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Research Article

Semistability of Nonlinear Impulsive Systems with Delays

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This paper is concerned with the stability analysis and semistability theorems for delay impulsive systems having a continuum of equilibria. We relate stability and semistability to the classical concepts of system storage functions to impulsive systems providing a generalized hybrid system energy interpretation in terms of storage energy. We show a set of Lyapunov-based sufficient conditions for establishing these stability properties. These make it possible to deduce properties of the Lyapunov functional and thus lead to sufficient conditions for stability and semistability. Our proposed results are evaluated using an illustrative example to show their effectiveness.

1. Introduction

Due to their numerous applications in various fields of sciences and engineering, impulsive differential systems have become a large and growing interdisciplinary area of research. In recent years, the issues of stability in impulsive differential equations with time delays have attracted increasing interest in both theoretical research and practical applications [1–9], while difficulties and challenges remain in the area of impulsive differential equations [10], especially those involving time delays [11]. Various mathematical models in the study of biology, population dynamics, ecology and epidemic, and so forth can be expressed by impulsive delay differential equations. These processes and phenomena, for which the adequate mathematical models are impulsive delay differential equations, are characterized by the fact that there is sudden change of their state and that the processes under consideration depend on their prehistory at each moment of time. In the transmission of the impulse information, input delays are often encountered. Control and synchronization of chaotic systems are considered in [12, 13]. By utilizing impulsive feedback control, all the solutions of the Lorenz chaotic system will converge to an equilibrium point. The application of networked control systems is considered in [14–17], while in [14], when analyzing the asymptotic stability for discrete-time neural networks, the activation functions are not required to be differentiable or strictly monotonic. The existence of the equilibrium point is first proved under mild conditions. By constructing a new Lyapnuov-Krasovskii functional, a linear matrix inequality (LMI) approach is developed to establish sufficient conditions for the discrete-time neural networks to be globally asymptotically stable. In [18], Razumikhin-type theorems are established which guarantee ISS/iISS for delayed impulsive systems with external input affecting both the continuous dynamics and the discrete dynamics. It is shown that when the delayed continuous dynamics are ISS/iISS but the discrete dynamics governing the impulses are not, the ISS/iISS property of the impulsive system can be retained if the length of the impulsive interval is large enough. Conversely, when the delayed continuous dynamics are not ISS/iISS but the discrete dynamics governing the impulses are, the impulsive system can achieve ISS/iISS. In [19, 20], the authors consider linear time invariant uncertain sampled-data systems in which there are two sources of uncertainty: the values of the process parameters can be unknown while satisfying a polytopic condition and the sampling intervals can be uncertain and variable. They model such systems as linear impulsive systems and they apply their theorem to the analysis and state-feedback stabilization. They find a positive constant which determines an upper bound on the sampling intervals for which the stability of the closed loop is guaranteed. Population growth and biological systems are considered in [21, 22]. Stochastic systems are considered in [23–25], and so forth. However, the corresponding theory for impulsive systems with time delays having a continuum of equilibria has been relatively less developed.

The purpose of this paper is to study the stability and semistability properties for nonlinear delayed impulsive systems with continuum of equilibria. Examples of such systems include mechanical systems having rigid-body modes and isospectral matrix dynamical systems [26]. Such systems also arise in chemical kinetics, compartmental modeling, and adaptive control. Since every neighborhood of a nonisolated equilibrium contains another equilibrium, a nonisolated equilibrium cannot be asymptotically stable. Thus asymptotic stability is not the appropriate notion of stability for systems having a continuum of equilibria. Two notions that are of particular relevance to such systems are convergence and semistability. Convergence is the property whereby every solution converges to a limit point that may depend on the initial condition. Semistability is the additional requirement that all solutions converge to limit points that are Lyapunov stable. More precisely, an equilibrium is semistable if it is Lyapunov stable, and every trajectory starting in a neighborhood of the equilibrium converges to a (possibly different) Lyapunov stable equilibrium. It can be seen that, for an equilibrium, asymptotic stability implies semistability, while semistability implies Lyapunov stability. We will employ the method of Lyapunov function for the study of stability and semistability of impulsive systems with time delays. Several stability criteria are established. A set of Lyapunov-based sufficient conditions is provided for stability criteria, then we extend the notion of stability to develop the concept of semistability for delay impulsive systems. Finally, an example illustrates the effectiveness of our approach.

2. Preliminaries

Let $\mathbb{N}$ denote the set of positive integer numbers. Let PC_t denote the set of piecewise right continuous functions $\phi : [t-r, t] \to \mathbb{R}^n$ with the norm defined by $\|\phi\|_r^t = \sup_{-r\leq s\leq 0}\|\phi(t+s)\|$. For simplicity, define $\|\phi\|_r = \|\phi\|_r^0$, for $\phi \in PC_0$. For given $r > 0$, if $x \in PC([t_0 - r, +\infty), \mathbb{R}^n)$, then for each $t \geq t_0$, we define $x_t, x_{t^-} \in \text{PC}_0$ by $x_t(s) = x(t+s)$ $(-r \leq s \leq 0)$ and $x_{t^-}(s) = x(t+s)$ $(-r \leq s < 0)$, respectively. A function $\alpha : \mathbb{R}^+ \to \mathbb{R}^+$ is of class $\mathcal{K}$, if α is continuous, strictly increasing, and $\alpha(0) = 0$. For a given scalar $\rho \geq 0$, let $\mathcal{B}(\rho) = \{x \in \mathbb{R}^n;\ \|x\| \leq \rho\}$.

Let $\Omega \in \mathbb{R}^n$ be an open set and $\mathfrak{B}(\rho) \subset \Omega$ for some $\rho > 0$. Given functionals $f : \mathbb{R}^+ \times \text{PC}([-r,0],\Omega) \to \mathbb{R}^n,\ g : \mathbb{R}^+ \times \Omega \to \mathbb{R}^n$, satisfying $f(t,0) = 0,\ g(0,0) = 0$. Considering the following nonlinear time-delay impulsive system Σ_t described by the state equation

$$\dot{x}(t) = f(t, x_t), \quad t > t_0,\ t \neq t_k,\ k \in \mathbb{N}, \tag{2.1}$$

$$x(t^+) = g(t, x(t)), \quad t = t_k,\ k \in \mathbb{N}, \tag{2.2}$$

$$x(t_0 + \theta) = \phi(\theta), \quad \theta \in [-r, 0], \tag{2.3}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $\dot{x}(t)$ denotes the right-hand derivative of $x(t)$, $x(t^+)$ and $x(t^-)$ denote the limit from the right and the limit from the left at point t, respectively. t_0 is the initial time. Here we assume that the solutions of system Σ_t are right continuous, that is, $x(t^+) = x(t)$. $\{t_k\},\ k \in \mathbb{N}$ is a strictly increasing sequence of impulse times in (t_0, ∞) where $\lim_{k\to\infty} t_k = \infty$.

Definition 2.1. The function $f : \mathbb{R} \times \text{PC} \to \mathbb{R}^n$ is said to be composite-PC, if for each $t_0 \in \mathbb{R}$ and $\alpha > 0$, $x \in \text{PC}([t_0 - r, t_0 + \alpha], \mathbb{R}^n)$ and x is continuous at each $t \neq t_k$ in $[t_0, t_0 + \alpha]$, then the composite function $h(x) = f(t, x_t) \in \text{PC}([t_0 - r, t_0 + \alpha], \mathbb{R}^n)$.

Definition 2.2. The function $f : \mathbb{R} \times \text{PC} \to \mathbb{R}^n$ is said to be quasi-bounded, if for each $t_0 \in \mathbb{R}^+$, $\alpha > 0$, and for each compact set $F \in \mathbb{R}^n$, there exists some $M > 0$, such that $\|f(t,\psi)\| \leq M$ for all $(t,\psi) \in [t_0, t_0 + \alpha] \times \text{PC}([-r,0], F)$.

Definition 2.3. The function $x : [t_0 - r, t_0 + \alpha] \to \mathbb{R}^n$ with $\alpha > 0$ is said to be a solution of Σ_t if

(i) x is continuous at each $t \neq t_k$ in $(t_0, t_0 + \alpha]$;

(ii) the derivative of x exists and is continuous at all but at most a finite number of points t in $t \in [t_0, t_0 + \alpha)$;

(iii) the right-hand derivative of x exists and satisfies (2.1) in $t \in [t_0, t_0 + \alpha]$, while for each $t_k \in [t_0, t_0 + \alpha]$, (2.2) holds;

(iv) Equation (2.3) holds, that is, $x(t_0 + \theta) = \phi(\theta),\ \theta \in [-r, 0]$.

We denote by $x(t, t_0, \phi)$ (or $x(t)$, if in not confusing) the solution of Σ_t. $x(t)$ is said to be a solution defined on $[t_0 - r, \infty)$ if all above conditions hold for any $\alpha > 0$.

We make the following assumptions on system Σ_t.

(A1) $f(t,\psi)$ is composite-PC, quasi-bounded and locally Lipschitzian in ψ.

(A2) For each fixed $t \in \mathbb{R}^+$, $f(t,\psi)$ is a continuous function of ψ on $\text{PC}([-\tau,0],\mathbb{R}^n)$.

Under the assumptions above, it was shown in [11] that for any $\phi \in \text{PC}([-r,0],\mathbb{R}^n)$, system Σ_t admits a solution $x(t,t_0,\phi)$ that exists in a maximal interval $[t_0 - r, t_0 + b)$ $(0 < b \leq +\infty)$ and the zero solution of the system exists.

Definition 2.4. An equilibrium point of Σ_t is a point $x_e \in \text{PC}([t_0 - r, t_0 + \alpha], \mathbb{R}^n)$ satisfying $x(t, t_0, \phi) = x_e$ for all $t \geq 0$ where $x(t, t_0, \phi)$ is the solution of Σ_t. Let $\mathcal{E}$ denote the set of equilibrium points of Σ_t.

Definition 2.5. Consider the delay impulsive system Σ_t.

(i) An equilibrium point $x(t) \equiv x_e$ of Σ_t is Lyapunov stable if for any $\varepsilon > 0$ there exists $\delta(\varepsilon, t_0)$, such that $\|\phi - \phi_e\|_r < \delta$ implies $\|x(t) - x_e\| < \varepsilon$ for all $t \geq t_0$, where ϕ_e is the initial function for x_e. An equilibrium point x is uniformly Lyapunov stable, if, in addition, the number δ is independent of t_0.

(ii) An equilibrium point x of Σ_t is semistable if it is Lyapunov stable and there exists an open subset of Ω containing x such that for all initial conditions in Ω the trajectory of Σ_t converges to a Lyapunov stable equilibrium point, that is, $\lim_{t\to\infty} x(t, t_0, \phi) = y,\ \phi \in \Omega$, where y is a Lyapunov stable equilibrium point.

(iii) System Σ_t is said to be uniformly asymptotically stable in the sense of Lyapunov with respect to the zero solution, if it is uniformly stable and $\lim_{t\to\infty}\|x(t)\| = 0$.

Definition 2.6. The function $V : [t_0, +\infty) \times \mathrm{PC}([-\tau, 0], \mathcal{B}(\rho)) \to \mathbb{R}^+$ is said to belong to the class $\mathcal{V}_0$ if

(i) V is continuous in each of the sets $[t_{k-1}, t_k) \times \mathrm{PC}([-\tau, 0], \mathcal{B}(\rho))$ and for each $k \in \mathbb{N}, \lim_{(t,y)\to(t_k^-,x)} V(t, y) = V(t_k^-, x)$ exists;

(ii) $V(t, x)$ is locally Lipschitzian in $x \in \mathrm{PC}([-\tau, 0], \mathcal{B}(\rho))$, and for all $t \geq t_0$, $V(t, 0) \equiv 0$.

Definition 2.7. Let $V \in \mathcal{V}_0$. For any $(t, \psi) \in [t_0, +\infty) \times \mathrm{PC}([-\tau, 0], \mathcal{B}(\rho))$, the upper right-hand derivative of V with respect to system Σ_t is defined by

$$D^+V(t, \psi(0)) := \lim_{h\to 0^+} \sup \frac{1}{h}\{V(t + h, \psi(0) + hf(t, \psi)) - V(t, \psi(0))\}. \tag{2.4}$$

3. Main Results

In the following, we will establish several sufficient conditions for Lyapunov stability and semistability for impulsive differential system Σ_t with time delays.

Theorem 3.1. *System Σ_t is uniformly stable, and the zero solution of Σ_t is asymptotically stable if there exists a Lyapunov function $V \in \mathcal{V}_0$ which satisfies the following.*

(i) *$\exists a, b \in \mathcal{K}$ such that*

$$a(\|x\|) \leq V(t, x) \leq b(\|x\|). \tag{3.1}$$

(ii) *For any $t \in [t_0, +\infty)$, $t \neq t_k$ and $\psi \in PC([-r, 0], \mathbb{R}^n)$, there exists $c > 0$, such that*

$$D^+V(t, \psi(0)) \leq -cV(t, \psi(0)). \tag{3.2}$$

(iii) *There exist a μ $(0 < \mu < 1)$ and a subsequence $\{t_{k_j}\}$ of the impulsive moments $\{t_k\}$ such that*

$$\left\|V\left(t_{k_{j+1}}, x\right)\right\|_r \leq \mu\left\|V\left(t_{k_j}, x\right)\right\|_r. \tag{3.3}$$

(iv) *For any* m, $0 \le m \le k_0$, *for all* $x_e \in \mathcal{E}$ *there exists a function* $\alpha \in \mathcal{K}$, *such that*

$$\|V(t_m, x - x_e)\|_r \le \alpha\left(\|\phi - \phi_e\|_r\right). \tag{3.4}$$

(v) *For any* $m \in \mathbb{N}$, $k_j \le m < k_{j+1}$, $j = 0, 1, 2, \ldots$, *there exists a function* $\beta \in \mathcal{K}$ *such that*

$$\|V(t_m, x - x_e)\|_r \le \beta\left\|V\left(t_{k_j}, x - x_e\right)\right\|_r. \tag{3.5}$$

Proof. Let x_e be an equilibrium point of the system Σ_t. We first prove that x_e is uniformly stable, that is, for for all $\varepsilon > 0$, there exists $\delta = \delta(\varepsilon) > 0$ such that $\|\phi - \phi_e\|_r < \delta$ implies $\|x(t) - x_e\| < \varepsilon$ for all $t \ge t_0$.

For all $\varepsilon > 0$, let $0 < \delta < \varepsilon$ such that

$$a(\varepsilon) > \max\{\alpha(\delta), \beta(\alpha(\delta))\}. \tag{3.6}$$

For any $\|\phi - \phi_e\|_r < \delta$, by condition (3.4), we get

$$\|V(t_m, x - x_e)\|_r \le \alpha\left(\|\phi - \phi_e\|_r\right) \le \alpha(\delta), \quad 0 \le m \le k_0. \tag{3.7}$$

By (3.3), it is clear that $\|V(t, x)\|_r$ is nonincreasing along the subsequence $\{t_{k_j}\}$, so we have

$$\left\|V\left(t_{k_j}, x - x_e\right)\right\|_r \le \|V(t_{k_0}, x - x_e)\|_r \le \alpha(\delta), \quad j = 0, 1, 2, \ldots. \tag{3.8}$$

For any m, $k_j \le m < k_{j+1}$, $j = 0, 1, 2, \ldots$, by (3.5), we get

$$\|V(t_m, x - x_e)\|_r \le \beta(\alpha(\delta)). \tag{3.9}$$

Combining (3.7), (3.8), and (3.9), we conclude that

$$\|V(t_k, x - x_e)\|_r < a(\varepsilon), \quad k = 1, 2, \ldots. \tag{3.10}$$

By condition (3.2), for any $t \in [t_k, t_{k+1})$, $k = 0, 1, 2 \ldots$, we have

$$V(t, x - x_e) \le V(t_k, x - x_e) < a(\varepsilon), \tag{3.11}$$

and then, by (3.10), for any $t \ge t_0$ we derive that $V(t, x - x_e) < a(\varepsilon)$. Hence, by (3.1) we obtain that $a(\|x - x_e\|) \le V(t, x - x_e) < a(\varepsilon)$. Since $a \in \mathcal{K}$, we get

$$\|x(t) - x_e\| < \varepsilon, \quad t \ge t_0, \tag{3.12}$$

which implies that system Σ_t is uniformly Lyapunov stable.

Next, we will prove that the zero solution of Σ_t is asymptotically stable.

Since system Σ_t is uniformly stable, from (3.1), there must exist a real number $M > 0$ such that $\|V(t,x)\|_r \leq M,\ t \geq t_0$. Hence, there exists a $v \geq 0$ such that

$$\limsup_{t\to\infty} \|V(t,x)\|_r = v \leq M. \tag{3.13}$$

In the following, we will show that $v = 0$. Without loss of generality, we can suppose that there exists a sequence $\{t_n\} \subset [t_0,\infty),\ n = 1,2,\ldots$, such that

$$\lim_{n\to\infty} \|V(t_n,x)\|_r = \limsup_{n\to\infty} \|V(t,x)\|_r = v. \tag{3.14}$$

From (3.3) we get

$$\left\|V\left(t_{k_j},x\right)\right\|_r < \mu^j \|v(t_{k_0},x)\|_r. \tag{3.15}$$

Since $0 < \mu < 1$, we obtain

$$\lim_{j\to\infty} \left\|V\left(t_{k_j},x\right)\right\|_r = 0. \tag{3.16}$$

If the sequence $\{t_n\} \subset [t_0,\infty),\ n = 1,2,\ldots$ is the same as the sequence $\{t_{k_j}\},\ j = 0,1,2,\ldots$, then it is obvious that $v = 0$. If $0 \leq n < k_0$, it follows from the assumptions above that (3.16) holds. Otherwise, we assume that $n \geq k_0$; there exists a $j \in \mathbb{N}$ such that $k_j \leq n < k_{j+1}$. Then from condition (3.5) we get

$$\|V(t_n,x)\|_r \leq \beta\left(\left\|V\left(t_{k_j},x\right)\right\|_r\right). \tag{3.17}$$

So

$$\lim_{n\to\infty} \|V(t_n,x)\|_r \leq \lim_{j\to\infty} \beta\left(\left\|V\left(t_{k_j},x\right)\right\|_r\right) = 0, \tag{3.18}$$

which implies $v = 0$.

Hence, we derive that $\lim_{t\to\infty}\|V(t,x)\| = 0$. Finally, by (3.1), we have $\lim_{t\to\infty}\|x(t)\| = 0$ which implies that the zero solution of the system Σ_t is asymptotically stable. The proof is completed. □

Next, we present a sufficient condition for semistability for system Σ_t.

Let $\mathfrak{L}_1 := \{f : [0,\infty) \to \mathbb{R};\ f$ is measurable and $\int_0^\infty |f(t)|dt < \infty\}$.

Theorem 3.2. *Consider the system Σ_t; assume that there exists nonnegative-definite continuous function $W : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ such that*

$$D^+V(t,\psi(0)) \leq -W(t,\psi(0)). \tag{3.19}$$

Let $W^{-1}(0) := \{x \mid W(t,x) \equiv 0,\ \text{for all } t \geq t_0\}$. If every equilibrium point of system Σ_t is Lyapunov stable, then every point in $W^{-1}(0)$ is semistable.

Proof. Define

$$\varphi(t) := \begin{cases} W(t, \psi(0)), & t \neq t_k,\ k \in \mathbb{N}, \\ 0, & t = t_k,\ k \in \mathbb{N}. \end{cases} \tag{3.20}$$

It follows from (3.19) and (3.3) that

$$\int_0^t \varphi(s)ds \leq V(x(t_1)) - V(x(t)) \leq V(x(t_1)) \ . \tag{3.21}$$

Since $\varphi(\cdot)$ is nonnegative, it follows that $\varphi(\cdot) \in \mathfrak{L}_1$. Next, we show that $\varphi(t) \to 0$ as $t \to \infty$.

If it is not true, then there exists $\varepsilon > 0$ and an infinite sequence of times $\tau_1, \tau_2, \ldots$ such that $|\varphi(\tau_i)| \geq \varepsilon$. By definition of $\varphi(\cdot)$ we have $\tau_i,\ i = 1, 2\ldots$ that does not belong to the set of impulsive times $\{t_k\}$.

Note that from (3.19), it follows from Proposition 3.1 of [26] that $x(t)$ is bounded for all $t \geq 0$. Hence, it follows from the Lipschitz continuity of $f(\cdot)$ that $\dot{x}(t)$ is bounded for all $t \geq 0$; thus, $\varphi(\cdot)$ is uniformly continuous on $[t_0, +\infty) \setminus \{t_n\}$. So, there exists $\delta > 0$ such that every τ_i is contained in some interval of $I_i,\ \tau_i \in I_i$ of length δ on which $\varphi(t) \geq \varepsilon/2,\ t \in I_i$. This contradicts $\varphi(\cdot) \in \mathfrak{L}_1$. Hence $\varphi(t) \to 0$ as $t \to \infty$. It follows that $W(t, \psi(0)) \to 0$ as $t \to \infty$. Since $x(t)$ is bounded, we get $x(t) \to W^{-1}(0)$ (as $t \to \infty$).

Next, let $x_e \in W^{-1}(0)$. For every open neighborhood U and $x_0 \in U$, $x(t) \to W^{-1}(0)$ (as $t \to \infty$), it follows from Proposition 5.1 of [26] that there exists $y \in W^{-1}(0)$ such that $\lim_{t\to\infty} x(t) = y$. Since every point in $\mathcal{E}$ is Lyapunov stable, and hence y is a Lyapunov stable equilibrium of Σ_t, it follows that x_e is semistable. Finally, since $x_e \in W^{-1}(0)$ is arbitrary, this implies every point in $W^{-1}(0)$ is semistable. The proof is completed. □

4. Numerical Example

In this section, we give an example about compartmental systems to illustrate the effectiveness of the proposed method. Compartmental systems involve dynamical models that are characterized by conservation laws (e.g., mass and energy) capturing the exchange of material between coupled macroscopic subsystems known as compartments. Each compartment is assumed to be kinetically homogeneous, that is, any material entering the compartment is instantaneously mixed with the material of the compartment.

Example 4.1. Consider the nonlinear two-compartment time-delay impulsive systems given by

$$\begin{gathered}
\dot{x}_1(t) = -x_1(t) + x_2(t)\{1 - \sin(x_1(t-r))\} + x_2^3(t) - x_1^3(t), \quad t \neq t_k,\ k \in \mathbb{N}, \\
\dot{x}_2(t) = -x_2(t) + x_1(t)\{1 + \sin(x_1(t-r))\} + x_1^3(t) - x_2^3(t), \quad t \neq t_k,\ k \in \mathbb{N}, \\
x_1(t^+) = 0.8x_1(t), \quad t = t_k,\ k \in \mathbb{N}, \\
x_2(t^+) = 0.9x_2(t), \quad t = t_k,\ k \in \mathbb{N}, \\
x(t_0 + \theta) = \phi(\theta) = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \quad \theta \in [-0.2, 0],
\end{gathered} \tag{4.1}$$

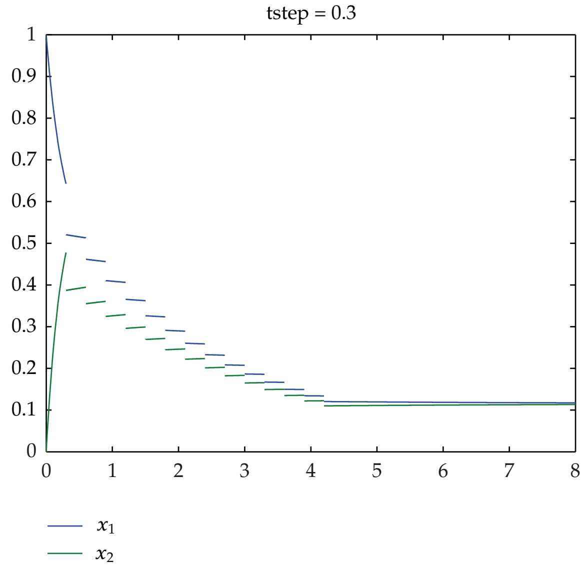

Figure 1: State trajectory: semistable.

where $r \geq 0$. Let Lyapunov function $V(\psi(0)) = (1/2)\psi_1^2(0) + (1/2)\psi_2^2(0)$, then for any $\psi \in \mathrm{PC}([-r, 0], \mathcal{B}(\rho))$ we have

$$\begin{aligned} D^+V(\psi(0)) &= -\frac{1}{2}\left(x_1^2 + x_2^2\right) - \frac{1}{2}(x_1 - x_2)^2\left(x_1^2 + x_1x_2 + x_2^2\right) \\ &\leq -\frac{1}{2}\left(x_1^2 + x_2^2\right) = -V(\psi(0)). \end{aligned} \tag{4.2}$$

Let $c = 1$, $a(\|x\|) = b(\|x\|) = (1/2)\|x\|$, $\mu = 0.9$, and $\beta(\|x\|) = \|x\|$, then the conditions of Theorem 3.1 are satisfied, which means the equilibrium points of the system are Lyapunov stable, and

$$\begin{aligned} D^+V(\psi(0)) &= -\frac{1}{2}(x_1 - x_2)^2\left[1 + \frac{3}{4}x_1^2 + \left(\frac{1}{2}x_1 + x_2\right)^2\right] \\ &\leq -\frac{1}{2}(x_1 - x_2)^2. \end{aligned} \tag{4.3}$$

Let $W(x_1, x_2) = (1/2)(x_1 - x_2)^2$ then we derive that $D^+V(\psi(0)) \leq -W(\psi(0))$; it follows from Theorem 3.2 that every point in $W^{-1}(0)$ is semistable.

The simulation result is depicted in Figure 1, where the length of the impulsive intervals is $T = 0.3$ second and the time delay $r = 0.1$ second.

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