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Research Article

Robust Observer Design for Takagi-Sugeno Fuzzy Systems with Mixed Neutral and Discrete Delays and Unknown Inputs

Hamid Reza Karimi[1] and Mohammed Chadli[2]

[1] *Department of Engineering, Faculty of Engineering and Science, University of Agder, 4898 Grimstad, Norway*
[2] *Laboratory of Modeling, Information and Systems, University of Picardie Jules Verne, Amiens, France*

Correspondence should be addressed to Hamid Reza Karimi, hamid.r.karimi@uia.no

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A robust observer design is proposed for Takagi-Sugeno fuzzy neutral models with unknown inputs. The model consists of a mixed neutral and discrete delay, and the disturbances are imposed on both state and output signals. Delay-dependent sufficient conditions for the design of an unknown input T-S observer with time delays are given in terms of linear matrix inequalities. Some relaxations are introduced by using intermediate variables. A numerical example is given to illustrate the effectiveness of the given results.

1. Introduction

In recent years, there have been rapidly growing interests in stability analysis and synthesis of fuzzy control systems; several works concerning stability and state estimation for a class of systems described by Takagi-Sugeno (T-S) fuzzy models [1–7] have been carried out. Furthermore, some recent applications of fuzzy theory in engineering are reported in [8–10]. Based on the Lyapunov method, design conditions for controllers and observers are given in linear matrix inequalities (LMIs) formulation (see among others [11–19]). In the literature of the study, all of the existing results concern T-S fuzzy models with known inputs (see, e.g., [15, 17]). However, it is well known that state estimation for dynamic systems with time delays and unknown inputs or disturbances is an interesting research topic in the fields of robust control, system supervision, and fault-tolerant control [20–23]. Recently, the problem of H_∞ model reduction for Takagi-Sugeno (TS) fuzzy stochastic systems in [24], the problem of H_∞ model approximation for discrete-time Takagi-Sugeno (T-S) fuzzy time-delay systems in [25], and the problem of filtering for a class of discrete-time T-S fuzzy time-varying delay systems in [26] have been fully investigated.

On the other hand, stability of neutral systems proves to be a more complex issue because the system involves the derivative of the delayed state. Especially, in the past few decades, increased attention has been devoted to the problem of robust delay-dependent stability and stabilization via different approaches for linear neutral systems with delayed state and/or input and parameter uncertainties [27–30]. Recently, the problem of network-based feedback control for systems with mixed delays based on quantization and dropout compensation has been studied in [31].

However, to the best of our knowledge, the class of unknown input T-S neutral models has not yet been fully investigated in the past and remains to be important and challenging. This motivates the present study. Thus, the contributions of this paper are two-fold: (i) design of observers for T-S fuzzy models with mixed neutral and discrete time delays and unknown inputs which influence states and outputs simultaneously and (ii) for the addressed problem, the observer gains are computed by solving a convex optimization technique.

This paper is organized as follows. First, the considered observer structure for T-S fuzzy model with mixed time delays and unknown inputs is given. In Section 3, the main results are given for T-S models. Section 4 gives a numerical example to show the validity of the given results. At last, we conclude the paper in Section 5.

Notation. Throughout this paper, the notation $X > Y$, where X and Y are symmetric matrices, means that $X - Y$ is positive definite, and $\mathbf{R}^n$ and $\mathbf{R}^{n\times m}$ denote, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices respectively. Superscript "T" denotes matrix transposition, I is the identity matrix with compatible dimensions, the symbol $(*)$ denotes the transpose elements in the symmetric positions, $I_M = \{1, 2, \ldots, M\}$, and Σ^+ denotes any generalized inverse of matrix Σ with $\Sigma\Sigma^+\Sigma = \Sigma$. The operator $\operatorname{diag}\{\cdots\}$ represents a block diagonal matrix, and the operator $\operatorname{sym}(A)$ represents $A + A^T$.

2. Problem Formulation

Now, consider the following T-S fuzzy models with unknown inputs and different neutral and discrete time delays:

$$\begin{gathered}
\dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left(A_i x(t) + A_{\tau_i}\dot{x}(t-\tau) + A_{h_i}x(t-h) + B_i u(t) + R_i d(t) + H_i w(t)\right) \\
x(t) = \phi(t), \quad t \in [-\kappa, 0], \\
y(t) = Cx(t) + Fd(t) + Jw(t),
\end{gathered} \tag{2.1}$$

with

$$\mu_i(\xi(t)) \geq 0, \quad \sum_{i=1}^{M} \mu_i(\xi(t)) = 1, \tag{2.2}$$

where M is the number of submodels, $x(t) \in \mathbf{R}^n$ is the state vector, $u(t) \in \mathbf{R}^m$ is the input vector, $d(t) \in \mathbf{R}^{q_d}$ is the unknown input, $w(t) \in \mathbf{R}^{q_w}$ is the external disturbance vector, and

$y \in \mathbf{R}^p$ is the measured output. $A_i \in \mathbf{R}^{n\times n}$, $A_{\tau_i} \in \mathbf{R}^{n\times n}$, $B_i \in \mathbf{R}^{n\times m}$, and $C \in \mathbf{R}^{p\times n}$ define the ith local model. Matrices $R_i \in \mathbf{R}^{n\times q_d}$ and $F \in \mathbf{R}^{p\times q_d}$ represent the influence matrices of the unknown inputs, and $H_i \in \mathbf{R}^{n\times q_w}$ and $J \in \mathbf{R}^{p\times q_w}$ represent the influence matrices of the disturbances. The *activation functions* $\mu_i(\cdot)$ depend on the decision vector $\xi(t)$ assumed to depend on measurable variables. It can depend on the measurable state variables and be a function of the measurable outputs of the system and possibly of the known inputs [1, 32]. The time-varying function $\phi(t)$ is continuous vector-valued initial function, and τ and h are constant time delays with $\kappa := \max\{\tau, h\}$.

In this paper, we are concerned with the reconstruction of state variable $x(t)$ of unknown inputs T-S model (2.1) using measurable signals, that is, known input $u(t)$ and measured output $y(t)$. In order to estimate the state of the unknown input T-S fuzzy model (2.1), the considered unknown input observer structure has the following form:

$$\begin{gathered}\dot{z}(t) = \sum_{i=1}^{M}\mu_i(\xi(t))\left(N_i z(t) + N_{\tau_i}\dot{z}(t-\tau) + N_{h_i}z(t-h) + G_i u(t) + L_i y(t)\right)\\ z(t) = \varphi(t), \quad t \in [-\kappa, 0],\\ \widehat{x}(t) = z(t) - Ey(t),\end{gathered} \tag{2.3}$$

where the observer considers the same activation functions $\mu_i(\cdot)$ as used for the T-S model (2.1). The variables $N_i \in \mathbf{R}^{n\times n}$, $N_{\tau_i} \in \mathbf{R}^{n\times n}$, $G_i \in \mathbf{R}^{n\times m}$, $L_i \in \mathbf{R}^{n\times p}$, and $E \in \mathbf{R}^{n\times p}$ are the observer gains to be determined in order to estimate the state of the unknown input T-S model (2.1). The time-varying function $\varphi(t)$ is continuous vector-valued initial function. Now let us define the state estimation error

$$e(t) = x(t) - \widehat{x}(t). \tag{2.4}$$

From estimation error (2.4) with the expression of $\widehat{x}(t)$ given by the observer (2.3) and T-S model (2.1), we get

$$e(t) = (I + EC)x(t) - z(t) + EFd(t) + EJw(t). \tag{2.5}$$

The dynamic of state estimation error is then given by

$$\begin{aligned}\dot{e}(t) = \sum_{i=1}^{M}\mu_i(\xi)(&N_i e(t) + (TA_i - K_iC - N_i)x(t) + TA_{\tau_i}\dot{x}(t-\tau) - N_{\tau_i}\dot{z}(t-\tau)\\ &+ TA_{h_i}x(t-h) - N_{h_i}z(t-h) + (TB_i - G_i)u(t)\\ &+(TR_i - K_iF)d(t) + (TH_i - K_iJ)w(t)) + EF\dot{d}(t) + EJ\dot{w}(t),\end{aligned} \tag{2.6}$$

with

$$T = I + EC, \qquad K_i = N_iE + L_i. \tag{2.7}$$

Then, we get

$$\dot{e}(t) = \sum_{i=1}^{M}\mu_i(\xi(t))(N_i e(t) + N_{\tau_i}\dot{e}(t-\tau) + N_{h_i}e(t-h)), \tag{2.8}$$

if the following constraints hold

$$N_i = TA_i - K_i C, \tag{2.9}$$

$$TA_{\tau_i} - N_{\tau_i}T = 0, \tag{2.10}$$

$$TA_{h_i} - N_{h_i}T = 0, \tag{2.11}$$

$$TB_i - G_i = 0, \tag{2.12}$$

$$TR_i - K_i F = 0, \tag{2.13}$$

$$E[F \ \ J] = 0, \tag{2.14}$$

$$TH_i - K_i J = 0. \tag{2.15}$$

For description brevity, (2.8) can be written as

$$\dot{e}(t) = N_\mu e(t) + N_{\mu_\tau}\dot{e}(t-\tau) + N_{\mu_h}e(t-h), \tag{2.16}$$

where $N_\mu = \sum_{i=1}^{M}\mu_i(\xi(t))N_i$, $N_{\mu_h} = \sum_{i=1}^{M}\mu_i(\xi(t))N_{h_i}$, and $N_{\mu_\tau} = \sum_{i=1}^{M}\mu_i(\xi(t))N_{\tau_i}$.

Remark 2.1. It is noting that if E is determined, we get T from (2.7) and deduce directly G_i, N_{h_i}, and N_{τ_i} from (2.9)–(2.15). Then, it suffices to guarantee the stability of the dynamic system (2.8) under the constraint (2.9)–(2.15). Furthermore, (2.5) is simplified to $e(t) = (I + EC)x(t) - z(t)$.

In the following, LMIs design conditions satisfying $e(t) \to 0$ when $t \to \infty$ are given for continuous-time systems.

3. Synthesis Conditions

This section deals with the continuous-time T-S models. Sufficient LMIs conditions guaranteeing the global asymptotic convergence of state estimation error (2.4) are given by using slack variable to introduce relaxation.

Theorem 3.1. *The observer* (2.3) *converges asymptotically to the state of the continuous-time T-S model* (2.1), *if there exist matrices* $\overline{Q}_j > 0$, $j = 1,\ldots,4$, $\overline{S}_1, \overline{U}_l > 0$, $l = 1,\ldots,3$, *and* $\overline{P}_2, \overline{P}_3, \overline{N}_i, \overline{N}_{h_i}, \overline{N}_{d_i}$ *such that the conditions* (2.9)–(2.15) *and the following hold for all* $i \in I_M$*:*

$$\begin{bmatrix} \widehat{\Sigma}_{11_i} & \widehat{\Sigma}_{12_i} & \widehat{\Sigma}_{13_i} & -\overline{Q}_3 + \overline{Q}_4^T & \overline{N}_{\tau_i} \\ (*) & \widehat{\Sigma}_{22} & \overline{N}_{h_i} & 0 & \overline{N}_{\tau_i} \\ (*) & (*) & \widehat{\Sigma}_{33} & 0 & 0 \\ (*) & (*) & (*) & \widehat{\Sigma}_{44} & 0 \\ (*) & (*) & (*) & (*) & -\overline{U}_2 \end{bmatrix} < 0, \tag{3.1}$$

where $\widehat{\Sigma}_{11_i} = \mathrm{sym}\{\overline{N}_i + \overline{Q}_1 + \overline{Q}_3\} + \overline{U}_1 + \overline{S}_1$, $\widehat{\Sigma}_{12_i} = -\overline{P}_2^T + \overline{N}_i^T$, $\widehat{\Sigma}_{13_i} = \overline{N}_{h_i} - \overline{Q}_1 + \overline{Q}_2^T$, $\widehat{\Sigma}_{22} = \mathrm{sym}\{-\overline{P}_3\} + \overline{U}_2 + d\overline{U}_3 + h\overline{S}_1$, $\widehat{\Sigma}_{33} = -\overline{S}_1 - \mathrm{sym}\{\overline{Q}_2\}$, *and* $\widehat{\Sigma}_{44} = -\overline{U}_1 - \mathrm{sym}\{\overline{Q}_4\}$.

Proof. To investigate the delay-dependent asymptotically stable analysis of the error system (2.8), we define a class of Lyapunov-Krasovskii functions as follows:

$$V(t) = \sum_{i=1}^{3} V_i(t), \tag{3.2}$$

where

$$\begin{aligned} V_1(t) &= e^T(t)P_1 e(t), \\ V_2(t) &= \int_{t-h}^{t} e^T(\theta)S_1 e(\theta)d\theta + \int_{-h}^{0}\int_{t+\tau}^{t} \dot{e}^T(\theta)S_2\dot{e}(\tau)d\theta\, d\tau, \\ V_3(t) &= \int_{t-\tau}^{t} e^T(\theta)U_1 e(\theta)d\theta + \int_{t-\tau}^{t} \dot{e}^T(s)U_2\dot{e}(s)ds + \int_{-\tau}^{0}\int_{t+\tau}^{t} \dot{e}^T(\theta)U_3\dot{e}(\tau)d\theta\, d\tau. \end{aligned} \tag{3.3}$$

Time derivative of $V_1(t)$ along the system trajectory (2.16) becomes

$$\dot{V}_1(t) = 2e^T(t)P_1\left(N_\mu e(t) + N_{\mu_\tau}\dot{e}(t-\tau) + N_{\mu_h} e(t-h)\right), \tag{3.4}$$

Then by taking the time derivative of $V_2(t)$ and $V_3(t)$, one can read

$$\dot{V}_2(t) = \&e^T(t)S_1 e(t) - e^T(t-h)S_1 e(t-h) + h\dot{e}^T(t)S_1\dot{e}(t) - \int_{t-h}^{t} \dot{e}^T(\theta)S_1\dot{e}(\theta)d\theta, \tag{3.5}$$

$$\begin{aligned} \dot{V}_3(t) = {} & e^T(t)U_1 e(t) - e^T(t-\tau)U_1 e(t-\tau) + \dot{e}^T(t)U_2\dot{e}(t) - \dot{e}^T(t-\tau)U_2\dot{e}(t-\tau) \\ & + \tau\dot{e}^T(t)U_3\dot{e}(t) - \int_{t-\tau}^{t} \dot{e}^T(\tau)U_3\dot{e}(\tau)d\tau. \end{aligned} \tag{3.6}$$

Moreover, from (2.16), the following equation holds for any matrices P_2 and P_3 with appropriate dimensions:

$$2\left(e^T(t)P_2^T + \dot{e}^T(t)P_3^T\right) \times \left(-\dot{e}(t) + N_\mu e(t) + N_{\mu_\tau}\dot{e}(t-\tau) + N_{\mu_h} e(t-h)\right) = 0. \tag{3.7}$$

Furthermore, from the Leibniz-Newton formula, the following equations hold for any matrices $Q_j, j = 1, \ldots, 4$, with appropriate dimensions:

$$
\begin{aligned}
&2\left(e^T(t)Q_1 + e^T(t-h)Q_2\right)\left(e(t) - e(t-h) - \int_{t-h}^{t} \dot{e}(s)ds\right) = 0, \\
&2\left(e^T(t)Q_3 + e^T(t-\tau)Q_4\right)\left(e(t) - e(t-\tau) - \int_{t-\tau}^{t} \dot{e}(s)ds\right) = 0.
\end{aligned}
\tag{3.8}
$$

From the obtained derivative terms in (3.4)–(3.6) and adding the left-hand side of (3.7)–(24) into $\dot{V}(t)$, we obtain the following result for $\dot{V}(t)$:

$$
\begin{aligned}
\dot{V}(t) = {} & \chi^T(t)\Pi\chi(t) \\
& - \int_{t-h}^{t} \left(\chi^T(t)\nu_1 + \dot{e}(s)S_1\right)\overline{S}_1\left(\chi^T(t)\nu_1 + \dot{e}(s)S_1\right)^T ds \\
& - \int_{t-\tau}^{t} \left(\chi^T(t)\nu_2 + \dot{e}(s)U_3\right)\overline{U}_3\left(\chi^T(t)\nu_2 + \dot{e}(s)U_3\right)^T ds,
\end{aligned}
\tag{3.9}
$$

or equivalently,

$$
\dot{V}(t) = \chi^T(t)\Pi\chi(t), \tag{3.10}
$$

with $\Pi := \Xi + h\nu_1\overline{S}_1\nu_1^T + \tau\nu_2\overline{U}_3\nu_2^T$, $\overline{S}_1 := S_1^{-1}$, $\overline{U}_3 := U_3^{-1}$, $\chi(t) = \text{col}\{e(t), \dot{e}(t), e(t-h), e(t-\tau), \dot{e}(t-\tau)\}$, $\nu_1 = \text{col}\{Q_1, 0, Q_2, 0, 0\}$, $\nu_2 = \text{col}\{Q_3, 0, 0, Q_4, 0\}$, and

$$
\Xi = \begin{bmatrix}
\Sigma_{11} & \Sigma_{12} & \Sigma_{13} & -Q_3 + Q_4^T & \Sigma_{15} \\
(*) & \Sigma_{22} & P_3^T N_{\mu_h} & 0 & P_3^T N_{\mu_\tau} \\
(*) & (*) & \Sigma_{33} & 0 & 0 \\
(*) & (*) & (*) & \Sigma_{44} & 0 \\
(*) & (*) & (*) & (*) & -U_2
\end{bmatrix},
\tag{3.11}
$$

where $\Sigma_{11} = \text{sym}\{(P_1 + P_2^T)N_\mu + Q_1 + Q_3\} + U_1 + S_1$, $\Sigma_{12} = -P_2^T + N_\mu^T P_3$, $\Sigma_{13} = (P_1 + P_2^T)N_{\mu_h} - Q_1 + Q_2^T$, $\Sigma_{15} = (P_1 + P_2^T)N_{\mu_\tau}$, $\Sigma_{22} = \text{sym}\{-P_3\} + U_2 + \tau U_3 + hS_1$, $\Sigma_{33} = -S_1 - \text{sym}\{Q_2\}$, and $\Sigma_{44} = -U_1 - \text{sym}\{Q_4\}$.

If a constant scalar $\kappa > 0$ satisfies the following condition:

$$
\overline{\Pi} = \Pi + \widetilde{\Pi} < 0, \tag{3.12}
$$

where

$$\widetilde{\Pi} := \begin{bmatrix} \kappa I & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \tag{3.13}$$

then

$$\begin{aligned} \dot{V}(t) &\leq \chi^T(t)\Pi\chi(t) = \chi^T(t)\left(\overline{\Pi} - \widetilde{\Pi}\right)\chi(t) \\ &< -\chi^T(t)\widetilde{\Pi}\chi(t). \end{aligned} \tag{3.14}$$

From the previous inequality, we can easily obtain, for all $\hat{e}(t) \neq 0$,

$$\dot{V}(t) < -\kappa\|\hat{e}(t)\|^2 < 0. \tag{3.15}$$

Based on Lyapunov stability theory, the error system (2.16) is asymptotically stable.

However, the condition (3.12) is not strict LMI due to the nonconvex constraints in the matrix indices, and, thus, one always has difficulties to get solutions satisfying the constraint. In order to find the solutions of (3.12), the obtained sufficient condition is now changed by some manipulations. The inequality $\overline{\Pi} < 0$ yields (by Schur complements)

$$\sum_{i=1}^{M}\mu_i(\xi(t)) \begin{bmatrix} \Xi_i + \widetilde{\Pi} & h\nu_1 & \tau\nu_2 \\ (*) & -hS_1 & 0 \\ (*) & (*) & -\tau U_3 \end{bmatrix} < 0, \tag{3.16}$$

with

$$\Xi_i = \begin{bmatrix} \Sigma_{11_i} & \Sigma_{12_i} & \Sigma_{13_i} & -Q_3 + Q_4^T & \Sigma_{15_i} \\ (*) & \Sigma_{22} & P_3^T N_{h_i} & 0 & P_3^T N_{\tau_i} \\ (*) & (*) & \Sigma_{33} & 0 & 0 \\ (*) & (*) & (*) & \Sigma_{44} & 0 \\ (*) & (*) & (*) & (*) & -U_2 \end{bmatrix}, \tag{3.17}$$

where $\Sigma_{11_i} = \mathrm{sym}\{(P_1 + P_2^T)N_i + Q_1 + Q_3\} + U_1 + S_1$, $\Sigma_{12_i} = -P_2^T + N_i^T P_3$, $\Sigma_{13_i} = (P_1 + P_2^T)N_{h_i} - Q_1 + Q_2^T$, and $\Sigma_{15_i} = (P_1 + P_2^T)N_{\tau_i}$. It is clear that $\Xi_i < 0$ result in LMI stabilization conditions.

It can be easily seen that the matrices $(P_1 + P_2)^T$ and P_3 are nonsingular. Let $\varsigma :=$ $\mathrm{diag}\{((P_1 + P_2)^T)^{-1}, P_3^{-1}, ((P_1 + P_2)^T)^{-1}, ((P_1 + P_2)^T)^{-1}, P_3^{-1}\}$. By premultiplying ς, postmultiplying ς^T to $\Xi_i < 0$, and using the definitions $\overline{N}_i := N_i(P_1 + P_2)^{-1}$, $\overline{N}_{h_i} := N_{h_i}(P_1 + P_2)^{-1}$, $\overline{N}_{\tau_i} :=$ $N_{\tau_i}P_3^{-1}$, $\overline{Q}_i := (P_1 + P_2^T)^{-1}Q_i(P_1 + P_2)^{-1}$, $\overline{U}_1 := (P_1 + P_2^T)^{-1}U_1(P_1 + P_2)^{-1}$, $\overline{U}_2 := P_3^{-1}U_1(P_3^T)^{-1}$, $\overline{P}_2 := P_3^{-1}P_2(P_1 + P_2)^{-1}$, and $\overline{P}_3 := P_3^{-1}$, one can obtain the LMI (3.1). This completes the proof. □

Remark 3.2. The equality constraints (2.13)–(2.15) can be rewritten in the following equivalent form:

$$[E \ \ \mathbf{K}]\begin{bmatrix} C\mathbf{R} & C\mathbf{H} & [F \ J] \\ -\mathbf{F} & -\mathbf{J} & 0 \end{bmatrix} = [-\mathbf{R} \ \ -\mathbf{H} \ \ 0], \tag{3.18}$$

with

$$\mathbf{H} = [H_1 \ \ H_2 \ \ \cdots \ \ H_M], \qquad \mathbf{K} = [K_1 \ \ K_2 \ \ \cdots \ \ K_M],$$
$$\mathbf{R} = [R_1 \ \ R_2 \ \ \cdots \ \ R_M],$$
$$\mathbf{F} = \left.\begin{bmatrix} F & 0 & \cdots & 0 \\ 0 & F & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & F \end{bmatrix}\right\}, \qquad \mathbf{J} = \left.\begin{bmatrix} J & 0 & \cdots & 0 \\ 0 & J & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & J \end{bmatrix}\right\} M \text{ times}, \tag{3.19}$$

where $\mathbf{F} \in \mathbf{R}^{M \cdot p \times M \cdot q_d}$, $\mathbf{H} \in \mathbf{R}^{n \times M \cdot q_w}$, $\mathbf{R} \in \mathbf{R}^{n \times M \cdot q_d}$, and $\mathbf{K} \in \mathbf{R}^{n \times M \cdot p}$. A necessary and sufficient condition for the existence of a solution $[E, \mathbf{K}]$ to (3.18) is [33]

$$\operatorname{rank}\begin{bmatrix} C\mathbf{R} & C\mathbf{H} & [F \ J] \\ -\mathbf{F} & -\mathbf{J} & 0 \\ -\mathbf{R} & -\mathbf{H} & 0 \end{bmatrix} = \operatorname{rank}\begin{bmatrix} C\mathbf{R} & C\mathbf{H} & [F \ J] \\ -\mathbf{F} & -\mathbf{J} & 0 \end{bmatrix}. \tag{3.20}$$

Note that under this condition, a solution is obtained by

$$\begin{aligned} [E \ \ \mathbf{K}] &= [-\mathbf{R} \ \ -\mathbf{H} \ \ 0]\begin{bmatrix} C\mathbf{R} & C\mathbf{H} & [F \ J] \\ -\mathbf{F} & -\mathbf{J} & 0 \end{bmatrix}^{+} \\ &\quad - Z\left(I - \begin{bmatrix} C\mathbf{R} & C\mathbf{H} & [F \ J] \\ -\mathbf{F} & -\mathbf{J} & 0 \end{bmatrix}\begin{bmatrix} C\mathbf{R} & C\mathbf{H} & [F \ J] \\ -\mathbf{F} & -\mathbf{J} & 0 \end{bmatrix}^{+}\right), \end{aligned} \tag{3.21}$$

where Z is an arbitrary matrix [33].

Remark 3.3. According to Theorem 3.1, the observer parameters can be calculated in the following:

(1) calculate $N_{\tau_i} = \overline{N}_{\tau_i} P_3$,

(2) compute T by solving (2.10) and calculate E from $T = I + EC$,

(3) compute N_{h_i} by solving (2.11),

(4) compute G_i by solving (2.12),

(5) compute $N_i = \overline{N}_i \overline{N}_{h_i}^{-1} N_{h_i}$,

(6) compute K_i by solving (2.9),

(7) compute L_i from $L_i = K_i - N_i E$.

Remark 3.4. It is worth noting that the number of the variables to be determined in the LMI (3.1) is $3n^2(M+2)+4n$.

Remark 3.5. The reduced conservatism of Theorem 3.1 benefits from the construction of the Lyapunov-Krasovskii functional in (3.2), introducing some free weighting matrices to express the relationship among the system matrices, and neither the model transformation approach nor any bounding technique is needed to estimate the inner product of the involved crossing terms. It can be easily seen that results of this paper are quite different from existing results in the literature in the following perspective. The structures at most of references, for instance [20], consider a delay-free T-S fuzzy system and in comparison to our case do not center on time delays, that is, the results in the previous reference cannot be directly applied to the T-S fuzzy models with unknown inputs and different neutral and discrete-time delays.

4. Numerical Example

To show the validness of the proposed results, a numerical example is proposed for the discrete-time T-S model (2.1) with the following data:

$$
\begin{gathered}
A_1 = \begin{bmatrix} -0.4 & 0.2 & 0.3 \\ 0.3 & -0.6 & 0.3 \\ 0.4 & 0.2 & 0.6 \end{bmatrix}, \\
A_2 = \begin{bmatrix} -0.45 & 0.375 & 0.375 \\ 0.15 & -0.45 & 0 \\ 0.75 & 0.75 & -0.45 \end{bmatrix}, \\
A_{h_1} = \begin{bmatrix} -0.01 & 0.03 & 0.02 \\ 0.01 & -0.005 & 0 \\ 0.005 & 0.07 & -0.03 \end{bmatrix}, \\
A_{h_2} = \begin{bmatrix} -0.005 & 0.01 & 0.04 \\ 0.01 & -0.025 & 0.02 \\ 0.05 & 0.03 & -0.01 \end{bmatrix}, \\
A_{\tau_1} = \begin{bmatrix} 0.0045 & -0.0037 & -0.0037 \\ -0.0015 & 0.0045 & 0 \\ -0.0075 & -0.0075 & 0.0045 \end{bmatrix}, \\
A_{\tau_2} = \begin{bmatrix} 0.0170 & -0.0115 & -0.0135 \\ -0.0090 & 0.0210 & -0.0060 \\ -0.0230 & -0.0190 & -0.0030 \end{bmatrix}, \\
B_1 = \begin{bmatrix} 1.0 \\ -0.5 \\ -0.5 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -0.5 \\ 1.0 \\ -0.5 \end{bmatrix}, \qquad F = \begin{bmatrix} 1 \\ 1.2 \end{bmatrix}, \\
R_1 = \begin{bmatrix} 0.4 \\ -0.4 \\ 0.4 \end{bmatrix}, \qquad R_2 = \begin{bmatrix} 0.4 \\ 0.4 \\ -0.8 \end{bmatrix}, \; C = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \\
J = 0, \qquad H = 0, \qquad \tau = 0.05, \qquad h = 0.1, \qquad A_{\tau_1} = 0.1 I_3, \qquad A_{\tau_2} = 0.7 I_3.
\end{gathered}
\tag{4.1}
$$

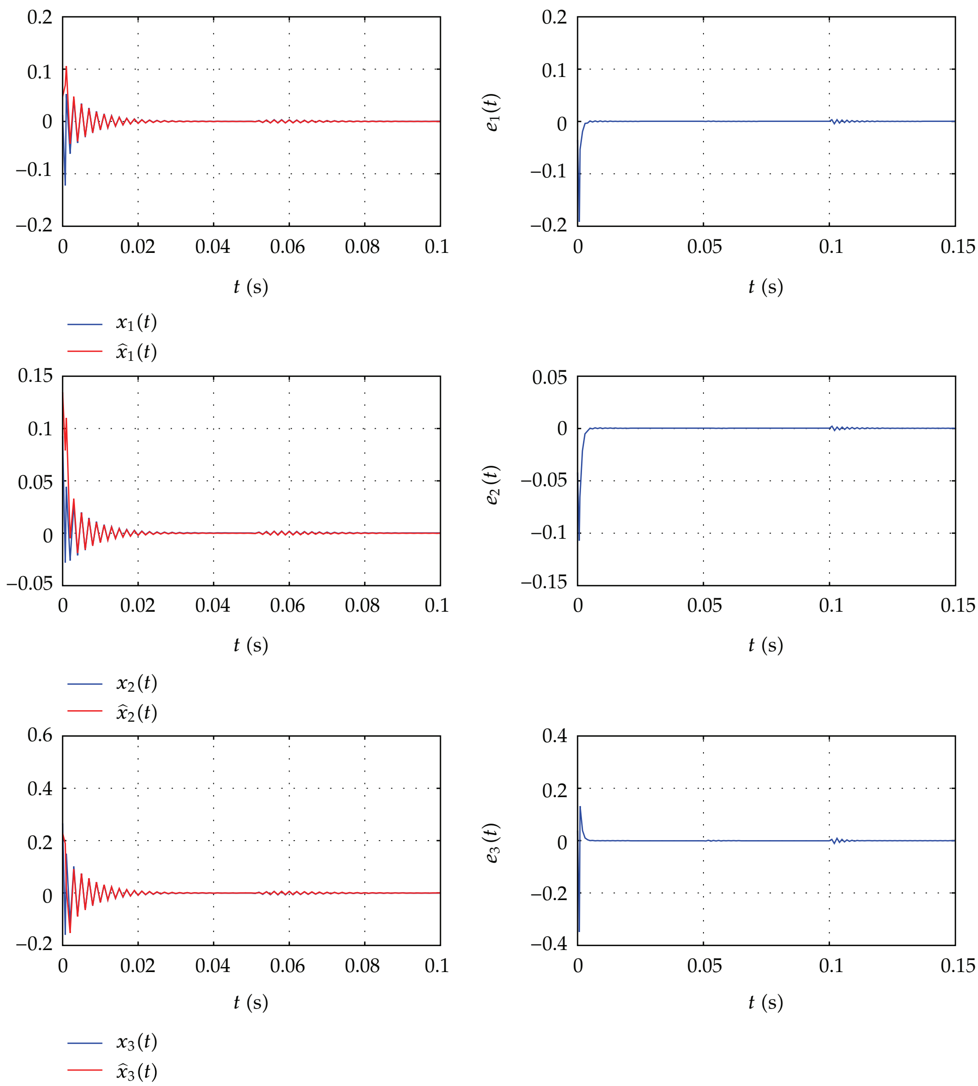

Figure 1: Time behaviour of states and corresponding error signals.

In the light of Theorem 3.1 and Remark 3.2, we solved LMIs and obtained the following observer gains by using Matlab LMI Control Toolbox [34]

$$E = \begin{bmatrix} 0.1945 & -0.1508 \\ -0.7474 & 0.6300 \\ -0.5524 & 0.4682 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 0.7363 \\ 0.5659 \\ 0.2927 \end{bmatrix}, \qquad G_2 = \begin{bmatrix} -0.2518 \\ -0.0032 \\ -1.2461 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} -0.2699 & -0.2149 & 0.0543 \\ -0.2053 & -0.1778 & 0.0849 \\ -0.0775 & -0.0530 & -0.2330 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} -0.2734 & -0.1996 & -0.0930 \\ -0.4198 & -0.3263 & -0.1422 \\ 1.1909 & 0.9204 & 0.3656 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 0.1565 & 0.0973 \\ 0.2284 & -0.1071 \\ 0.6159 & 0.1294 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 0.3221 & 0.0557 \\ 0.0335 & 0.3480 \\ 0.8122 & -1.3116 \end{bmatrix}. \tag{4.2}$$

For simulation purpose, we simply choose $w(t) = t/(1+t^2)$ as the disturbance, $u(t) = e^{-t}\sin(t)$ as the input signal, and $\mu_1(\xi) = (1 - \sin(x_2(t)))/2$ and $\mu_2(\xi) = (1 + \sin(x_2(t)))/2$ as activation functions. The error signals for an input are depicted in Figure 1. It is seen that the state estimation for the systems is performed as well.

5. Conclusion

In this paper, a robust observer design was proposed for Takagi-Sugeno (T-S) fuzzy neutral models with unknown inputs. The model consists of a mixed neutral and discrete delay, and the disturbances are imposed on both state and output signals. Delay-dependent sufficient conditions for the design of an unknown input T-S observer with time delays were given in terms of linear matrix inequalities (LMIs). Some relaxations were introduced by using intermediate variables. A numerical example was given to illustrate the effectiveness of the given results. Extension to the case of unmeasured decision variables is considered as a challenging problem. A numerical example has shown the effectiveness of the proposed results. Future work will investigate fault detection and Markovian jump systems for fuzzy systems with unknown inputs and time delays (see for instance [25–27, 35–37]).

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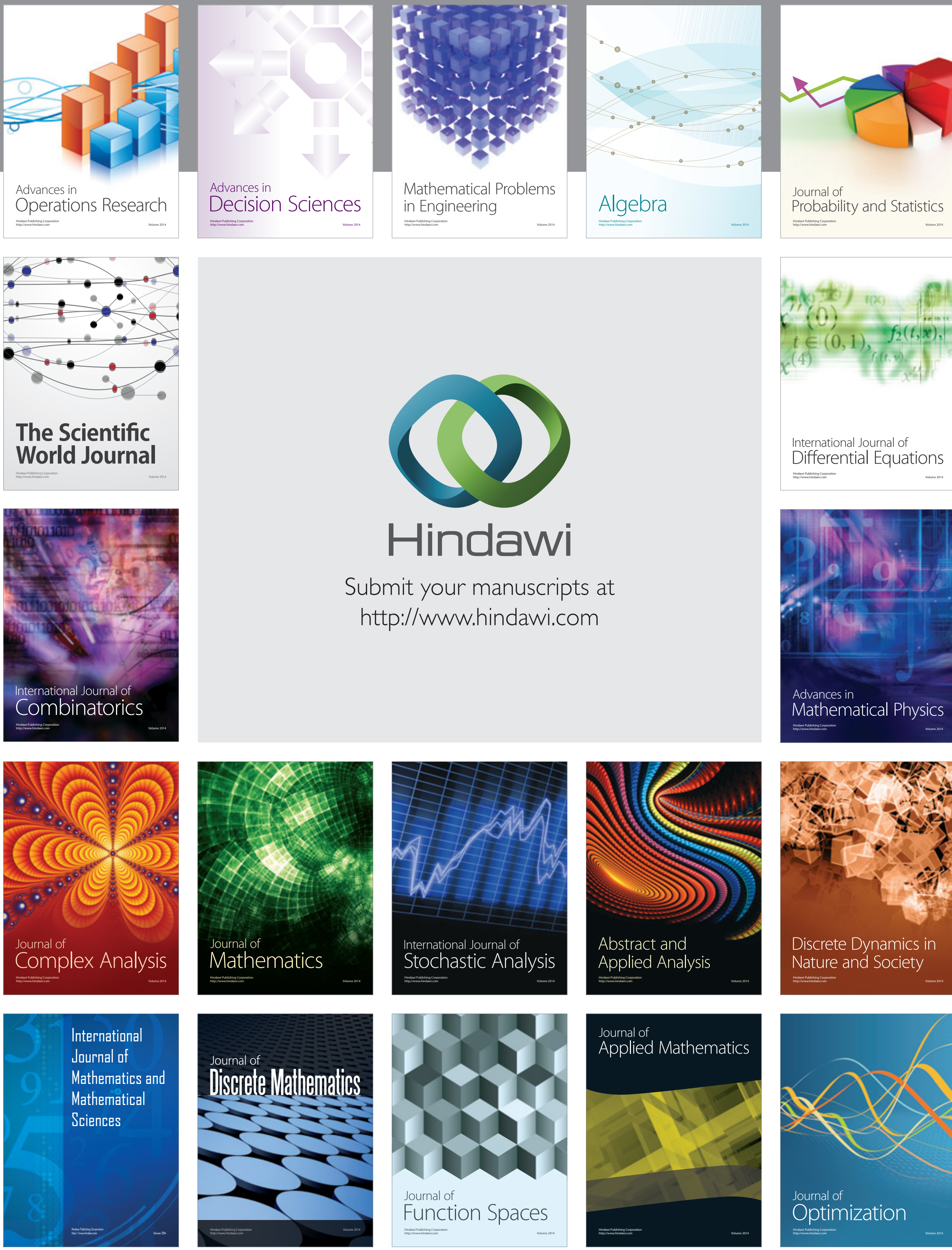

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