Research Article

A New Numerical Solution of Maxwell Fluid over a Shrinking Sheet in the Region of a Stagnation Point

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The mathematical model for the incompressible two-dimensional stagnation flow of a Maxwell fluid towards a shrinking sheet is proposed. The developed equations are used to discuss the problem of being two dimensional in the region of stagnation point over a shrinking sheet. The nonlinear partial differential equations are transformed to ordinary differential equations by first-taking boundary-layer approximations into account and then using the similarity transformations. The obtained equations are then solved by using a successive linearisation method. The influence of the pertinent fluid parameters on the velocity is discussed through the help of graphs.

1. Introduction

Many engineering fluid mechanical problems have been solved using this boundary-layer theory rendering results which compare well with experimental observations—at least as far as Newtonian fluids are concerned. In spite of the success of this theory for Newtonian fluids, an extension of the theory to non-Newtonian fluids has turned out to be a rather formidable task [1–3]. The main difficulty in reaching to a general boundary-layer theory for non-Newtonian fluids lies obviously in the diversity of these fluids in their constitutive behavior, simultaneous viscous, and elastic properties such that differentiating between those effects which arise as a result of a fluid's shear-dependent viscosity from those which are attributable to the fluid's elasticity becomes virtually impossible. But some mathematical models have been proposed to fit well with the experimental observations. The simplest model for the rheological effects of viscoelastic fluids is the Maxwell model where the dimensionless relaxation time is small. However, in some more concentrated polymeric fluids the Maxwell

model is also used for large dimensionless relaxation time. Some recent investigations which deal with the flows of Maxwell fluids are given in [4–8].

The viscous flows due to a stretching sheet have attracted the attention of many researchers. This is due to their several applications in polymer processing industries, environmental pollution, biological process, aerodynamic extrusion of plastic sheets, glass fiber production of the boundary layer along a liquid film and condensation process, the cooling, and/or drying of paper and textiles. In 1966, Erickson et al. [9] investigated the problem of heat and mass transfer on a moving continuous plate with suction and injection. In view of above applications, Crane [10] extended Sakiadis [11, 12] study of boundary layer flow for stretching sheet. The literature on the stretching sheet topic is quite extensive and hence cannot be described in detail. However, some most recent works of eminent researchers regarding the flow over a stretching sheet may be mentioned in the studies [13–20]. Literature survey indicates that no attention has been given to the shrinking flow [21–25] for Maxwell fluid for two-dimensional stagnation flow and axisymmetric stagnation flow towards an axisymmetric shrinking over a shrinking sheet. There are few situations like rising shrinking balloon, and so forth, the standard stretching phenomena is not useful therefore, the shrinking phenomena are used.

Wang [26] has discussed the stagnation flow towards a shrinking sheet. According to him, solution does not exist for a shrinking sheet in an otherwise still fluid since vorticity could not be confined in a boundary layer. However, in the presence of stagnation flow to contain the vorticity, similarity solution may exist. Mention may be made to the works of [27–33]. The objective of this paper is two-fold: first, to formulate the Maxwell fluid for two-dimensional stagnation flow towards a shrinking sheet; second, to calculate the numerical solution of transformed nonlinear ordinary differential equations via the successive linearisation method. The numerical results for boundary layer Maxwell fluid in the stagnation flow induced by shrinking sheet by means of SLM are yet not available in the literature.

2. Mathematical Modeling

For the steady, incompressible stagnation flow equations for Maxwell fluid towards a shrinking sheet are of the following form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$
 (2.1)

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + \lambda \left(u^2 \frac{\partial^2 u}{\partial x^2} + 2uw \frac{\partial^2 u}{\partial x \partial z} + w^2 \frac{\partial^2 u}{\partial z^2} \right) = U\frac{dU}{dx} + v\frac{\partial^2 u}{\partial z^2}, \tag{2.2}$$

where u, v, and w are the velocity components along x, y, and z-axis, U = ax is the free stream velocity, v is the kinematic viscosity, and λ is the relaxation time.

The boundary conditions for the present problem are given by

$$u = b(x + c), \quad w = 0, \quad \text{at } y = 0,$$

$$u = ax, \quad w = -az, \quad \text{as } y \longrightarrow \infty,$$
(2.3)

where *a* is the strength of the stagnation flow, *b* is stretching rate (shrinking is b < 0), and *c* is the location of the stretching origin.

We define the following similarity variables [26]:

$$\eta = \sqrt{\frac{a}{v}}z, \qquad u = axf'(\eta) + bch(\eta), \qquad v = 0, \qquad w = -\sqrt{av}f(\eta).$$
(2.4)

Equation (2.1) is satisfied identically, and (2.2) yields

$$f''' + ff'' - f^{'2} + 1 - \beta \left(f^2 f''' - 2ff'f'' \right) = 0,$$
(2.5)

$$h'' + fh' - f'h - \beta \left(f^2 h'' - fhf'' \right) = 0.$$
(2.6)

Here primes denote differentiation with respect to η , and $\beta = \lambda c$ is the Deborah number. The boundary conditions are

$$f(0) = 0, \qquad f'(0) = \alpha, \qquad f'(\infty) = 1,$$
 (2.7)

$$h(0) = 1, \qquad h(\infty) = 0,$$
 (2.8)

where $\alpha = b/a$, $\alpha > 0$, and $\alpha < 0$ correspond to stretching and shrinking sheets, while $\alpha = 0$ correspond to planar stagnation flow towards a stationary sheet. Moreover, when $\alpha = 1$, the flow will be with no boundary layer.

3. Solution Method

In this section the implementation of the successive linearisation method (SLM) is solving the governing boundary value problems. The SLM (see, e.g., [34–38] for details) is based on transforming the governing nonlinear boundary value problem into an iterative scheme made up of linear differential equations which are subsequently solved using analytical or numerical methods wherever possible. Details of the formulae for implementing the SLM in one dimensional equations are given in [36].

In applying the SLM on (2.5), we set

$$f(\eta) = f_i(\eta) + \sum_{m=0}^{i-1} f_m(\eta), \quad i = 1, 2, 3, \dots,$$
(3.1)

where f_i are unknown functions that are obtained by iteratively solving the linearized version of the governing equations assuming that f_i ($0 \le m \le i - 1$) are known from previous iterations. The algorithm starts with an initial approximation $f_0(\eta)$ which is chosen to satisfy the boundary conditions (2.7). A suitable initial guess in this example is

$$f_0(\eta) = \eta + (\alpha - 1)(1 - e^{-\eta}). \tag{3.2}$$

To investigate the SLM application on (2.5), we write the equation as

$$\mathcal{L}(f, f', f'', f''', f'''') + \mathcal{N}(f, f', f'', f''', f'''') = 0,$$
(3.3)

where

$$\mathcal{L}(\dots) = f''',$$

$$\mathcal{N}(\dots) = ff'' - f^{'2} + 1 - \beta \left(f^2 f''' - 2ff'f'' \right).$$
(3.4)

Substituting (3.1) in (2.5) and linearising the resulting equation, we obtain the following governing equation:

$$f_i''' + a_{0,i-1}f_i''' + a_{1,i-1}f_i'' + a_{2,i-1}f_i' + a_{3,i-1}f_i = r_{i-1},$$
(3.5)

subject to the boundary conditions

$$f_i(0) = 0, \qquad f'_i(0) = 0, \qquad f'_i(\infty) = 0,$$
(3.6)

where $a_{p,i-1}$ (p = 0, 1, ..., 3) are defined using $\tilde{f}_m = \sum_{m=0}^{i-1} f_m$ (for compactness) as

$$a_{0,i-1} = -\beta \tilde{f}_{m}^{2},$$

$$a_{1,i-1} = \tilde{f}_{m} \left(1 + 2\tilde{f}_{m}^{\prime} \right),$$

$$a_{2,i-1} = -2\tilde{f}_{m}^{\prime} + 2\beta \tilde{f}_{m} \tilde{f}_{m}^{\prime\prime},$$

$$a_{3,i-1} = \tilde{f}_{m}^{\prime\prime} - 2\beta \tilde{f}_{m} \tilde{f}_{m}^{\prime\prime\prime} + 2\beta \tilde{f}_{m}^{\prime} \tilde{f}_{m}^{\prime\prime\prime}.$$
(3.7)

We solve (3.5) using the Chebyshev spectral collocation method. To allow for numerical implementation of the spectral method, the physical region $[0, \infty)$ is truncated to [0, L], where *L* is chosen to be sufficiently large. The truncated region is further transformed to the space [-1, 1] using the transformation

$$\xi = \frac{2}{L}\eta - 1.$$
(3.8)

As with any other numerical approximation method some sort of *discretization* is introduced to the interval [-1,1]. To this end, we choose the Gauss-Lobatto collocation points (see, e.g., [39-42]) to define the nodes in [-1,1] as

$$\xi_j = \cos\left(\frac{\pi j}{N}\right), \quad j = 0, 1, \dots, N, \tag{3.9}$$

where (N + 1) is the number of collocation points. The essence of the Chebyshev spectral collocation method is the concept of a differentiation matrix *D*. The differentiation matrix

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maps a vector of the function values $\mathbf{F} = [f(\xi_0), \dots, f(\xi_N)]^T$ at the collocation points to a vector \mathbf{F}' defined as

$$\mathbf{F}' = \sum_{k=0}^{N} D_{kj} f(\xi_k) = D\mathbf{F}.$$
(3.10)

In general, the derivative of order *p* for the function $f(\xi)$ can be expressed by

$$f^{(p)}(\xi) == D^p \mathbf{F}.$$
 (3.11)

The entries of D can be computed in different ways (see, e.g., [39–42]). In this work we use the method proposed by Trefethen [42] in the cheb.m Matlab m-file. Thus, applying the spectral method, with derivative matrices on linearised equations (3.5) and (3.6) leads to the following linear matrix system:

$$\mathbf{A}_{i-1}\mathbf{F}_i = \mathbf{R}_{i-1} \tag{3.12}$$

with the boundary conditions

$$f_i(\xi_N) = 0, \qquad \sum_{k=0}^N \mathbf{D}_{Nk} f_i(\xi_k) = 0, \qquad \sum_{k=0}^N \mathbf{D}_{0k} f_i(\xi_k) = 0, \qquad \sum_{k=0}^N \mathbf{D}_{0k}^2 f_i(\xi_k) = 0, \qquad (3.13)$$

where

$$\mathbf{A}_{i-1} = \mathbf{D}^3 + \mathbf{a}_{0,i-1}\mathbf{D}^3 + \mathbf{a}_{1,i-1}\mathbf{D}^2 + \mathbf{a}_{2,i-1}\mathbf{D} + \mathbf{a}_{3,i-1}$$
(3.14)

and $\mathbf{D} = (2/L)D$, $\mathbf{a}_{s,i-1}(s = 0, 1, ..., 3)$ are $(N + 1) \times (N + 1)$ diagonal matrices with $a_{s,i-1}(\xi_j)$, on the main diagonal and

$$\mathbf{F}_{i} = f_{i}(\xi_{j}), \qquad \mathbf{R}_{i} = r_{i}(\xi_{j}), \quad j = 0, 1, \dots, N.$$
 (3.15)

After imposing the boundary conditions (3.13), the solutions for f_i are obtained by iteratively solving the system (3.12) starting from the initial approximation (3.2). Once the solution for $f(\eta)$ has been found from solving (3.13), the Chebyshev pseudospectral method is directly applied on (2.6) which is now linear. This gives the following:

$$\mathbf{BH} = \mathbf{S} \tag{3.16}$$

with the boundary conditions

$$h(\xi_N) = 1, \qquad h(\xi_0) = 0,$$
 (3.17)

Table 1: Compa	arison of the re	esults of Wang [2	26] against	the present	SLM resul	ts for the o	case whe	$n \alpha >$	0
and $\beta = 0$.									

α	(Iter.)	0	0.1	0.2	1	2	5
	1	1.243490	1.156422	1.059691	0.000000	-1.861844	-9.792476
	2	1.232631	1.146594	1.051154	0.000000	-1.887189	-10.243081
f''(0)	3	1.232588	1.146561	1.051130	0.000000	-1.887307	-10.264639
	4	1.232588	1.146561	1.051130	0.000000	-1.887307	-10.264749
	5	1.232588	1.146561	1.051130	0.000000	-1.887307	-10.264749
Reference [26]		1.232588	1.14656	1.05113	0.000000	-1.88731	-10.26475
h'(0)		-0.811301	-0.863452	-0.913303	-1.253314	-1.589567	-2.338099
Reference [26]		-0.811301	-0.86345	-0.91330	-1.25331	-1.58957	-2.33810

Table 2: Comparison of the results of Wang [26] against the present SLM results for the case when $\alpha < 0$ and $\beta = 0$.

α	(Iter.)	-0.25	-0.5	-0.75	-1	-1.15	-1.15 (2nd Solution)
	1	1.413934	1.504746	1.494801	1.352084	1.182769	-0.267033
	2	1.402301	1.495718	1.489318	1.329038	1.088692	0.028223
	3	1.402241	1.495670	1.489298	1.328817	1.082272	0.109614
	4	1.402241	1.495670	1.489298	1.328817	1.082231	0.116650
	5	1.402241	1.495670	1.489298	1.328817	1.082231	0.116702
Reference [26]		1.40224	1.49567	1.48930	1.32882	1.08223	0.116702
<i>h</i> ′(0)		-0.668573	-0.501448	-0.293763	0	0.297995	0.2763445
Reference [26]		-0.66857	-0.50145	-0.29376	0	0.297995	0.276345

where

$$\mathbf{B} = \mathbf{D}^2 - \beta \mathbf{f}^2 \mathbf{D}^2 + \mathbf{f} \mathbf{D} - \mathbf{f}' + \beta \mathbf{f} \mathbf{f}'' = 0$$
(3.18)

and $\mathbf{H} = h(\xi_j)$, **S** is a vector of zeros, and all vectors in (3.18) are converted to a diagonal matrix. The boundary conditions (3.17) are placed on the first and last rows of **B** and **S**.

4. Results and Discussion

In this section we give results obtained by the successive linearisation method for selected values of important parameters of the Maxwell fluid over a shrinking sheet in the region of a stagnation point. To check the accuracy of the SLM employed in the current study, a comparison of the initial values is made with published results from literature. In particular, comparison is made with the skin friction coefficient f''(0) results and results for h'(0) of Wang [26] for the special case when $\beta = 0$. Tables 1 and 2 give the results for the friction coefficient f''(0) and h'(0) for $\alpha > 0$ and $\alpha < 0$, respectively. It can be seen from these tables that the present SLM results are in excellent agreement with those of Wang [26]. We also observe that SLM results converge rapidly with full convergence to the numerical results of [26] reached after only 3 or 4 iterations. This confirms the validity of the SLM method for solving the problem under study.



Figure 1: Effect of the shrinking parameter α ($\alpha < 0$) on $f'(\eta)$ and $h(\eta)$ when $\beta = -0.2$.



Figure 2: Effect of the stretching parameter α ($\alpha > 0$) on $f'(\eta)$ and $h(\eta)$ when $\beta = -0.2$.

For some negative values of α it was found that multiple solutions exist. To generate the multiple solution different initial approximations were used as first guesses of the SLM iteration scheme. The multiple solutions for f''(0) and h'(0) are resolved when $\alpha = -1.15$ and illustrated in Table 2 for the case $\beta = 0$. Again, good agreement with the results of [26] is observed. The presence of dual solutions has also been observed in related investigations stagnation flow in the presence of shrinking, (see, e.g., [26–33]).

The effect of the shrinking parameter α ($\alpha < 0$) of $f'(\eta)$ and $h(\eta)$ is depicted on Figure 1. We observe that shrinking of the sheet decelerates the fluid flow. Figure 2 shows the effect of the stretching parameter α ($\alpha > 0$) on $f'(\eta)$ and $h(\eta)$. We observe that the boundary layer thickness is decreased when values of α increase. Physically, this can be explained as follows: for fixed values of α corresponding to the stretching of the sheet, an increase in bimplies an increase in straining motion near the stagnation region resulting in the increased acceleration of the external stream, and this leads to the thinning of the boundary layer.

The influence of the Deborah number β , for both the shrinking ($\alpha < 0$) and stretching ($\alpha > 0$), can be seen from Figures 3 and 4. As shown in these figures, the velocity decreases with the increase of the Deborah number β or the relaxation time.



Figure 3: Effect of the parameter β on $f'(\eta)$ and $h(\eta)$ when $\alpha = -1$.



Figure 4: Effect of the parameter β on $f'(\eta)$ and $h(\eta)$ when $\alpha = 0.6$.

Figure 5 depicts the existence of the dual solutions for the velocity profiles. The effects of α and β on f''(0) and h'(0) are shown on Figure 6. In this figure, we clearly observe the domain of valid values for α which give rise to multiple solutions.

5. Concluding Remarks

The two-dimensional equations for Maxwell fluid are derived in this paper. The boundary layer in the region of stagnation point over a two-dimensional shrinking sheet is discussed for Maxwell fluid. The similarity transformations are used to transform the partial differential equations to ordinary ones, and hence a numerical solution is obtained using the SLM. The novel technique was compared against previous studies and excellent agreement was observed. The study also confirmed the existence of a dual solution. We also observed the relatively rapid convergence of the SLM. The results are presented graphically and the effects of the parameters are discussed.



Figure 5: Dual solutions: effect of the parameter β on $f'(\eta)$ and $h(\eta)$ when $\alpha = -1.15$.



Figure 6: Graphs of f''(0) and h'(0) against α for different values of β .

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