluids: a Newtonian fluid, for which $\bar{\alpha}_1 = 0$, $\bar{\beta}_i = 0$ $(i = 1, 2, 3)$, and $\bar{\gamma}_i = 0$ $(i = 1, 3, 4, 5)$, and a fourth-grade non-Newtonian fluid, in which we choose $\bar{\alpha}_1 = 1$, $\bar{\beta}_i = 1$ $(i = 2, 3)$, and $\bar{\gamma}_i = 1$ $(i = 3, 4, 5)$; however $\bar{\beta}_1 = 0$ and $\bar{\gamma}_1 = 0$ to avoid that the effect of the lower-order terms is counteracted by these higher-order terms. In addition, the alone influences of the additional second-, third-, and fourth-order terms on the first-order Newtonian fluid are also studied, respectively.

All numerical results are depicted in the η-coordinate of the computational domain. It should be pointed out that in η-coordinate the physical domain at infinity $(\bar{z} \to \infty)$ or far away from the disk (large $\bar{z}$) is massively compressed $(\eta \to 0)$, whilst the physical domain near the disk $(\bar{z} \to 0)$ is relatively expanded $(\eta \to 1)$.

The numerical results obtained by using various suction velocities $\bar{W}_0$ through the disk are illustrated in Figure 4.1. For both the Newtonian fluid (Figures 4.1(a) and 4.1(b)) and the fourth-grade fluid (Figures 4.1(c) and 4.1(d)), when the suction velocity $\bar{W}_0$ increases, the boundary layer near the disk $(\eta = 1$ or $\bar{z} = 0)$ tends to become thinner. In general, the boundary layer for the Newtonian fluid $(0.3 \le \eta \le 1$ or $0 \le \bar{z} \le 2.3)$ is much thinner than that for the fourth-grade fluid $(0.15 \le \eta \le 1$ or $0 \le \bar{z} \le 5.7)$.

On the contrary, for a blowing velocity through the disk, the boundary layer tends to become thicker, especially for the Newtonian fluid (Figures 4.2(a) and 4.2(b)), in which the boundary layer thickness increases from $\eta > 0.25$ (or $\bar{z} < 3$) to $\eta > 0.05$ (or $\bar{z} < 19$) if the blowing velocity increases from zero to $|\bar{W}_0| = 5$. However, for the fourth grade fluid, when the blowing velocity increases from $|\bar{W}_0| = 1$ to $|\bar{W}_0| = 5$, a thinning of boundary layer occurs. The change of the boundary layer thickness with increasing blowing velocity loses monotonicity for the fourth-grade fluid.

It is well known that in an inertial frame, for a Newtonian fluid, no steady asymptotic solution is possible for flow past a porous plate subjected to uniform blowing [7]. This is due to the fact that the blowing causes a thickening of the boundary layer, as shown in Figures 4.2(a) and 4.2(b), so that at a sufficiently large distance from the plate the boundary layer becomes so thick that it becomes turbulent and a steady solution is not possible. In a rotating frame, however, the situation is different. The Ekman boundary layer thickness is of the order of $(\mu/(\rho\Omega))^{1/2}$, which decreases with an increase in rotation. Thus, if the blowing is not too large, the thinning effect of rotation may just counterbalance the thickening effect of blowing so that the vorticity generated at the disk instead of being convected away from the disk by blowing remains confined near the disk and a steady solution is possible. It should be pointed out that for the problem under consideration, if $B_0 = 0$, steady numerical solutions exist only with fairly small values of the blowing velocity $|\bar{W}_0| \le 0.07$ for both the Newtonian fluid and the fourth-grade fluid, while when

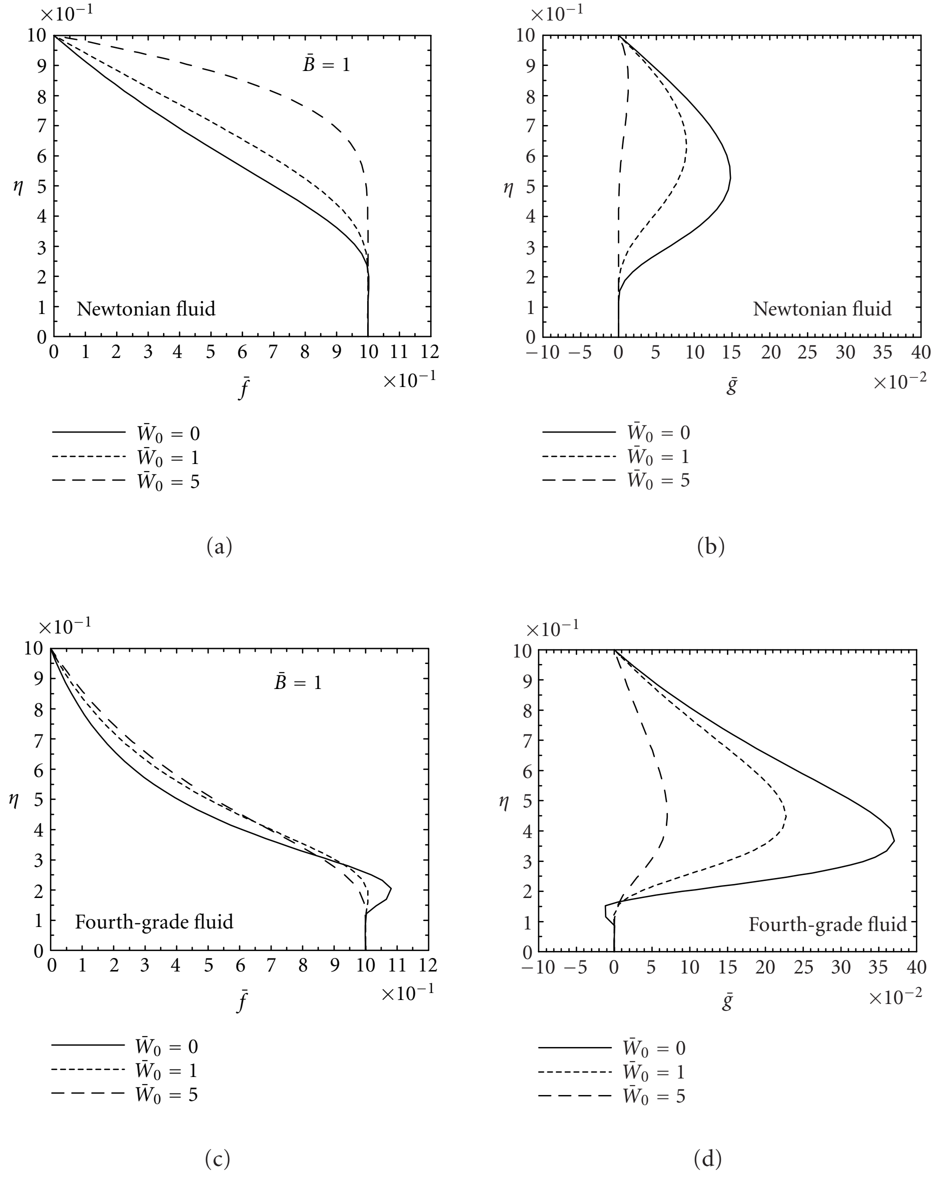

Figure 4.1. Profiles of dimensionless translational velocities $\bar{f}$ and $\bar{g}$ with three values of the suction velocity $\bar{W}_0 = 0, 1, 5$ for a Newtonian fluid (panels a, b) and a fourth-grade fluid (panels c, d).

$\bar{B} = 1$, steady solutions are still possible for much larger values of the blowing velocity, for example, for $|\bar{W}_0| = 5$, as shown in Figure 4.2. The reason is that the effect of the magnetic field causes a thinning of the boundary layer, as shown in Figure 4.3, which can compensate the thickening of the boundary layer caused by the blowing to some extent, so that the thickness of the boundary layer remains still at a low level.

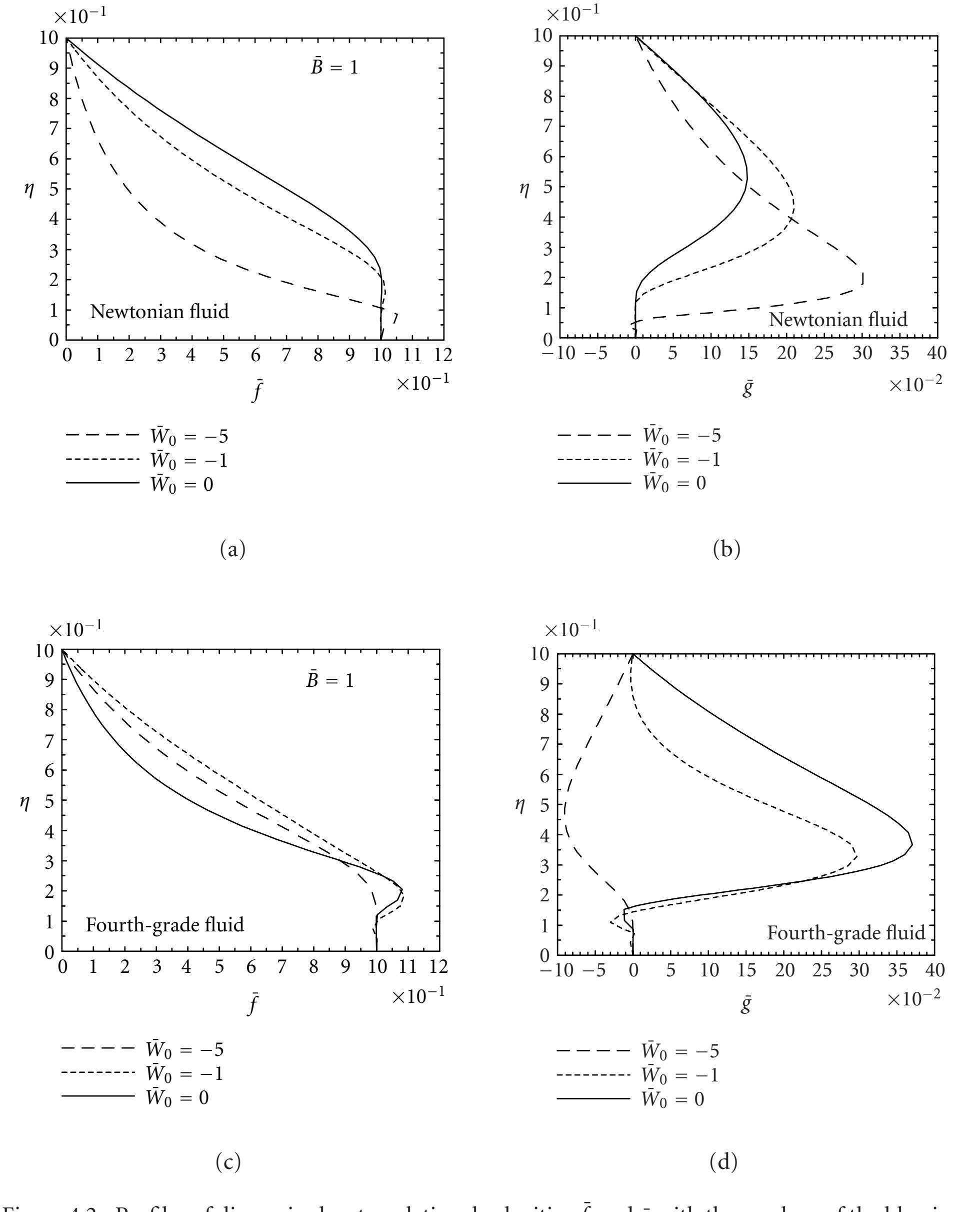

Figure 4.2. Profiles of dimensionless translational velocities $\bar{f}$ and $\bar{g}$ with three values of the blowing velocity $\bar{W}_0 = 0, -1, -5$ for a Newtonian fluid (panels a, b) and a fourth-grade fluid (panels c, d).

As has already been mentioned, similarly to a suction velocity, an applied magnetic field tends to restrict the shear layer to a thinner boundary layer near the disk ($\eta = 1$ or $\bar{z} = 0$) for both the fourth-grade fluid and the Newtonian fluid, as one can see in Figure 4.3. It means that the electromagnetic force provides some mechanism to control the boundary layer thickness.

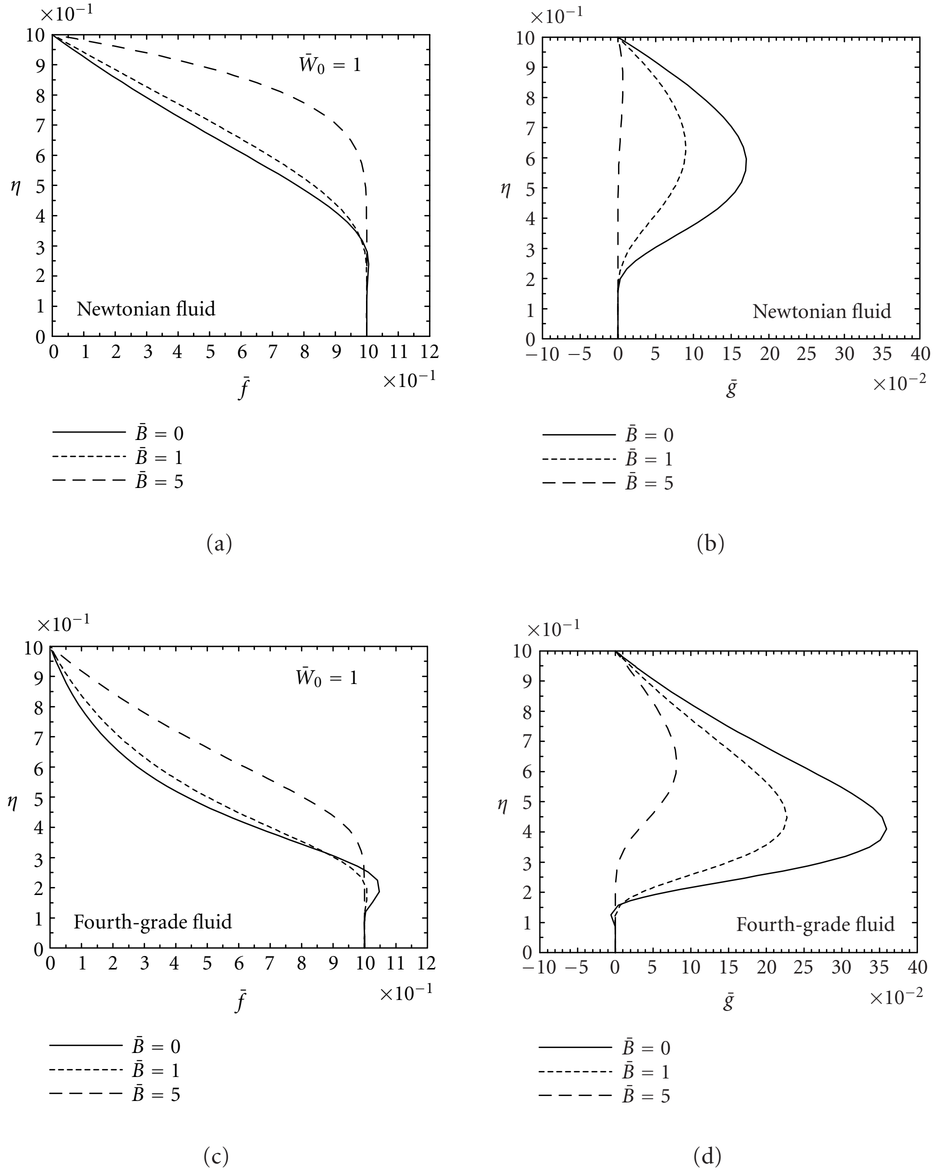

Figure 4.3. Profiles of dimensionless translational velocities $\bar{f}$ and $\bar{g}$ with three values of the magnetic parameter $\bar{B} = 0, 1, 5$ for a Newtonian fluid (panels a, b) and a fourth-grade fluid (panels c, d).

The effect of higher-order terms on the flow can be observed in Figures 4.4, 4.5, and 4.6, in which the variations of the dimensionless translational velocity components are represented for various values of the material parameters of the fluid. It is noted that the boundary layer thickness is increased by increasing these higher-order material parameters.

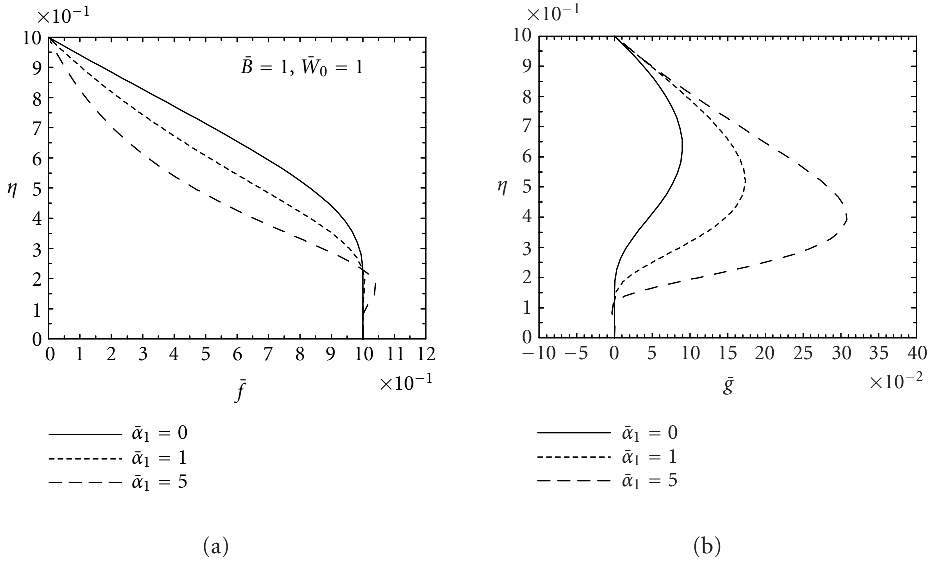

Figure 4.4. Profiles of dimensionless translational velocities $\bar{f}$ (panel a) and $\bar{g}$ (panel b) for a non-Newtonian fluid with various coefficients of the second-grade terms $\bar{\alpha}_1 = 0$ (a Newtonian fluid), 1, 5. The third-grade and fourth-grade terms are neglected ($\bar{\beta}_i = 0$, $i = 1, 2, 3$; $\bar{\gamma}_i = 0$, $i = 1, 3, 4, 5$).

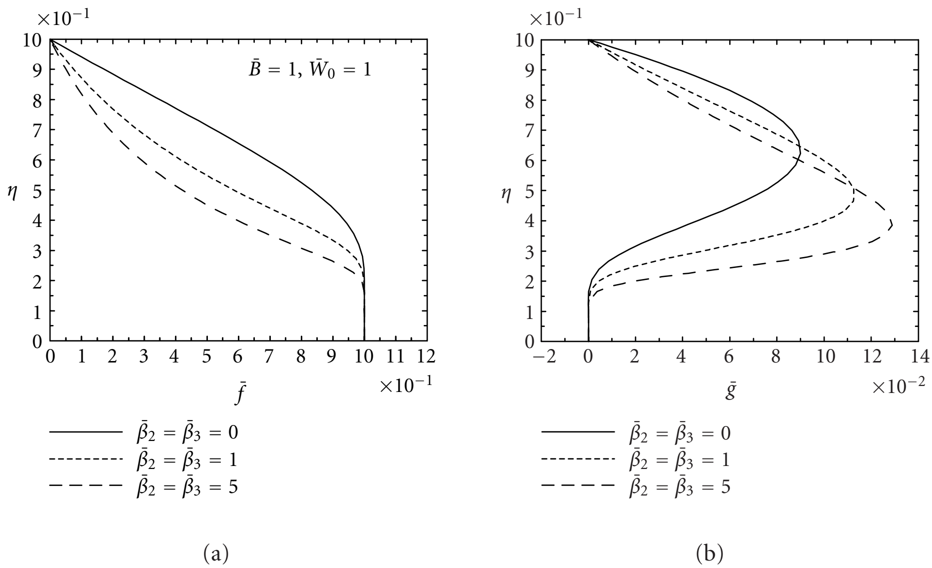

Figure 4.5. Profiles of dimensionless translational velocities $\bar{f}$ (panel a) and $\bar{g}$ (panel b) for a non-Newtonian fluid with various coefficients of the third-grade terms $\bar{\beta}_{2,3} = 0$ (a Newtonian fluid), 1, 5. The second-grade and fourth-grade terms are neglected ($\bar{\alpha}_1 = 0$; $\bar{\gamma}_i = 0$, $i = 1, 3, 4, 5$).

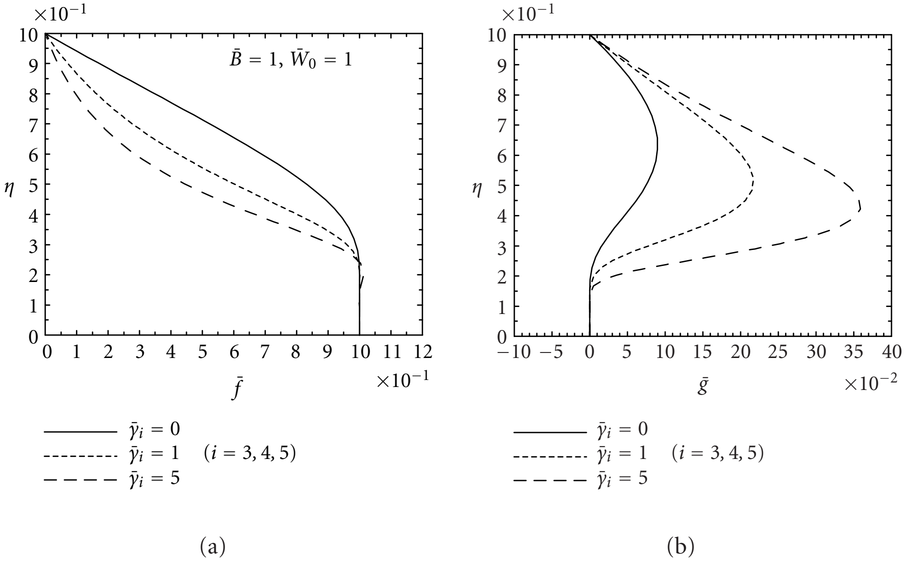

Figure 4.6. Profiles of dimensionless translational velocities $\bar{f}$ (panel a) and $\bar{g}$ (panel b) for a non-Newtonian fluid with various coefficients of the fourth-grade terms $\bar{\gamma}_i = 0$ (a Newtonian fluid), 1, 5 $(i = 3, 4, 5)$. The second-grade and third-grade terms are neglected ($\bar{\alpha}_1 = 0$; $\bar{\beta}_i = 0$, $i = 1, 2, 3$).

5. Concluding remarks

The flow due to noncoaxially rotations of a porous disk and a fourth-grade fluid at infinity subjected to a magnetic field has been studied in comparison with the flow of a Newtonian fluid. As the magnetic field is intensified and the suction velocity increases, the boundary layer near the rotating disk becomes thinner for both the fourth-grade fluid and the Newtonian fluid. On the contrary, the blowing causes a thickening of the boundary layer, with the exception of strong blowing for the fourth-grade fluid, in which a thinning occurs. In general, the boundary layer of a Newtonian fluid is much thinner than the fourth-grade fluid. It is also clear that, as the second-, third-, and fourth-grade viscosity coefficients increase, the boundary layer tends to become thicker.

The purpose of the presented communication has been to examine the flow of a fourth-grade fluid with a view towards understanding its response characteristics. This has been put into practice here by imposing the boundary conditions (2.19), which were suggested from that the shear stresses are zero on infinity. As long as physical arguments do not allow us to constrain the boundary condition to exact statements, we do not see much hope that fourth-grade fluid flows can help us in understanding certain non-Newtonian behaviours. However, strong dependences of the velocity profiles on suction, magnetic field, and fourth-grade parameters may shed some light in this regard.

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Tasawar Hayat: Department of Mathematics, Quaid-i-Azam University, Islamabad 45320, Pakistan
E-mail address: t_pensy@hotmail.com

Yongqi Wang: Institute of Mechanics, Darmstadt University of Technology, Hochschulstrasse 1, 64289 Darmstadt, Germany
E-mail address: wang@mechanik.tu-darmstadt.de