Research Article

Robust ESO Two-Degree-of-Freedom Control Design for Permanent Magnet Synchronous Motor

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A robust two-degree-of-freedom control scheme is proposed for permanent magnet synchronous motor (PMSM) using extended state observer (ESO). The robustness is achieved based on the ESO. Parameter perturbation and external disturbances in PMSM drive system are treated as disturbance variable, and then the motion model of PMSM is transformed into an extended state model by introducing this disturbance variable. To estimate the disturbance variable, an ESO is constructed. Estimator is compensated into the control system to improve robustness and adaptability of 2DOF controller against parameter perturbation and external disturbances. The effectiveness of the proposed control scheme is demonstrated with simulation results.

1. Introduction

The PMSM plays an important role in industrial application due to its high efficiency, high power density, low inertia, no need for maintenance, and high air gap magnetic density [1, 2]. To achieve the high-performance control, the vector control of PMSM drive is developed. In the vector control technique, the Proportional-integral (PI) controller provides the efficient solution to real-world control problems and is applied to many industrial applications [3, 4]. However, the PI controller had many problems in high-performance applications requiring fast and precise speed response, quick speed recovery under any disturbances, and insensitivity to the machine parameters, and it cannot give good command tracking and load regulation property simultaneously because the closed loop zeros cannot be placed arbitrarily. The speed controller that uses a conventional PI controller has poor robustness to nonlinearity, strong coupling, and dynamic uncertainty of PMSM drive systems [5, 6]. Many robust control techniques are proposed to overcome the disturbances of systematic nonlinearity and systematic uncertainty in order to achieve fast speed response and robustness and adaptability to the parameter variations [7–15]. According to the output

error correction, observer used in these strategies can be fall into two categorical types: classical state observer and variable structure observer. The classical state observer such as the Luenberger observer using output error linear correction and all state variable feedback control can only be used in linear and certain systems and fail to nonlinear and uncertain systems. The variable structure observer uses nonsmooth structure to improve its robustness to both system uncertainties and measurement errors but with chattering phenomenon. To overcome the drawbacks of the PI controller, 2DOF controller is investigated to treat the command tracking and disturbance regulation specifications, separately. However, the 2DOF controller is sensitive to the machine parameters. When the parameter variation exceeds certain range, performances of the 2DOF controller become worse.

The extended state observer uses the extended state variable to observe the system uncertainties and external disturbances. The control system is compensated by the estimator of the extended state variable to improve its robustness.

In this paper, a robust ESO-2DOF control scheme is introduced. Regarding parameter perturbation and external disturbances as system disturbance variable, which are observed by the ESO and compensated into 2DOF controller to cancel influences of the disturbances, simulation results show that the proposed strategy effectively improves the robustness of the 2DOF controller as well as good command tracking and strong disturbance rejection characteristic simultaneously.

2. Mathematical Model of PMSM

On the basis of assumptions that the stator windings generate sinusoidal magnetic field, air gap is uniform and saturation is negligible. With reference to synchronous rotating reference frame, the voltage and torque equations of an IPMSM may be expressed as follows [16]:

$$u_{d} = R_{s}i_{d} + L_{d}\frac{di_{d}}{dt} - \omega_{r}L_{q}i_{q},$$

$$u_{q} = R_{s}i_{q} + L_{q}\frac{di_{q}}{dt} + \omega_{r}(L_{d}i_{d} + \psi_{f}),$$

$$T_{e} = 1.5n_{p}[\psi_{f}i_{q} + (L_{d} - L_{q})i_{d}i_{q}].$$
(2.2)

Equation of motion

$$\frac{d\omega_r}{dt} = \frac{1}{J} \left(n_p T_e - n_p T_L - B\omega_r \right), \tag{2.3}$$

where u_d and u_q are stator d- and q-axis voltages; L_d and L_q are stator d- and q-axis inductances; R_s is resistance of the stator windings; i_d and i_q are stator d- and q-axis currents; Ψ_f is amplitude of the flux induced by the permanent magnets of the rotor in the stator phases; n_p is number of pole pairs; T_e is electromagnetic torque; ω_r is angular speed of the motor; J is inertia of moment; T_L is the load torque; B is viscous friction coefficient.

3. Design of ESO-2DOF Controller

3.1. Extended State Observer

The function of state observer is to reconstruct system state that bases on known inputs and measured outputs. The control systems generally have some disturbances such as parameter perturbation, unmodeled-dynamic, and measurement error. The ESO is used to estimate the uncertainty of system and external disturbances. By compensating the estimator into the control system, the nonlinear and uncertain system can be approximated linearization and certainty. As an example, the procedure of developing ESO for first-order system is given.

A class of first-order system is described as follows:

$$\frac{dx_1}{dt} = f(x_1, t, w(t)) + bu,$$

$$y = x_1,$$
(3.1)

where x_1 is state variable; t is time variable; w(t) is unknown disturbance; $f(x_1, t, w(t))$ is unknown function; b is system parameter, which denotes the known part of system; u is control input; y is output signal.

Let function $f(x_1, t, w(t))$ be the extended state variable of system, let $x_2 = f(x_1, t, w(t))$, and let unknown disturbance w(t) be the differential of extended state variable. Equation (3.1) can be rearranged into following equation

$$\dot{x}_1 = x_2 + bu,$$

$$\dot{x}_2 = w(t),$$

$$y = x_1.$$

(3.2)

If x_2 is bounded, the ESO of (3.2) can be defined as

$$e = z_1 - y,$$

$$\dot{z}_1 = z_2 - \beta_{01} f_1(e) + bu,$$

$$\dot{z}_2 = -\beta_{02} f_2(e),$$
(3.3)

where β_{01} and β_{02} are system gains; $f_1(e)$ and $f_2(e)$ are correcting function, which satisfy $e \cdot f_1(e) > 0$ and $e \cdot f_2(e) > 0$, $\forall e \neq 0$.

If $f_i(e)$ (i = 1, 2) in (3.3) is replaced with e, the model denoted by (3.3) has the transitional Luenberger observer form. If $f_i(e) = e + k_i \operatorname{sign}(e)$, the model denotes variable structure observer.

The extended state variable z_2 is the real-time action of the unknown disturbances [17]. z_1 is estimator of x_1 .



Figure 1: 2DOF Control Configuration.

3.2. 2DOF Controller

The 2DOF controller does not consist of two individual Proportional-integral-derivative (PID) controllers, but it can set independently the parameters of each PID controller to achieve good command tracking and disturbance rejection characteristic simultaneously.

This paper uses advanced 2DOF controller [18]. It is shown in Figure 1.

In Figure 1, H(s) is a compensation unit; $G_c(s)$ is a primary controller; $G'_c(s)$ is a derivative unit; $G_p(s)$ is the controlled object; α, γ are 2DOF proportional and derivative coefficients, separately; r is reference signal.

For the PMSM regulation speed system, the expressions are given by

$$G_{c}(s) = K_{p} \left(1 + \frac{1}{T_{i}s}\right),$$

$$H(s) = \frac{1}{1 + \beta T_{i}s'},$$

$$G_{c}'(s) = \frac{k_{p}T_{d}s}{1 + \eta T_{d}s'},$$
(3.4)

where β is 2DOF integral coefficient; $1/\eta$ is derivative gain; K_p is proportional gain; T_i is integral time; T_d is derivative time.

The transfer function of 2DOF controller can be obtained as

$$C(S) = K_p \left(\alpha + \frac{\alpha \gamma T_d S}{1 + \eta T_d S} + \frac{1}{T_i S} - \frac{(1 - \alpha)(\beta - 1)}{1 + \beta T_i S} \right).$$
(3.5)

From Figure 1, it is seen that the advanced 2DOF controller mainly consists of reference signal filter and PID controller. The adjustable parameters of the reference signal filter are α , β , and γ , the change of which influences the command tracking performance of advanced 2DOF controller. The dynamic processes are controlled by the PID controller, the parameters of which can be set according to the traditional PID adjusting methods.

3.3. Robust ESO-2DOF Controller

To overcome the impact of disturbances to the system, this paper uses the ESO to estimate and compensate the disturbances into the 2DOF controller. Thus, motion model of PMSM is transformed into the extended state model.

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Substituting (2.2) into (2.3) yields

$$\dot{\omega}_r = \frac{1.5n_p\psi_f}{J}i_q + \lambda,\tag{3.6}$$

where $\lambda = [(L_d - L_q)n_p i_d i_q - n_p T_L - B\omega_r]/J$.

The variable λ is regarded as the extended state variable of system. Let the differential of λ be g, and (3.6) can be rewritten in the following form:

$$\dot{\omega}_r = \lambda + \frac{1.5n_p \psi_f}{J} i_{qr}$$

$$\dot{\lambda} = g.$$
(3.7)

Referring to (3.3), one can obtain the ESO of (3.7):

$$e = z_1 - \omega_r,$$

$$\dot{z}_1 = z_2 - \beta_{01} \text{fal}(e, \alpha_{01}, \delta) + b_0 u,$$

$$\dot{z}_2 = -\beta_{02} \text{fal}(e, \alpha_{02}, \delta),$$
(3.8)

where $0 < \alpha_{01} < 1$; $0 < \alpha_{02} < 1$; δ is the length of linear segment of the fal(·) function which is given by (3.9); ω_r is the output rotating speed of the PMSM; b_0 denotes known part of model, the expression of which is $1.5n_p\psi_f/J$; z_1 is estimator of speed; z_2 is estimator of the extended state variable.

For (3.8), the correcting function adopts the $fal(\cdot)$ to achieve quickly smooth convergence property and good stability at equilibrium point:

$$fal(e, \alpha, \delta) = \begin{cases} |e|^{\alpha} \operatorname{sign}(e), & |e| > \delta, \\ \\ \frac{e}{\delta^{1-\alpha}}, & |e| \le \delta, \end{cases}$$
(3.9)

where sign(\cdot) is a symbolic function.

The error dynamic model of (3.8) can be described as

$$e_{1} = z_{1} - \omega_{r},$$

$$e_{2} = z_{2} - \lambda,$$

$$\dot{e}_{1} = e_{2} - \beta_{01} \operatorname{fal}(e_{1}, \alpha_{01}, \delta),$$

$$\dot{e}_{2} = g - \beta_{02} \operatorname{fal}(e_{1}, \alpha_{02}, \delta).$$
(3.10)



Figure 2: 2DOF Control Configuration Base on ESO.

Table 1: Parameters of the PMSM.

Name	Value
Rated power	1.5 kW
Rated speed	2800 r/min
Number of poles	8
Rated current	22A
Stator resistance	0.093Ω
<i>d</i> -axis inductance	0.4 mH
<i>q</i> -axis inductance	0.5 mH
magnetic flux	0.026 Wb

If $|g| < w_0$, that is, the disturbances are bounded, one can obtain the following:

$$\lim_{t \to \infty} |z_1(t) - \omega_r(t)| < \beta_{01} \left(\frac{w_0}{\beta_{02}}\right)^{1/\alpha_{02}},\tag{3.11}$$

$$\lim_{t \to \infty} |z_2(t) - \lambda(t)| < \beta_{02} \cdot \left(\frac{w_0}{\beta_{02}}\right)^{\alpha_{01}/\alpha_{02}}.$$
(3.12)

The extended state variable z_2 can better estimate the disturbances by adjusting the parameters of (3.11) and (3.12).

By using the extended state variable, the estimator of the disturbances can be obtained. The ESO-2DOF controller is developed through compensating the disturbances into 2DOF controller. In the light of the 2DOF controller structure, the estimator is compensated before the primary controller.

The diagram of the ESO-2DOF controller is shown in Figure 2. *r* is speed reference.

4. Simulation Results and Analysis

In order to verify the validity of the proposed ESO-2DOF controller, a computer simulation model is developed in MATLAB/Simulink software according to Figure 2. Table 1 lists the parameters of the tested PMSM.

The parameters in ESO-2DOF controller fall into two groups. One group is β_{01} , β_{02} , α_{01} , α_{02} and δ . These parameters belong to the ESO. The others belong to advanced 2DOF controller. The robustness of the ESO to both disturbance and parametric variation is strong when α_{01} and α_{02} are small [19, 20]. α_{01} and α_{02} are set to 0.25 and 0.5 in simulation,



Figure 3: Dynamic antidisturbance curves of PMSM system.

respectively. δ denotes the length of linear segment of the nonlinear function. Slope of the linear segment can be described by differential of δ . In general, δ is less than 5, or else the performances of nonlinear feedback and disturbance rejection performances of system will get worse. δ is set to 0.05 in simulation.

By selecting appropriate values for β_{01} and β_{02} , the tracking performances of the state variables in the ESO can be improved. If β_{01} is too small, the tracking performances of z_1 and z_2 can be degraded. On the contrary, if β_{02} has a high value, the steady-state characteristic of system becomes poor.

The simulation results show that β_{01} can be set according to the sampling frequency. The recommendation is that β_{01} is one to two times sampling frequency and β_{02} is one to ten times β_{01} .

Due to limitation of space, only two simulation results are presented to illustrate the characteristics of the proposed ESO-2DOF controller. In the first test condition, a step command speed n = 1500 rpm is applied to the PMSM drive starting from rest at t = 0 s and with no load. At t = 0.04 s, the load disturbance is used. The second test condition is that the moment of inertia is changed into 150% of the original value and the load disturbance is used at t = 0.03 s and the unloading is occurred at 0.05 s.

Figure 3 shows the speed responses with load disturbance at t = 0.04 s when the motor speed is 1500 rpn. The fast tracking and disturbance rejection performance of the ESO-2DOF controller is achieved well.

Figure 4 shows the estimator of the disturbance. As can be observed from Figure 5, the ESO can effectively estimate the disturbances.

Figures 5 and 6 show speed response with load disturbance and with unloading when the moment of inertia is changed into 150% of the original value. The results show that the ESO-2DOF controller has strong robustness to parameter perturbation.

Figure 7 shows the estimator of the unloading disturbance. As can be observed from Figure 7, the ESO can effectively estimate the disturbances.



Figure 4: Disturbance curves of system.



Figure 5: Dynamic speed response curves of PMSM system with parameter perturbation.

5. Conclusions

To improve the robustness of 2DOF controller, the ESO-2DOF controller is proposed. The parameter perturbation and external disturbances are estimated simultaneously by using the ESO. The estimator is compensated into 2DOF controller to guarantee the



Figure 6: Dynamic speed response curves of PMSM system with unloading.



Figure 7: Disturbance curves of system with parameter perturbation.

strong robustness, the good command tracking, and disturbance rejection characteristic simultaneously. Simulation results are given to demonstrate the validity of the proposed control scheme.

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