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Research Article

Special Approach to Near Set Theory

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The aim of this paper is to introduce two approaches to near sets by using a special neighbourhood. Some fundamental properties and characterizations are given. We obtain a comparison between these new set approximations as well as set approximations introduced by Peters (2011, 2009, 2007, 2006).

1. Introduction

Rough set theory, proposed by Pawlak in 1982 [1, 2], can be seen as a new mathematical approach to vagueness. The rough set philosophy is founded on the assumption that with every object of the universe of discourse we associate some information (data, knowledge). For example, if objects are patients suffering from a certain disease, symptoms of the disease form information about patients. Objects characterized by the same information are indiscernible (similar) in view of the available information about them. The indiscernibility relation generated in this way is the mathematical basis of rough set theory. This understanding of indiscernibility is related to the idea of Gottfried Wilhelm Leibniz that objects are indiscernible if and only if all available functionals take on identical values (Leibniz's Law of Indiscernibility: The Identity of Indiscernibles) [3]. However, in the rough set approach, indiscernibility is defined relative to a given set of partial functions (attributes).

Any set of all indiscernible (similar) objects is called an elementary set and forms a basic granule (atom) of knowledge about the universe. Any union of some elementary sets is referred to as a crisp (precise) set. A set which is not crisp is called rough (imprecise, vague) set.

Consequently, each rough set has boundary region cases, that is, objects that cannot with certainty be classified either as members of the set or of its complement. Obviously, crisp sets have no boundary region elements at all. This means that boundary region cases cannot be properly classified by employing available knowledge.

Thus, the assumption that objects can be seen only through the information available about them leads to the view that knowledge has a granular structure. Due to the granularity of knowledge, some objects of interest cannot be discerned and appeared as the same (identical or similar). Consequently, vague concepts, in contrast to precise concepts, cannot be characterized in terms of information about their elements.

Ultimately, there is interest in selecting probe functions [4] that lead to descriptions of objects that are minimally near each other. This is an essential idea in the near set approach [5–7] and differs markedly from the minimum description length (MDL) proposed in 1983 by Jorma Rissanen. MDL depends on the identification of possible data models and possible probability models. By contrast, NDP deals with a set X that is the domain of a description used to identify similar objects. The term similar is used here to denote the presence of objects that have descriptions that match each other to some degree.

The near set approach leads to partitions of ensembles of sample objects with measurable information content and an approach to feature selection. The proposed feature selection method considers combinations of n probe functions taken r at a time in searching for those combinations of probe functions that lead to partitions of a set of objects that has the highest information content.

In this paper, we assume that any vague concept is replaced by a pair of precise concepts, called the lower and the upper approximations of the vague concept. The lower approximation consists of all objects which surely belong to the concept, and the upper approximation contains all objects which possibly belong to the concept. The difference between the upper and the lower approximation constitutes the boundary region of the vague concept. These approximations are two basic operations in rough set theory. There is a chance to be useful in the analysis of sample data. The proposed approach does not depend on the joint probability of finding a feature value for input vectors that belong to the same class. In addition, the proposed approach to measuring the information content of families of neighborhoods differs from the rough set approach. The near set approach does not depend on preferential ordering of value sets of functions representing object features. The contribution of this research is the introduction of a generalization of the near set approach to feature selection.

2. Preliminaries

Rough set theory expresses vagueness, not by means of membership, but by employing a boundary region of a set. If the boundary region of a set is empty, it means that the set is crisp, otherwise the set is rough (inexact). The nonempty boundary region of a set means that our knowledge about the set is not sufficient to define the set precisely.

Suppose we are given a set of objects U called the universe and an indiscernibility relation $E \subseteq U \times U$, representing our lack of knowledge about elements of U. For the sake of simplicity, we assume that E is an equivalence relation and X is a subset of U. We want to characterize the set X with respect to E. To this end we will need the basic concepts of rough set theory given below [2].

The equivalence class of E determined by element x is $[x]_E(x) = \{y \in X : E(x) = E(x')\}$. Hence E-lower, upper approximations and boundary region of a subset $\mathrm{X} \subseteq U$ are

$$\begin{aligned}\underline{E}(\mathrm{X}) &= \cup\{[x]_E : [x]_E \subseteq \mathrm{X}\};\\ \overline{E}(\mathrm{X}) &= \cup\{[x]_E : [x]_E \cap \mathrm{X} \neq \phi\};\\ \mathrm{BND}_E(\mathrm{X}) &= \overline{E}(\mathrm{X}) - \underline{E}(\mathrm{X}).\end{aligned} \tag{2.1}$$

It is easily seen that approximations are in fact interior and closure operations in a topology generated by the indiscernibility relation [8].

The rough membership function $\mu_X^E(x)$ is a measure of the degree that x belongs to X in view of information expressed by E. It is defined as [9]

$$\mu_X^E(x) : U \longrightarrow (0,1), \qquad \mu_X^E(x) = \frac{|X \cap [x]_E|}{|[x]_E|}, \tag{2.2}$$

where $|*|$ denotes the cardinality of $*$.

A rough set can also be characterized numerically by the accuracy measure of an approximation [1] that is defined as

$$\alpha_E(X) = \frac{\left|\underline{E}(X)\right|}{\left|\overline{E}(X)\right|}. \tag{2.3}$$

Obviously, $0 \leq \alpha_E(X) \leq 1$. If $\alpha_E(X) = 1$, X is crisp with respect to E (X is precise with respect to E), and otherwise, if $\alpha_E(X) < 1$, X is rough with respect to E (X is vague with respect to E).

Underlying the study of near set theory is an interest in classifying sample objects by means of probe functions associated with object features. More recently, the term feature is defined as the form, fashion, or shape (of an object).

Let F denote a set of features for objects in a set X. For any feature $a \in F$, we associate a function f_a that maps X to some set V_{f_a} (range of f_a).

The value of $f_a(x)$ is a measurement associated with feature a of an object $x \in X$. The function f_a is called a probe function [4, 10].

The following concepts were introduced by Peters in [5–7].

$\mathrm{GAS} = (U, F, N_r, \nu_B)$ is a generalized approximation space, where U is a universe of objects, F is a set of functions representing object features, N_r is a neighbourhood family function defined as

$$N_r(F) = \bigcup_{A \subseteq P_r(F)} [x]_A, \quad \text{where } P_r\ (F) = \{A \subseteq F : |A| = r,\ 1 \leq r \leq |F|\}, \tag{2.4}$$

and ν_{B_r} is an overlap function defined by

$$\nu_{B_r} : P(U) \times P(U) \longrightarrow [0,1], \qquad \nu_{B_r}(Y, N_r(B)_* X) = \frac{|Y \cap N_r(B)_* X|}{|N_r(B)_* X|}, \tag{2.5}$$

where $N_r(B)_* X \neq \phi$, Y is a member of the family of neighbourhoods $N_r(B)$ and $\nu_{B_r}(Y, N_r(B)_* X)$ is equal to 1, if $N_r(B)_* X = \phi$.

The overlap function ν_{B_r} maps a pair of sets to a number in $[0,1]$, representing the degree of overlap between the sets of objects with features B_r.

$N_r(B)$-lower, upper approximations and boundary region of a set X with respect to r features from the probe functions B are defined as

$$N_r(B)_*X = \bigcup_{x:[x]_{B_r}\subseteq X} [x]_{B_r};$$
$$N_r(B)^*X = \bigcup_{x:[x]_{B_r}\cap X\neq\phi} [x]_{B_r}; \tag{2.6}$$
$$\mathrm{BND}_{N_r(B)}X = N_r(B)^*X - N_r(B)_*X.$$

Peters introduces the following meanings [5, 6].

Objects x and x' are minimally near each other if $\exists f \in B$ such that $f(x') = f(x)$. Set X to be near to X' if $\exists x \in X$, $x' \in X'$ such that x and x' are near objects. A set X is termed a near set relative to a chosen family of neighborhoods $N_r(B)$ if $|\mathrm{BND}_{N_r(B)}X| \geq 0$.

3. Approach to Near Set Theory

We aim in this section to introduce a generalized approach to near sets by using new neighbourhoods. Deduce a modification of some concepts.

Definition 3.1. Let $B \subseteq F$ be probe functions on a nonempty set X, $\phi_i \in B$. A general neighbourhood of an element $x \in X$ is

$$(x)_{\phi_i,r} = \{y \in X : |\phi_i(y) - \phi_i(x)| < r\}, \tag{3.1}$$

where $|*|$ is the absolute value of $*$ and r is the length of a neighbourhood with respect to the feature ϕ_i.

Remark 3.2. We will replace the equivalence class in the approximations of near set theory defined by Peters [5, 6] by the general neighbourhood defined in Definition 3.1.

Definition 3.3. Let $\phi_i \in B$ be a general relation on a nonempty set X. Hence, we can deduce a special neighbourhood of an element $x \in X$ as

$$x_{[\phi_i]} = \bigcap_{\phi_i\in B} \left\{(y)_{\phi_i,r} : x \in (y)_{\phi_i,r}\right\}. \tag{3.2}$$

Remark 3.4. Let $\phi_i \in B$ be a general relation on a nonempty set X, where $1 \leq i \leq |B|$. The special neighbourhood of an element x with respect to two features is defined as

$$x_{[\phi_i\phi_j]} = x_{[\phi_i]} \cap x_{[\phi_j]}, \quad i \neq j. \tag{3.3}$$

Consequently,

$$x_{[\phi_1\cdots\phi_{|B|}]} = x_{[\phi_1]} \cap x_{[\phi_2]} \cap \cdots \cap x_{[\phi_{|B|}]}. \tag{3.4}$$

Definition 3.5. Let B be probe functions defined on a nonempty set X. The family of special neighbourhoods with respect to one feature is defined as

$$N_{[1]}(B) = \cup\{x_{[\phi_i]} : x \in X,\ \phi_i \in B\}. \tag{3.5}$$

Remark 3.6. The family of neighbourhoods with respect to two features is defined as

$$N_{[2]}(B) = \cup\left\{x_{[\phi_i\phi_j]} : x \in X,\ \phi_i, \phi_j \in B,\ i \neq j\right\}. \tag{3.6}$$

Consequently,

$$N_{[|B|]}\ (B) = \cup\{x_{[\phi_1\cdots\phi_{|B|}]} : x \in X\}. \tag{3.7}$$

Definition 3.7. Let $B \subseteq F$ be probe functions representing features of $x, y \in X$. Objects x and y are minimally near each other if $\exists f \in B$ such that $|f(y) - f(x)| < r$, where r is the length of a general neighbourhood defined in Definition 3.1 with respect to the feature $f \in B$ (denoted by $xN_f y$).

Definition 3.8. Let $Y, Y' \subseteq X$ and $B \subseteq F$. Set Y to be minimally near to Y' if $\exists x \in Y$, $x' \in Y'$ and $f \in B$ such that $xN_f x'$ (Denoted by $YN_f Y'$).

Remark 3.9. We can determine a degree K of the nearness between the two sets X, Y as

$$K = \frac{\left|f \in B : XN_f Y\right|}{|B|}. \tag{3.8}$$

Theorem 3.10. *Let $B \subseteq F$ be probe functions representing features of $x, y \in X$. Then x is near to y if $y \in x_{[\phi_i]}$, where $\phi_i \in B$, $1 \leq i \leq |B|$.*

Proof. Obvious. □

Theorem 3.11. *Any subset of X is near to X.*

Proof. From Definitions 3.7 and 3.8, we get the proof obviously. □

Postulation 1. Every set X is a near set (near to itself) as every element $x \in X$ is near to itself.

Definition 3.12. Let B be probe functions on a nonempty set X. The lower and upper approximations for any subset $A \subseteq X$ by using the special neighbourhood are defined as

$$\begin{aligned} \underline{N}_{[i]}(A) &= \cup\{y \in N_{[i]}(B) : y \subseteq A\}; \\ \overline{N}_{[i]}(A) &= \cup\{y \in N_{[i]}(B) : y \cap A \neq \phi\}. \end{aligned} \tag{3.9}$$

Consequently, the boundary region is

$$b_{N_{[i]}} = \overline{N}_{[i]}(A) - \underline{N}_{[i]}(A), \quad \text{where } 1 \leq i \leq |B|. \tag{3.10}$$

Definition 3.13. Let B be probe functions on a nonempty set X. The accuracy measure for any subset $A \subseteq X$ by using the special neighbourhood with respect to i features is

$$\alpha_{[i]}(A) = \frac{\left|\underline{N}_{[i]}(A)\right|}{\left|\overline{N}_{[i]}(A)\right|}, \quad \overline{N}_{[i]}(A) \neq \phi. \tag{3.11}$$

Remark 3.14. $0 \leq \alpha_{[i]}(A) \leq 1$, $\alpha_{[i]}(A)$ measures the degree of exactness of any subset $A \subseteq X$. If $\alpha_{[i]}(A) = 1$ then A is exact set with respect to i features.

Definition 3.15. Let B be probe functions on a nonempty set X. The new generalized lower rough coverage of any subset Y of the family of special neighbourhoods is defined as

$$\nu_{[i]}\left(Y, \underline{N}_{[i]}(D)\right) = \frac{\left|Y \cap \underline{N}_{[i]}(D)\right|}{\left|\underline{N}_{[i]}(D)\right|}, \quad \underline{N}_{[i]}(D) \neq \phi. \tag{3.12}$$

If $\underline{N}_{[i]}(D) = \phi$, then $\nu_{[i]}(Y, \underline{N}_{[i]}(D)) = 1$.

Remark 3.16. $0 \leq \nu_{[i]}(Y, \underline{N}_{[i]}(D)) \leq 1$, $\nu_{[i]}(Y, \underline{N}_{[i]}(D))$ means the degree that the subset Y covers the sure region (acceptable objects).

4. Modification of Our Approach to Near Sets

In this section, we introduce a modification of our approach introduced in Section 3. We deduce some of generalized concepts. Finally, we prove that our modified approach in this section is the best.

Definition 4.1. Let B be probe functions on a nonempty set X. The modified near lower, upper, and boundary approximations for any subset $A \subseteq X$ are defined as

$$\begin{gathered}\underline{N}'_{[i]}(A) = \cup\left\{y \in N_{[i]}(B) : y \subseteq A\right\}; \\ \overline{N}'_{[i]}(A) = \left[\underline{N}'_{[i]}(A^c)\right]^c; \\ b_{N'_{[i]}} = \overline{N}'_{[i]}(A) - \underline{N}'_{[i]}(A), \quad \text{where } 1 \leq i \leq |B|.\end{gathered} \tag{4.1}$$

Definition 4.2. Let B be probe functions on a nonempty set X. The new accuracy measure for any subset $A \subseteq X$ is

$$\alpha'_{[i]}(A) = \frac{\left|\underline{N}'_{[i]}(A)\right|}{\left|\overline{N}'_{[i]}(A)\right|}, \quad \overline{N}'_{[i]}(A) \neq \phi. \tag{4.2}$$

Table 1: The values of the three features.

	s	a	r
x_1	0.51	1.2	0.53
x_2	0.56	3.1	2.35
x_3	0.72	2.8	0.72
x_4	0.77	2	0.95

Theorem 4.3. *Let $A \subseteq X$, then*

(1) $\underline{N}_{[i]}(A)$ *is near to* $\overline{N}'_{[i]}(A)$ *and* $\overline{N}_{[i]}(A)$;

(2) $b_{N'_{[i]}}(A)$ *is near to* $\overline{N}_{[i]}(A)$ *and* $\overline{N}'_{[i]}(A)$;

(3) $\overline{N}'_{[i]}(A)$ *is near to* $\overline{N}_{[i]}(A)$;

(4) $b_{N'_{[i]}}(A)$ *is near to* $b_{N_{[i]}}(A)$.

Proof. Obvious. □

Remark 4.4. A set A is called a near set if $|b_{N'_{[i]}}(A)| \geq 0$.

Definition 4.5. Let B be probe functions on a nonempty set X. The new generalized lower rough coverage of any subset Y of the family of special neighbourhoods is defined as

$$\nu'_{[i]}\left(Y, \underline{N}'_{[i]}(D)\right) = \frac{\left|Y \cap \underline{N}'_{[i]}(D)\right|}{\left|\underline{N}'_{[i]}(D)\right|}, \quad \underline{N}'_{[i]}(D) \neq \phi. \tag{4.3}$$

If $\underline{N}'_{[i]}(D) = \phi$, then $\nu'_{[i]}(Y, \underline{N}'_{[i]}(D)) = 1$.

Now, we give an example to explain these definitions.

Example 4.6. Let s, a, r be three features defined on a nonempty set $X = \{x_1, x_2, x_3, x_4\}$ as in Table 1.

If the length of the neighbourhood of the feature s (resp., a and r) equals to 0.2 (resp., 0.9 and 0.5), then

$$N_1(B) = \{\xi(s_{0.2}), \xi(a_{0.9}), \xi(r_{0.5})\}, \tag{4.4}$$

where $\xi(s_{0.2}) = \{\{x_1, x_2\}, \{x_1, x_2, x_3\}, \{x_2, x_3, x_4\}, \{x_3, x_4\}\}$; $\xi(a_{0.9}) = \{\{x_1, x_4\}, \{x_2, x_3\}, \{x_2, x_3, x_4\}, \{x_1, x_3, x_4\}\}$; $\xi(r_{0.5}) = \{\{x_1, x_3, x_4\}, \{x_2\}\}$. Hence,

$$N_{[1]}(B) = \{\{x_2\}, \{x_3\}, \{x_4\}, \{x_1, x_2\}, \{x_1, x_4\}, \{x_2, x_3\}, \{x_3, x_4\}, \{x_1, x_3, x_4\}\}. \tag{4.5}$$

Also, we get

$$N_2(B) = \{\xi(s_{0.2}, a_{0.9}), \xi(s_{0.2}, r_{0.5}), \xi(a_{0.9}, r_{0.5})\}, \tag{4.6}$$

where $\xi(s_{0.2}, a_{0.9}) = \{\{x_1\}, \{x_2, x_3\}, \{x_2, x_3, x_4\}, \{x_3, x_4\}\}$; $\xi(s_{0.2}, r_{0.5}) = \{\{x_1\}, \{x_2\}, \{x_3, x_4\}\}$; $\xi(a_{0.9}, r_{0.5}) = \{\{x_1, x_4\}, \{x_2\}, \{x_3, x_4\}, \{x_1, x_3, x_4\}\}$. Hence,

$$N_{[2]}(B) = N_{[3]}(B) = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \ldots\}. \tag{4.7}$$

Also, we find that

$$N_3(B) = \{\{x_1\}, \{x_2\}, \{x_3, x_4\}\}. \tag{4.8}$$

Theorem 4.7. *Every rough set is a near set but not every near set is a rough set.*

Proof. There are two cases to consider:

(1) $|b_{N'_{[i]}}(A)| > 0$. Given a set $A \subseteq X$ that has been approximated with a nonempty boundary, this means A is a rough set as well as a near set;

(2) $|b_{N'_{[i]}}(A)| = 0$. Given a set $A \subseteq X$ that has been approximated with an empty boundary, this means A is a near set but not a rough set. □

The following example proves Theorem 4.7.

Example 4.8. From Example 4.6, if $A = \{x_3, x_4\}$, then $N_3(B)^* A = N_3(B)_* A = A$, $\underline{N}_{[2]}(A) = \overline{N}_{[2]}(A) = A$, and $\underline{N}'_{[1]}(A) = \overline{N}'_{[1]}(A) = A$. Hence A is a near set in each case, but is not rough set with respect to three features by using the approximations introduced by Peters, with respect to two features by using our approach defined in Section 3, and with respect to only one feature by using our modified approach defined in Section 4.

Now the following example deduces a comparison between the classical and new general near approaches by using the accuracy measures of them.

Example 4.9. From Example 4.6, we introduce Table 2, where $Q(X)$ is a family of subsets of X and $\text{II} = \alpha_{[2]} = \alpha_{[3]} = \alpha'_{[2]} = \alpha'_{[3]}$.

From Table 2, we note that when we use our generalized set approximations of near sets with respect to one feature many of subsets become exact sets. Also, with respect to two features, all subsets become completely exact. Consequently we consider that our approximations are a start point of real-life applications in many fields of science.

5. Medical Application

If we consider that $B = \{a, s, r\}$ in Example 4.6 represents measurements for a kind of diseases and the set of objects $X = \{x_1, x_2, x_3, x_4\}$ are patients, then for any group of patients, we can determine the degree of this disease, by using the lower rough coverage based on the decision class D as in the following examples.

Example 5.1. In Example 4.6, if the decision class $D = \{x_1, x_3\}$ and we consider the following groups of the patients: $\{x_1, x_3\}$, $\{x_2, x_3\}$, $\{x_3, x_4\}$, $\{x_1, x_2, x_3\}$ and $\{x_2, x_3, x_4\}$, then, we get

Table 2: Comparison between traditional and modificated approaches.

$Q(X)$	α_1	α_2	α_3	$\alpha_{[1]}$	$\alpha'_{[1]}$	II
$\{x_1\}$	0	1/3	1	0	0	1
$\{x_2\}$	1/4	1/3	1	1/3	1	1
$\{x_3\}$	0	0	0	1/4	1	1
$\{x_4\}$	0	0	0	1/3	1	1
$\{x_1, x_2\}$	1/2	1/2	1	12	1	1
$\{x_1, x_3\}$	0	1/4	1/3	1/4	1/2	1
$\{x_1, x_4\}$	1/2	1/2	1/3	1/2	1	1
$\{x_2, x_3\}$	1/2	1/2	1/3	12	1	1
$\{x_2, x_4\}$	1/4	1/4	1/3	12	2/3	1
$\{x_3, x_4\}$	1/2	1/2	1	12	1	1
$\{x_1, x_2, x_3\}$	3/4	3/4	1/2	34	1	1
$\{x_1, x_2, x_4\}$	3/4	3/4	1/2	34	1	1
$\{x_1, x_3, x_4\}$	3/4	3/4	1	34	1	1
$\{x_2, x_3, x_4\}$	3/4	3/4	1	34	3/4	1

Table 3: The degree that some subsets $Q(X)$ cover the sure region.

$Q(X)$	ν_1	ν_2	ν_3	$\nu'_{[1]}$	II
$\{x_1, x_3\}$	1	1	1	1	1
$\{x_2, x_3\}$	1	0	0	1	12
$\{x_3, x_4\}$	1	0	0	1	12
$\{x_1, x_2, x_3\}$	1	1	1	1	1
$\{x_2, x_3, x_4\}$	1	0	0	1	12

the following results: $N_1(B)_*D = \phi$, $N_2(B)_*D = N_3(B)_*D = \{x_1\}$, $\underline{N}'_{[1]}(D) = \{x_3\}$, and $\underline{N}'_{[2]}(D) = \underline{N}'_{[3]}(D) = \{x_1, x_3\}$.

So these sets cover the acceptable objects by the following degrees in Table 3, where

$$\text{II} = \nu'_{[2]} = \nu'_{[3]}. \tag{5.1}$$

Remark 5.2. If we want to determine the degree that the lower of the decision class D covers the set Y, then we use the following formulas:

$$\begin{gathered}\nu_i^*(Y, N_r(B)_*D) = \frac{|Y \cap N_r(B)_*D|}{|Y|}, \quad Y \neq \phi; \\ \nu^{*\prime}_{[i]}\left(Y, \underline{N}'_{[i]}(D)\right) = \frac{\left|Y \cap \underline{N}'_{[i]}(D)\right|}{|Y|}, \quad Y \neq \phi.\end{gathered} \tag{5.2}$$

Example 5.3. In Example 4.6, if we are interested in the degree that the sure region (acceptable objects) covers these groups, we get Table 4, where $\text{II} = \nu^{*\prime}_{[2]} = \nu^{*\prime}_{[3]}$.

Table 4: The degree that the sure region covers some subsets $Q(X)$.

$Q(X)$	ν_1^*	ν_2^*	ν_3^*	$\nu^{*\prime}_{[1]}$	II
$\{x_1, x_3\}$	0	12	12	12	1
$\{x_2, x_3\}$	0	0	0	12	12
$\{x_3, x_4\}$	0	0	0	1/2	12
$\{x_1, x_2, x_3\}$	0	13	13	13	23
$\{x_2, x_3, x_4\}$	0	0	0	1/3	1/3

From this table, we can say that our modified approach is better than the classical approach of near set theory, as our lower approximations are increasing the acceptable objects.

For example, when we used classical approximations the group $\{x_1, x_3\}$ with respect to one feature has no disease and with respect to three features has this disease with ratio 50%, unless this group is itself the decision class of this disease.

But when we used our modified set approximations with respect to two or three features, we find the fact of this disease that the degree of disease in this group is 100%.

6. Conclusion

In this paper, we used a special neighborhood to introduce a generalization of traditional set approximations. In addition we introduce a modification of our special approach to near sets. Our approaches are mathematical tools to modify the traditional approximations. The suggested methods of near approximations open a way for constructing new types of lower and upper approximations.

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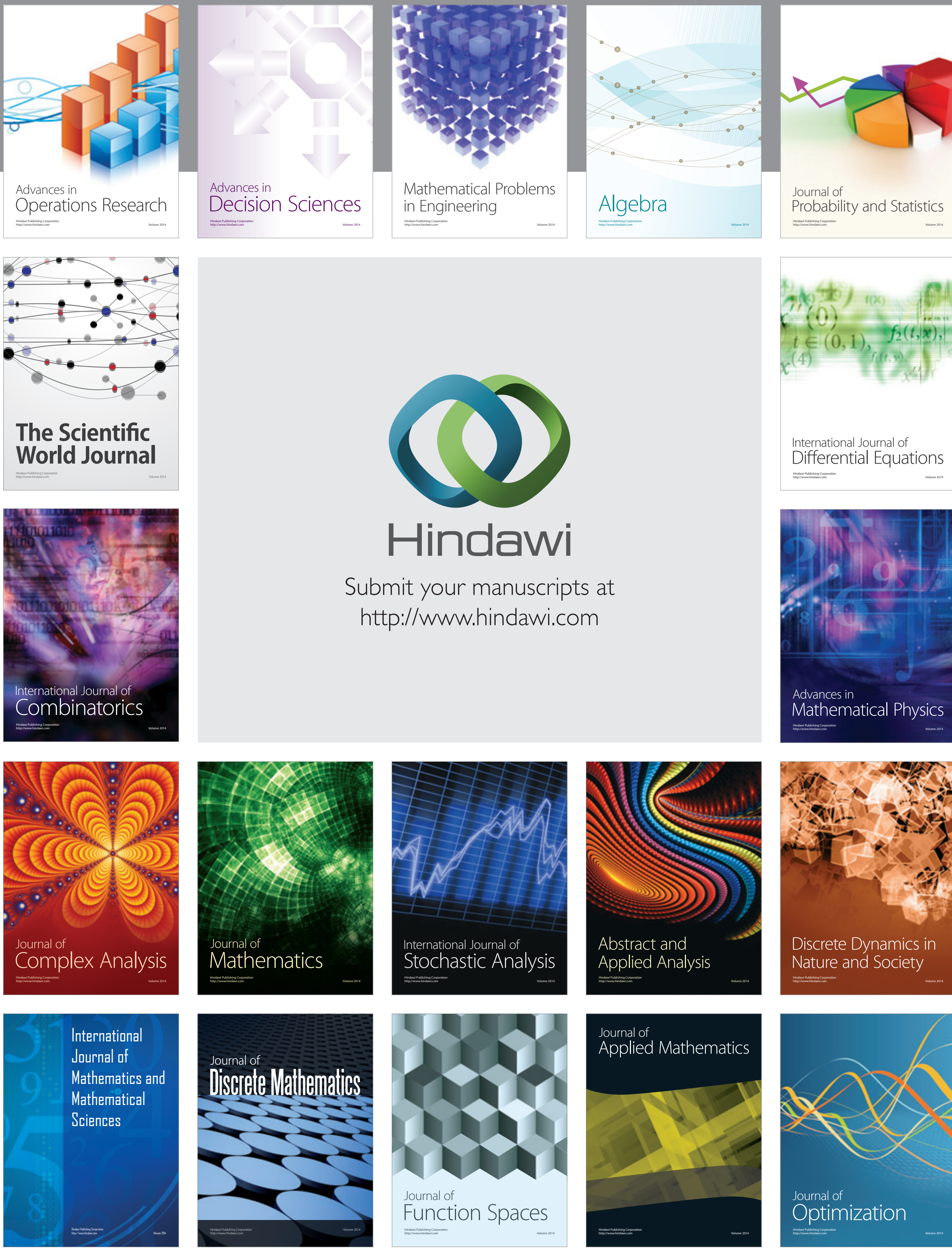
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