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Research Article

Nonlinear Vibration Analysis for a Jeffcott Rotor with Seal and Air-Film Bearing Excitations

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The nonlinear coupling vibration and bifurcation of a high-speed centrifugal compressor with a labyrinth seal and two air-film journal bearings are presented in this paper. The rotary shaft and disk are modeled as a rigid Jeffcott rotor. Muszynska's model is used to express the seal force with multiple parameters. For air-film journal bearings, the model proposed by Zhang et al. is adopted to express unsteady bearing forces. The Runge-Kutta method is used to numerically determine the vibration responses of the disk center and the bearings. Bifurcation diagrams for transverse motion of the rotor are presented with parameters of rotation speed and pressure drop of the seal. Multiple subharmonic, periodic, and quasiperiodic motions are presented with two seal-pressure drops. The bifurcation characteristics show inherent interactions between forces of the air-film bearings and the seal, presenting more complicated rotor dynamics than the one with either of the forces alone. Bifurcation diagrams are obtained with parameters of pressure drop and seal length determined for the sake of operation safety.

1. Introduction

The motion stability of high-speed rotor systems has drawn extensive attention throughout the past several decades. It is now well known that the stability of the rotor's equilibrium can be lost as a result of the Hopf bifurcation, which leads to finite-amplitude whirls of oil-film inside the bearings. The mechanism of oil whips developed from escalating whirling motions has been thoroughly investigated both experimentally and theoretically (see, e.g., [1–4]). Several models have been developed to investigate oil-film forces of short bearings and bearings with finite lengths [5–9]. Various studies of the oil-film forces were carried out to present nonlinear vibrations, for example, super- and subharmonic motions, of the rotor system related to the bearing dynamics [10–12]. Aside from the bearing forces, seal forces

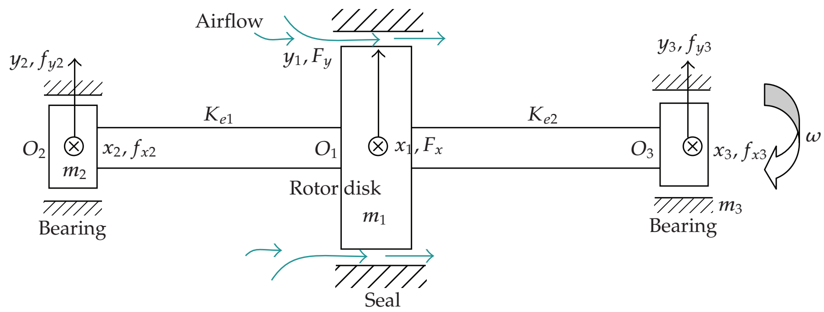


Figure 1: A Jeffcott rotor-seal-bearing system.

play significant roles in vibration and stability of air compressors and steam turbines. Seal forces are usually generated due to the fluid-solid interaction in the clearance between the shaft and the stator which may cause self-excited motions of the rotor. Previous investigations showed that the seal force provides not only supportive reactions to the rotor in the radial direction but also cross-coupling forces in the tangential direction that excites severe vibrations in some occasions. An effective model was proposed by Muszynska to express nonlinear seal forces based on experimental results [13, 14]. This model was later adopted by Ding et al. [15] in their study on the Hopf bifurcation of a symmetric rotor-seal system and by Hua et al. who numerically obtain the nonlinear vibration and bifurcation characteristics of an unbalanced rotor-seal system [16]. Similar research was provided in Zhang et al. [17] where subharmonic motions and bifurcation diagrams were demonstrated with parameter of rotation speed. In spite of the numerous publications that separately dealt with rotor-bearing and rotor-seal systems, very few literatures have been focused on the dynamics of rotor-seal-bearing systems which is a great concern of air-compressor and steam turbine engineers. It is worth emphasizing that the interaction between the seal and bearing excitations should not be ignored since complicated, large-amplitude motions can be developed for rotors of compressors and turbines.

The numerical analysis for nonlinear vibration and bifurcation behavior of a high-speed centrifugal compressor with a labyrinth seal and two journal bearings is presented in this paper. What differentiates the current rotor system from others is the application of air-film bearings rather than conventional oil-film journal bearings. Practically, compressors supported by this kind of bearings operate under circumstance where only inflammable lubricants (i.e., air or pure water) are allowed. It should be noticed that the air-film bearings complicate the dynamics of the rotor in two aspects: (1) since the viscosity of the air is very small, the amplitude of whirling orbit is remarkably large, which brings rich nonlinear characteristics into the rotor response; (2) the airflow inside the clearance of journals is much more irregular and turbulent than the oil-film bearings, which makes most of existent theories unable to provide realistic prediction of the bearing dynamics. In the first case, the vibration response is strongly nonlinear and must be solved numerically with consideration of both bearing and seal forces. In the second case, an effective model for unsteady air-film force should be adopted to express time-varying boundaries of the film that whirls rapidly around the journal center. In the present study, the oil-film force proposed by Zhang et al. [18, 19] is used to model the nonlinear, unsteady air-film excitation in the current study. For the seal force Muszynska's model is adopted with parameters of pressure drop, rotation speed, and seal length. The complexity in the rotor motion is demonstrated through bifurcation diagrams with those parameters as well as through the Poincaré maps, time history of displacement, and rotor orbits. For seal pressure drop of 0.2 MPa the bifurcation sequence is given with increasing rotation speed, showing subharmonic motions of periodic-1, 12, 11, 10, 9, 8, 7 and quasiperiodic motions. The results are compared to the ones without bearing forces to present the interaction between the air-film bearing and the seal forces. Periods-4 and -11 bifurcations and quasiperiodic motion are observed with a 0.4 MPa pressure drop. The bifurcation diagrams of motion with parameters of pressure drop and length of the seal provide suitable values of these quantities for improvement of operation safety of the machinery. The intricacy in the motion's bifurcation presents complicated dynamics of the system in contrast to the rotors with either of bearing forces or of the seal excitations.

2. Problem Modeling

A Jeffcott rotor with a rigid disk, a segment of labyrinth seal, and two supporting air-film journal bearings is shown in Figure 1, where o_1 is the geometric center of the disk; o_2 and o_3 are centers of the left and right bearings. Denote by (x_1, y_1), (x_2, y_2), and (x_3, y_3) the displacements of the disk center, the left bearing, and the right journal bearing, respectively. The equation of motion of the system is expressed as follows:

$$\begin{gathered}
m_1\ddot{x}_1 + D_e(\dot{x}_1 - \dot{x}_2) + D_e(\dot{x}_1 - \dot{x}_3) + K_{e1}(x_1 - x_2) + K_{e2}(x_1 - x_3) = F_x + m_1 e\omega^2 \cos \omega t, \\
m_1\ddot{y}_1 + D_e(\dot{y}_1 - \dot{y}_2) + D_e(\dot{y}_1 - \dot{y}_3) + K_{e1}(y_1 - y_2) + K_{e2}(y_1 - y_3) = F_y - m_1 g + m_1 e\omega^2 \sin \omega t, \\
m_2\ddot{x}_2 + D_e(\dot{x}_2 - \dot{x}_1) + K_{e1}(x_2 - x_1) = f_{x2}, \\
m_2\ddot{y}_2 + D_e(\dot{y}_2 - \dot{y}_1) + K_{e1}(y_2 - y_1) = f_{y2} - m_2 g, \\
m_3\ddot{x}_3 + D_e(\dot{x}_3 - \dot{x}_1) + K_{e2}(x_3 - x_1) = f_{x3}, \\
m_3\ddot{y}_3 + D_e(\dot{y}_3 - \dot{y}_1) + K_{e2}(y_3 - y_1) = f_{y3} - m_3 g,
\end{gathered} \tag{2.1}$$

where m_1 is the mass of the disk; m_2 and m_3 are masses of the left and the right bearings. K_{e1} and K_{e2} are equivalent stiffness coefficients of the left and the right shafts; D_e is the factor of viscous damping; e is the mass unbalance of the disk; F_x and F_y are directional components of the seal force; $f_{x2,x3}$ and $f_{y2,y3}$ are directional force components of the left and the right bearings, respectively. ω is the rotation speed and g is the gravitational acceleration. The symmetry of the fluid field inside the seal clearance is destroyed as the rotor is perturbed from its equilibrium position with a nonzero rotation speed. Muszynska's model [13, 14] is used to express the seal forces in both x- and y-directions, as

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = -\begin{bmatrix} K - m_f\tau^2\omega^2 & \tau\omega D \\ -\tau\omega D & K - m_f\tau^2\omega^2 \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \end{Bmatrix} - \begin{bmatrix} D & 2\tau m_f\omega \\ -2\tau m_f\omega & D \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{Bmatrix} - \begin{bmatrix} m_f & 0 \\ 0 & m_f \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \end{Bmatrix}, \tag{2.2}$$

where K and D are coefficients of stiffness and damping of the air that flows through the seal clearance, respectively; m_f is the effective mass of the air; τ is the factor of average angular speed of fluid that rotates along with the rotor, determined by

$$\tau = \tau_0(1-\varepsilon)^b, \tag{2.3}$$

where τ_0 is the average angular speed for the unperturbed rotor; b is an empirical coefficient; $\varepsilon = \sqrt{x_{2,3}^3 + y_{2,3}^2}$ is the nondimensional amplitude of whirling motion of the bearings. The model of the bearing force adopted in the current study is the one proposed by Zhang et al. [18, 19] for unsteady oil-film journal bearings, expressed as follows:

$$f_x = -C_1\dot{\varepsilon} - C_2\left(\dot{\varphi} - \frac{\omega}{2}\right)\varepsilon, \qquad f_y = -C_2\dot{\varepsilon} - C_3\left(\dot{\varphi} - \frac{\omega}{2}\right)\varepsilon, \tag{2.4}$$

where $\dot{\varphi}$ is the whirling speed of the journal; C_1, C_2, and C_3 are damping coefficients of the lubricant [20]. Unlike most existent bearing theories that handle time-invariant boundaries of the lubricant film with, for example, the Gümbel condition and the π-oil-film assumption, the unsteady force model of (2.4) is capable of dealing with time-varying boundary of the film arising from large whirling velocity of the journal center, which is appropriate for weakly viscous systems with air- or water-film bearings such as the present one. Introducing the following nondimensional parameters:

$$\begin{gathered}
T = \omega t, \qquad X_1 = \frac{x_1}{c}, \qquad Y_1 = \frac{y_1}{c}, \\
X_{2,3} = \frac{x_{2,3}}{\delta}, \qquad Y_{2,3} = \frac{y_{2,3}}{\delta}, \\
X_1' = \frac{\dot{x}_1}{(\omega c)}, \qquad Y_1' = \frac{\dot{y}_1}{(\omega c)}, \\
X_{2,3}' = \frac{\dot{x}_{2,3}}{(\omega \delta)}, \qquad Y_{2,3}' = \frac{\dot{y}_{2,3}}{(\omega \delta)},
\end{gathered} \tag{2.5}$$

where $(\cdot)' \triangleq d(\cdot)/dT$ denotes the derivative of a quantity with respect to T, and c and δ are clearances of the seal and the journal bearings, respectively, the equation of motion is then rewritten as

$$\begin{aligned}
X_1'' &+ \frac{2D_e + D}{(m_1+m_f)\omega}X_1' + \frac{2\tau m_f}{m_1+m_f}Y_1' - \frac{D_e\delta}{(m_1+m_f)\omega c}(X_2' + X_3') + \frac{K_{e1}+K_{e2}+K-m_f\tau^2\omega^2}{(m_1+m_f)\omega^2}X_1 \\
&+ \frac{\tau D}{(m_1+m_f)\omega}Y_1 - \frac{K_{e1}\delta}{(m_1+m_f)\omega^2 c}X_2 - \frac{K_{e2}\delta}{(m_1+m_f)\omega^2 c}X_3 = \frac{m_1 e}{(m_1+m_f)c}\cos T,
\end{aligned}$$

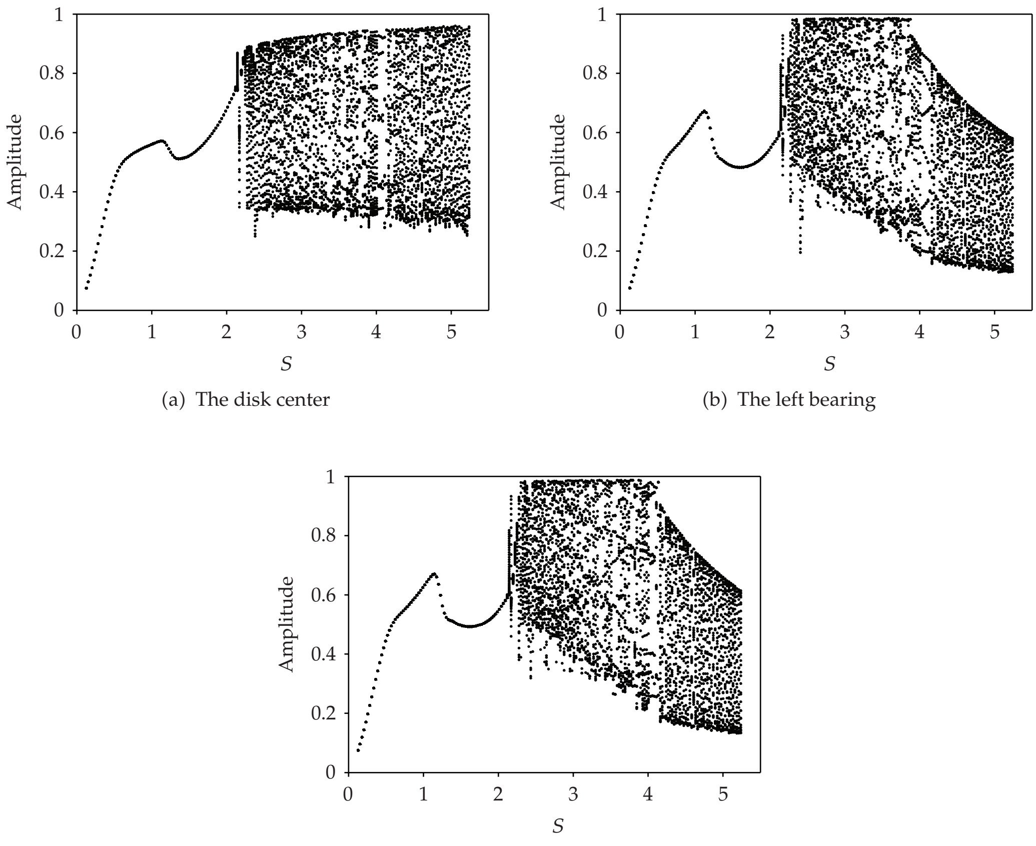
$$
\begin{aligned}
&Y_1'' - \frac{2\tau m_f}{m_1+m_f}X_1' + \frac{2D_e+D}{(m_1+m_f)\omega}Y_1' - \frac{D_e\delta}{(m_1+m_f)\omega c}\left(Y_2'+Y_3'\right) - \frac{\tau D}{(m_1+m_f)\omega}X_1 \\
&\quad + \frac{K_{e1}+K_{e2}+K-m_f\tau^2\omega^2}{(m_1+m_f)\omega^2}Y_1 - \frac{K_{e1}\delta}{(m_1+m_f)\omega^2 c}Y_2 \\
&\quad - \frac{K_{e2}\delta}{(m_1+m_f)\omega^2 c}Y_3 = \frac{m_1 e}{(m_1+m_f)c}\sin T - \frac{m_1 g}{(m_1+m_f)\omega^2 c}, \\
&X_2'' - \frac{D_e c}{m_2\omega\delta}X_1' + \frac{D_e\omega\delta+S_0C_{11L}}{m_2\omega^2\delta}X_2' + \frac{S_0C_{12L}}{m_2\omega^2\delta}Y_2' - \frac{K_{e1}c}{m_2\omega^2\delta}X_1 \\
&\quad + \frac{2K_{e1}\delta - C_{2L}S_0}{2m_2\omega^2\delta}X_2 + \frac{C_{3L}S_0}{2m_2\omega^2\delta}Y_2 = 0, \\
&Y_2'' - \frac{D_e c}{m_2\omega\delta}Y_1' + \frac{S_0C_{12L}}{m_2\omega^2\delta}X_2' + \frac{D_e\omega\delta+S_0C_{22L}}{m_2\omega^2\delta}Y_2' - \frac{K_{e1}c}{m_2\omega^2\delta}Y_1 \\
&\quad + \frac{2K_{e1}\delta - C_{2L}S_0}{2m_2\omega^2\delta}Y_2 - \frac{C_{3L}S_0}{2m_2\omega^2\delta}X_2 = -\frac{g}{\omega^2\delta}, \\
&X_3'' - \frac{D_e c}{m_3\omega\delta}X_1' + \frac{D_e\omega\delta+S_0C_{11R}}{m_3\omega^2\delta}X_3' + \frac{S_0C_{12R}}{m_3\omega^2\delta}Y_3' - \frac{K_{e2}c}{m_3\omega^2\delta}X_1 \\
&\quad + \frac{2K_{e2}\delta - S_0C_{2R}}{2m_3\omega^2\delta}X_3 + \frac{S_0C_{3R}}{2m_3\omega^2\delta}Y_3 = 0, \\
&Y_3'' - \frac{D_e c}{m_3\omega\delta}Y_1' + \frac{S_0C_{12R}}{m_3\omega^2\delta}X_3' + \frac{D_e\omega\delta+S_0C_{22R}}{m_3\omega^2\delta}Y_3' - \frac{K_{e2}c}{m_3\omega^2\delta}Y_1 \\
&\quad - \frac{S_0C_{3R}}{2m_3\omega^2\delta}X_3 + \frac{2K_{e2}\delta - S_0C_{2R}}{2m_3\omega^2\delta}Y_3 = -\frac{g}{\omega^2\delta},
\end{aligned}
\tag{2.6}
$$

where l and r are length and radius of the bearing, respectively; μ is the dynamic viscosity of the lubricant; superscripts L and R represent the left and the right bearings, respectively, and

$$
\begin{aligned}
C_{11} &= C_1\cos^2\varphi + C_3\sin^2\varphi - 2C_2\sin\varphi\cos\varphi, \\
C_{12} &= C_{21} = C_2\left(\cos^2\varphi - \sin^2\varphi\right) + (C_1 - C_3)\sin\varphi\cos\varphi, \\
C_{22} &= C_1\sin^2\varphi + C_3\cos^2\varphi + 2C_2\sin\varphi\cos\varphi, \\
S_0 &= 6\mu\omega l r^3\delta^{-2}.
\end{aligned}
\tag{2.7}
$$

3. Subharmonic Motions and Bifurcation Behavior

Notice that parameters K, D, and τ and coefficients C_1, C_2, and C_3 are functions of displacements of the disk centers and the bearings. Hence, (2.6) is a group of highly nonlinear



(a) The disk center

(b) The left bearing

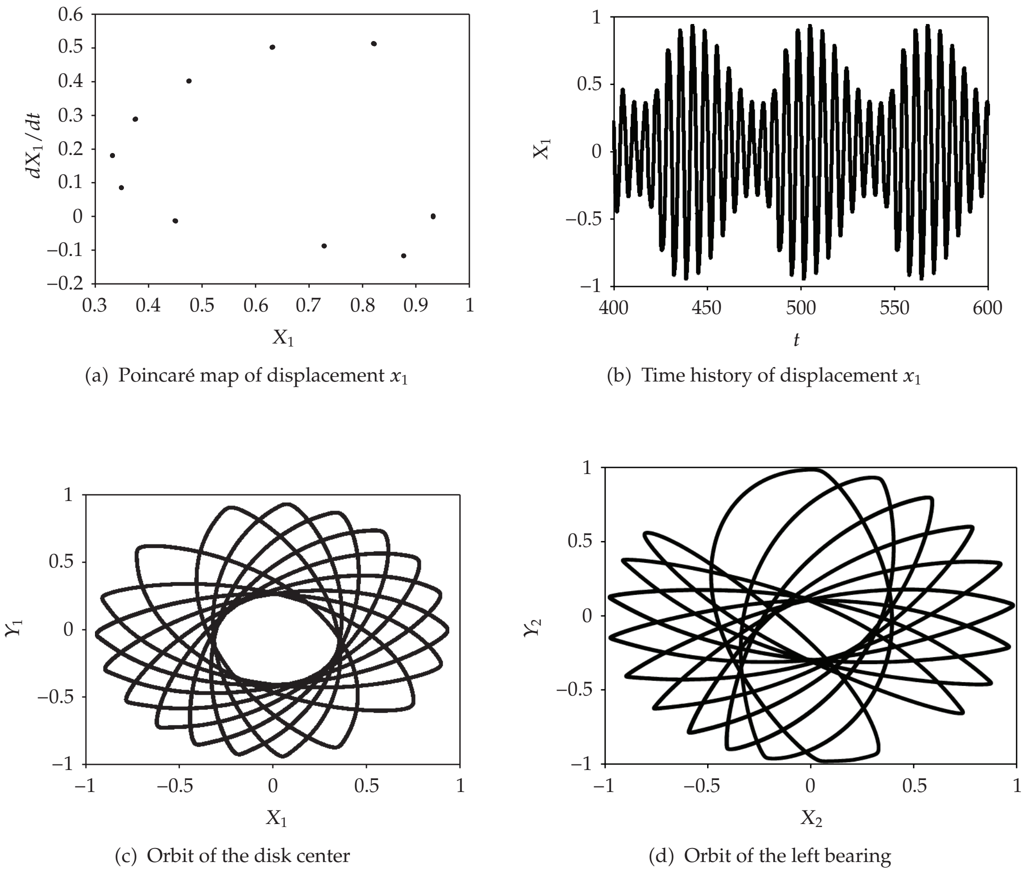
(c) The right bearing

Figure 2: Bifurcation diagrams of the rotor system with seal and air-film excitations. $\Delta P = 0.2\,\mathrm{MPa}$.

ordinary differential equations that can hardly be solved through conventional perturbation methods [21]. Instead, the vibration responses of the disk center and the two bearings are computed by using the fourth-order Runge-Kutta method with adaptive-step control to reduce local truncation error of every single step. The parameters selected for the current study are

$$
\begin{gathered}
m_1 = 50\,\mathrm{kg}, \qquad m_2 = 3.5\,\mathrm{kg}, \qquad m_3 = 3.5\,\mathrm{kg}, \qquad D_e = 3000\,\mathrm{N \cdot s/m}, \\
K_{e1} = 3.4635 \times 10^6\,\mathrm{N/m}, \qquad K_{e2} = 3.8127 \times 10^6\,\mathrm{N/m}, \\
e_1 = 0.2\,\mathrm{mm}, \qquad r = 0.035\,\mathrm{m}, \qquad l = 0.06\,\mathrm{m}, \qquad c = 0.3\,\mathrm{mm}, \qquad \delta = 0.3\,\mathrm{mm}, \\
\mu = 1.47 \times 10^{-5}\,\mathrm{Pa \cdot s}, \qquad \tau_0 = 0.4, \qquad b = 0.45.
\end{gathered}
\tag{3.1}
$$

Additionally, the length and radius of the seal are 0.102 m and 0.067 m, respectively. The system's parameters are chosen based on a single-staged centrifugal compressor manufactured by Shenyang Turbo-machinery Cooperation. To investigate the bifurcation we chose the rotation speed as the parameter under two pressure drops of the seal, that is, the



(a) Poincaré map of displacement x_1

(b) Time history of displacement x_1

(c) Orbit of the disk center

(d) Orbit of the left bearing

Figure 3: Motions of the rotor system with rotation speed $S = 3.355$.

pressure differences between the entrance and the exit of the seal. The initial displacements and velocities of the disk center and the two are $(0.01, 0)$.

Let pressure drop ΔP be 0.2 MPa. The bifurcation diagrams of displacement x are illustrated in Figure 2, where $S = \omega/\sqrt{(K_{e1} + K_{e2})/m_1}$ is the nondimensional rotation speed. In the current computation $S = 1$ corresponds to a rotation speed of 60.71 Hz or 3642.77 rpm.

It can be seen that the disk and the bearings are in motions of period-1, that is, motions with the same frequency as the rotation speed, when the rotation is slow. The primary resonance happens at $S = 1.1272$. The stability of the period-1 motions is lost at $S = 2.1496$, and the motion becomes quasiperiodic. Various subharmonic motions can be observed when the rotation speed is increased. A period-12 bifurcation takes place at $S = 3.1457$. Following that, the motions become quasiperiodic again with escalating rotation speed. At $S = 3.2505$ the displacements undergo a period-11 bifurcation and return quasiperiodic with higher S afterwards. A period-10 bifurcation is encountered with speed $S = 3.355$. The Poincaré map of displacement x_1 is presented in Figure 3(a) to show the existence of a periodic-10 motion. The time history of x_1 is illustrated in Figure 3(b), and the orbits of the disk center and the left bearing are shown in Figures 3(c) and 3(d), respectively. Further, a period-9 bifurcation is observed at $S = 3.5389$ followed by a period-8 bifurcation at $S = 3.8010$. The bifurcation cascade continues at $S = 4.0632$ when a period-7 bifurcation takes place. Following that,

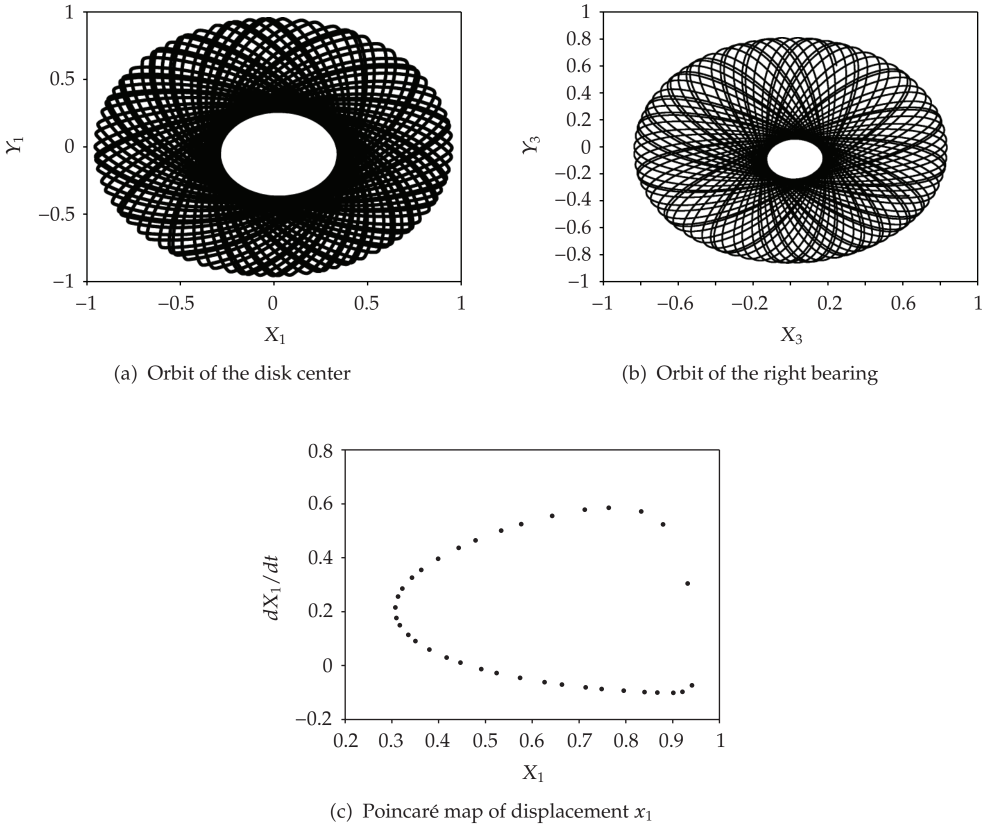


Figure 4: Motions of the rotor system with rotation speed $S = 4.352$.

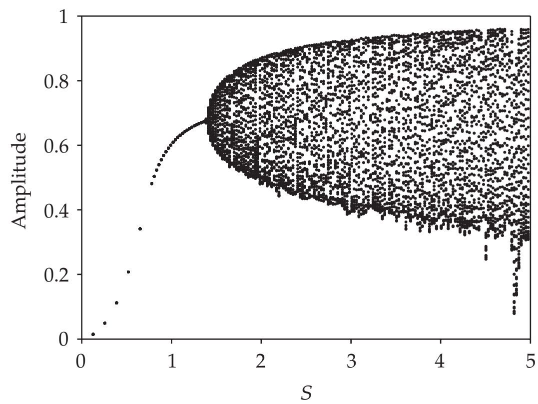
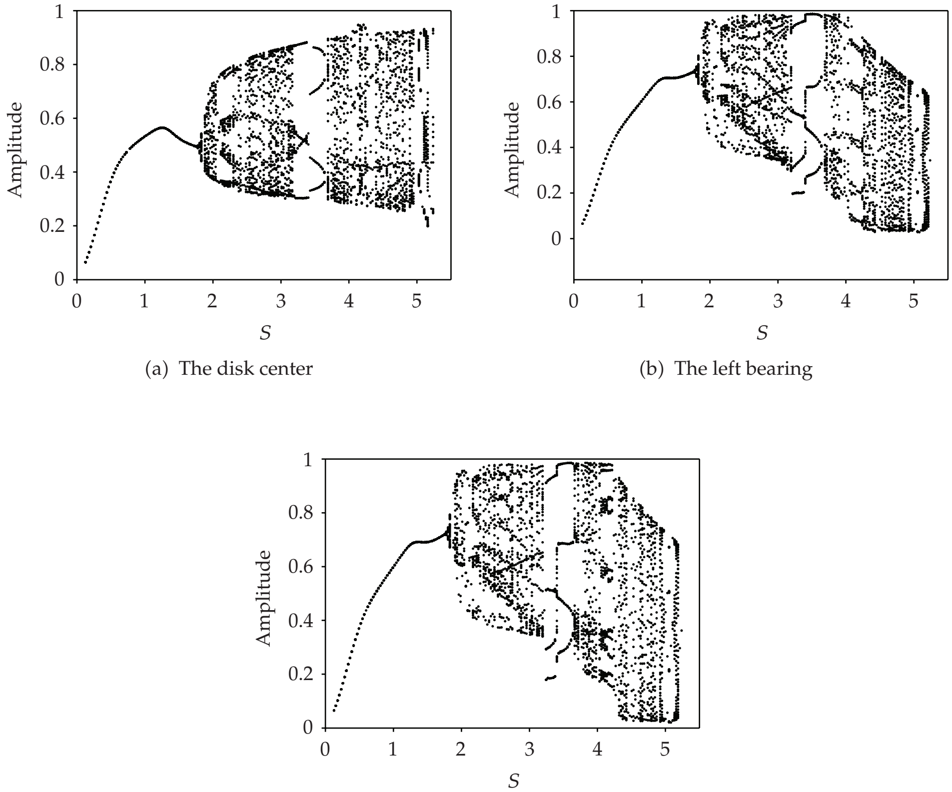


Figure 5: Bifurcation diagram of the disk center with rigid supports.

quasiperiodic motions are obtained with higher rotation speed. Figures 4(a), 4(b), and 4(c) depict the orbits of the disk center and the right bearing as well as the Poincaré map of displacement x_1 at $S = 4.352$, respectively.

To investigate the interaction between the bearing and the seal forces a comparative computation is carried out for a Jeffcott rotor with two rigid supports (hence, the seal force



(a) The disk center

(b) The left bearing

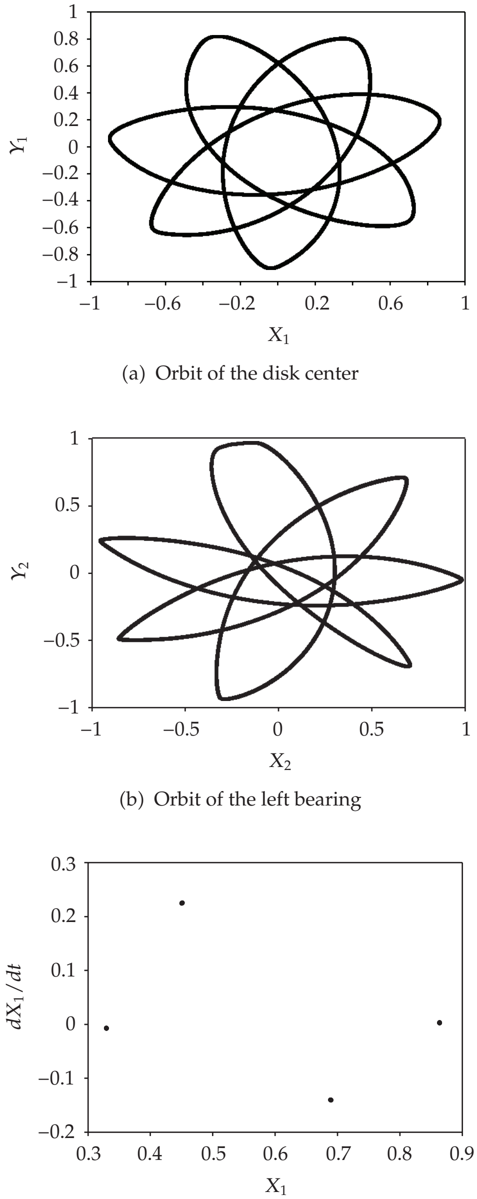
(c) The right bearing

Figure 6: Bifurcation diagrams of the rotor system with seal and air-film excitations. $\Delta P = 0.4\,\text{MPa}$.

is the only excitation of the system) and exactly the same geometrical and seal properties as aforementioned. The bifurcation diagram is shown in Figure 5. For rotation speed less than $S = 1.432$, the motion is period-1 with the same frequency as the rotation speed. With an increasing speed, the motion remains quasiperiodic up to $S = 3.52$, where a period-8 bifurcation is observed from the disk's displacements. The motions turns into quasiperiodic again with advancing rotation speed. The comparison between the responses to the coupling forces and to the seal force alone reveals rich bifurcating behavior of the system vibration: the interaction of the seal and the air-film forces results in more period-multiple bifurcations (see Figures 2(a) and 5).

We now change the pressure drop of the seal to 0.4 MPa. The bifurcation diagrams of displacement x of the disk and the two journal bearings are presented in Figure 6.

It is found that the x-directional displacements of the disk and the bearings are period-1 with small rotation speed. The primary resonance in the motion is found at $S = 1.2582$. Then, the bifurcation starts and the motions become quasiperiodic. A period-4 bifurcation takes place with speed $S = 2.0709$ followed by quasiperiodic motions as the rotor is accelerated. For speed $S \in [3.2243, 3.3816] \cup [3.4340, 3.6437]$ the motions are period-4. Figures 7(a) and 7(b) show the orbits of the disk center and the left bearing at $S = 3.4340$. Figure 7(c) depicts the Poincaré map of the disk motion. The motions become quasiperiodic with higher rotation speed. Figures 8(a) and 8(b) plot the orbits of the disk



(a) Orbit of the disk center

(b) Orbit of the left bearing

(c) Poincaré map of displacement x_1

Figure 7: Motions of the rotor system with rotation speed $S = 3.434$.

center and the right bearing at $S = 3.8797$. The Poincaré maps of displacements x_1 and x_3 are shown in Figures 8(c) and 8(d), respectively. A period-11 bifurcation is observed at $S = 4.2204$ followed by another series of quasiperiodic motions. With higher pressure-drop from the entrance to the exit of the seal, some previously notified bifurcations are not observed again. Nevertheless, the bifurcation behavior is still more complicated than the one with the seal force only.

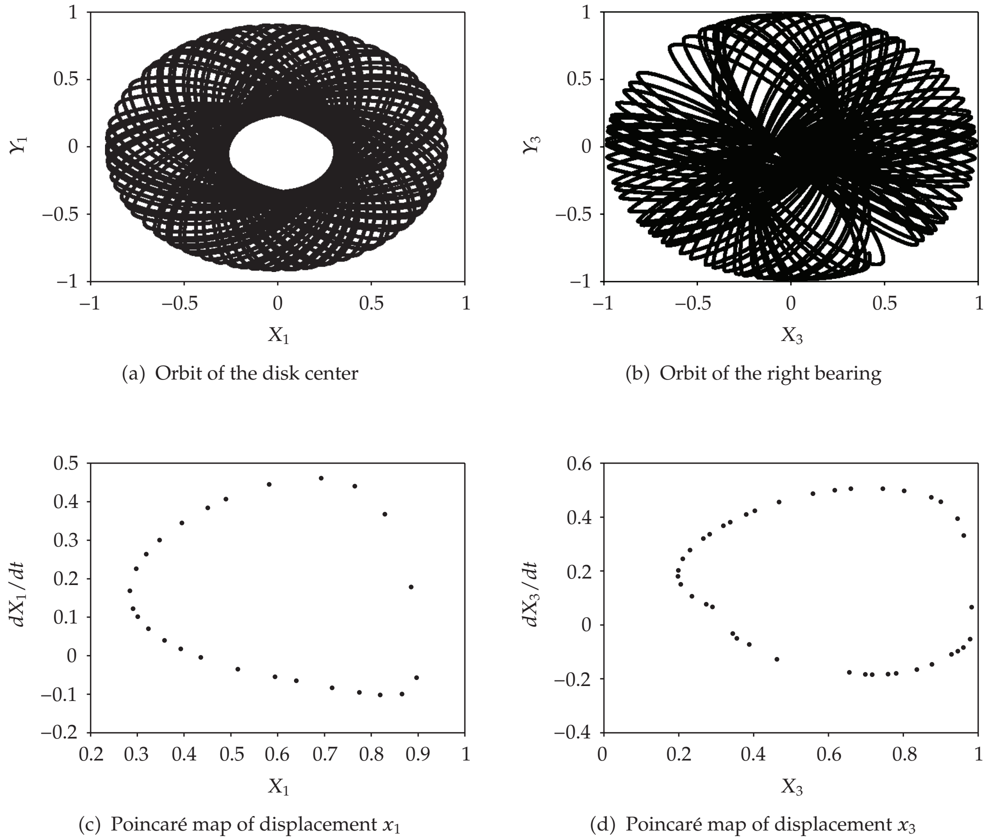
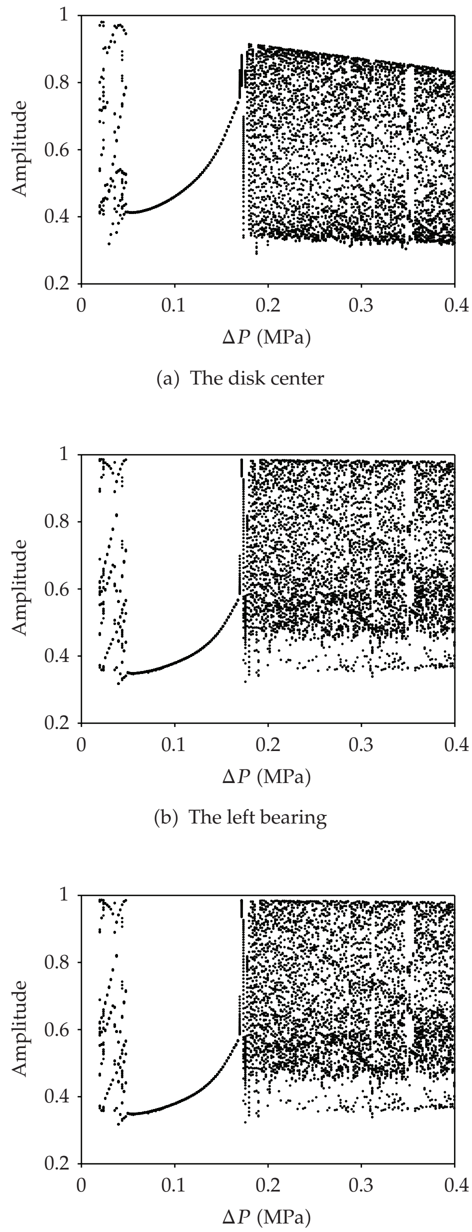


Figure 8: Motions of the rotor with rotation speed $S = 3.8797$.

In the following analysis we adopt the pressure drop as the bifurcation parameter. Let rotation speed ω be 1200 rad/s. The bifurcation diagrams of x-displacements of the disk center and the left and the right bearings are presented in Figure 9. For low-pressure drops the motions are found quasiperiodic with large amplitude until $\triangle P$ up to 0.048 MPa. The motions of the disk center and the bearings then become period-1, and the amplitudes step up with the advancing pressure drop. The synchronous motions are lost at a critical drop $\Delta P =$ 0.168 MPa without undergoing primary resonances in the motions. The vibrations afterwards are basically quasiperiodic, and it is very difficult to distinguish the bifurcation points. The average amplitudes of the displacements remain almost unchanged with increasing pressure drops, showing the remarkable air-film whip in the journal bearings. This implies that the whole system cannot be stabilized by increasing the pressure drops larger than the critical value.

Finally, the evolution of the bifurcation in the rotor motions is investigated by taking the length of the seal as the control parameter. Let rotation speed ω be 1200 rad/s and let pressure drop ΔP be 0.2 MPa. The bifurcation diagrams of x-displacements of the disk center and the left and the right bearings are depicted in Figure 10. The period-1 motion is found for length: 0.082 m $\leq l \leq$ 0.098 m, where the orbital whirling motions grow monotonously with the increasing seal-length. Beyond this range of length, the motions are mainly quasiperiodic



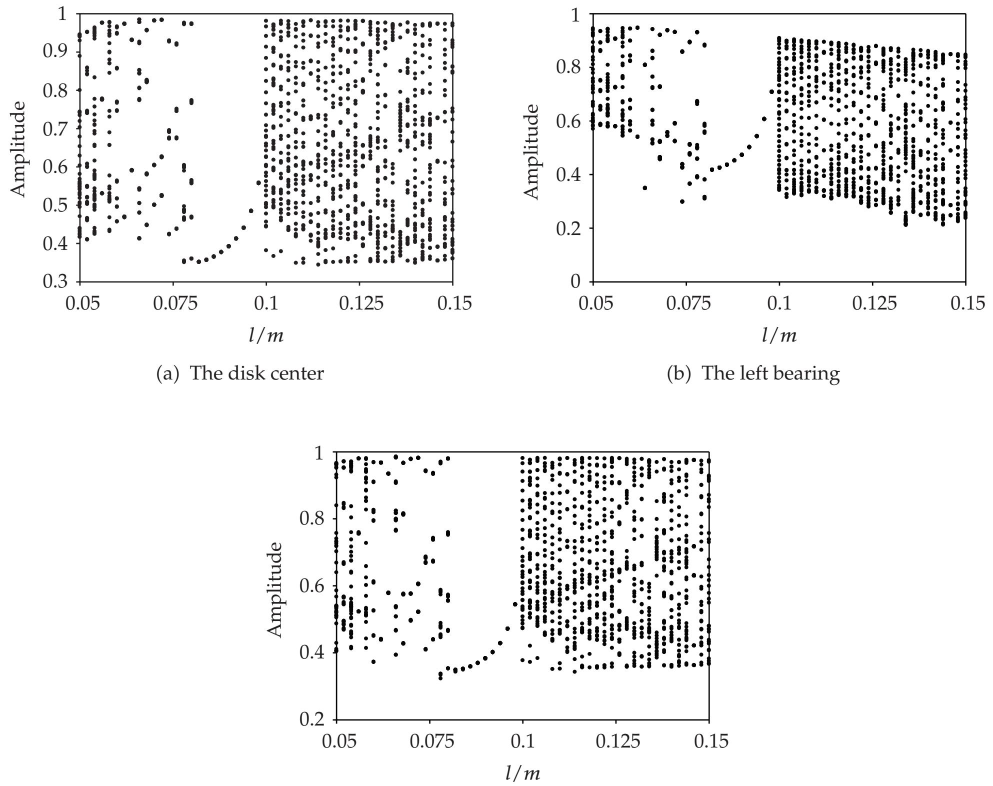
(a) The disk center

(b) The left bearing

(c) The right bearing

Figure 9: Bifurcation diagram of the rotor with varying pressure drop. $\omega = 1200\,\text{rad/s}$.

with considerably large amplitudes. Therefore, a suitable length of seal should be chosen between 0.082 m and 0.098 m to keep the rotor distant from strong vibration responses that may jeopardize the safety of the machine in operation. From the manufacturer's point of view, a labyrinth seal with a medium length of between 0.082 m and 0.098 m is feasible for it can be conveniently processed, assembled, and positioned by using conventional tools.



(a) The disk center

(b) The left bearing

(c) The right bearing

Figure 10: Bifurcation diagram of the rotor with varying seal-length. $\omega = 1200\,\mathrm{rad/s}$; $\Delta P = 0.2\,\mathrm{MPa}$.

4. Conclusions

The nonlinear coupling vibration excited by a labyrinth seal and two air-film journal bearing is investigated through numerical simulations for high-speed centrifugal compressors. The results obtained with various rotation speeds and seal pressure drops show complexity of nonlinear vibration and bifurcation behavior in the displacements of the rotor system. Further, the motions of the system reveal period-multiple bifurcations compared to the system excited only by the seal force, presenting an intricate interaction between the seal and the bearing forces. Suitable seal pressure drop and seal length are determined for the sake of operation safety through the bifurcation analysis for rotor displacements as well.

Acknowledgments

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References

[1] B. L. Newkirk and H. D. Taylor, “Shaft whipping due to oil action in journal bearing,” *General Electric Review*, vol. 28, pp. 559–568, 1925.
[2] J. W. Lund, “Spring and damping coefficients for the tilting pad journal bearing,” *Transactions of the ASAE*, vol. 7, pp. 342–352, 1964.
[3] D. W. Childs, *Turbomachinery Rotordynamics: Phenomena, Modeling, and Analysis*, John Wiley & Sons, New York, NY, USA, 1993.
[4] E. Kramer, *Dynamics of Rotors and Foundations*, Springer, Berlin, Germany, 1993.
[5] L. San Andres, “Dynamic force and moment coefficients for short length annular seals,” *Journal of Tribology*, vol. 115, no. 1, pp. 61–70, 1993.
[6] K. Kwanka, “Dynamic coefficients of stepped labyrinth gas seals,” *Journal of Engineering for Gas Turbines and Power*, vol. 122, no. 3, pp. 473–477, 2000.
[7] J. K. Scharrer and D. W. Childs, “Theory versus experiment for the rotordynamic coefficients of labyrinth gas seals—part I: a two control volume model,” *Journal of Vibration, Acoustics, Stress, and Reliability in Design*, vol. 110, no. 3, pp. 270–280, 1988.
[8] J. K. Scharrer and D. W. Childs, “Theory versus experiment for the rotordynamic coefficients of labyrinth gas seals—parts II: a comparison to experiment,” *Journal of Vibration, Acoustics, Stress, and Reliability in Design*, vol. 110, no. 3, pp. 281–287, 1988.
[9] O. R. Marquette and D. W. Childs, “An extended three-control-volume theory for circumferentially-grooved liquid seals,” *Journal of Tribology*, vol. 118, no. 2, pp. 276–285, 1996.
[10] F. F. Ehrich, “Subharmonic vibration of rotors in bearing clearance,” ASME Paper no. 66-MD-1, 1966.
[11] F. F. Ehrich, “High order subharmonic response of high speed rotor in bearing clearance,” *Journal of Vibration, Acoustics, Stress, and Reliability in Design*, vol. 110, pp. 695–702, 1988.
[12] D. E. Bently, “Forced subrotative speed dynamic action of rotating machinery,” ASME Paper no. 74-PET-16, 1994.
[13] A. Muszynska, “Whirl and whip-rotor/bearing stability problems,” *Journal of Sound and Vibration*, vol. 110, no. 3, pp. 443–462, 1986.
[14] A. Muszynska and D. E. Bently, “Frequency-swept rotating input perturbation techniques and identification of the fluid force models in rotor/bearing/seal systems and fluid handling machines,” *Journal of Sound and Vibration*, vol. 143, no. 1, pp. 103–124, 1990.
[15] Q. Ding, J. E. Cooper, and A. Y. T. Leung, “Hopf bifurcation analysis of a rotor/seal system,” *Journal of Sound and Vibration*, vol. 252, no. 5, pp. 817–833, 2002.
[16] J. Hua, S. Swaddiwudhipong, Z. S. Liu, and Q. Y. Xu, “Numerical analysis of nonlinear rotor-seal system,” *Journal of Sound and Vibration*, vol. 283, no. 3–5, pp. 525–542, 2005.
[17] Y.-H. Zhang, J. Hua, Q.-Y. Xu, and X.-L. Zhang, “A new high precision direct integration scheme for nonlinear rotor-seal system,” *Chinese Journal of Computational Mechanics*, vol. 22, no. 5, pp. 541–545, 2005 (Chinese).
[18] W. Zhang, H. S. Zhang, and X. F. Xu, “Study of general nonlinear formula of oil-film force acting on a journal with unsteady motion,” in *Proceedings of the Asia-Pacific Vibration Conference*, Kyungju, Korea, 1997.
[19] W. Zhang and X. Xu, “Modeling of nonlinear oil-film force acting on a journal with unsteady motion and nonlinear instability analysis under the model,” *International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 1, no. 3, pp. 179–186, 2000.
[20] D. W. Childs, “Dynamic analysis of turbulent annular seals based on Hirs’ lubrication equation,” *Journal of Lubrication Technology*, vol. 105, no. 3, pp. 429–436, 1983.
[21] J.-H. He, “A review on some new recently developed nonlinear analytical techniques,” *International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 1, no. 1, pp. 51–70, 2000.