

Research Article

Estimation for Stochastic Nonlinear Systems with Randomly Distributed Time-Varying Delays and Missing Measurements

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The estimation problem is investigated for a class of stochastic nonlinear systems with distributed time-varying delays and missing measurements. The considered distributed time-varying delays, stochastic nonlinearities, and missing measurements are modeled in random ways governed by Bernoulli stochastic variables. The discussed nonlinearities are expressed by the statistical means. By using the linear matrix inequality method, a sufficient condition is established to guarantee the mean-square stability of the estimation error, and then the estimator parameters are characterized by the solution to a set of LMIs. Finally, a simulation example is exploited to show the effectiveness of the proposed design procedures.

1. Introduction

In the past decades, estimation techniques have been extensively investigated in many complex dynamical processes of networks such as target tracking [1], advanced aircrafts, and manufacturing processes. A number of estimation methods have been proposed in the literature, most of them are under the assumption that the measurements always contain true signals with the disturbances and the noises, see for example, [2–9]. But, in practical applications, the measurements may contain missing measurements due to many reasons such as the sensor temporal failures, network congestion, multipath fading, and high maneuverability of the tracked targets. Because of the clear engineering signification, the estimation problems with missing measurements have received attention, see for example [10–22].

Recently, with the rapid development of networks, novel methods and flexible models have been devoted, but the research of missing measurements is still a challenge, and the Bernoulli-based distributed model has still been a hot approach to modeling the missing observation cases. For example, in [10], the missing probability for each sensor is governed by an individual random variable satisfying a certain probabilistic distribution over the interval $[0, 1]$. Packet dropouts and communication delays are considered simultaneously in [12]. The variance-constrained dissipative control problem for a class of stochastic nonlinear systems with multiple degraded measurements in [13], where the degraded probability for each sensor is governed by an individual random variable satisfying a certain probabilistic distribution over a given interval. The H_∞ filtering problem has been addressed in [20] for a class of nonlinear systems with randomly occurring incomplete information, where the considered incomplete information includes both the sensor saturations and the missing measurements, a regional sensor model has been designed to account for both the randomly occurring sensor saturation and missing measurement in a unified representation, based on this sensor model, a newfangled H_∞ filter with a certain ellipsoid constraint has been researched such that the filtering error dynamics is locally mean-square asymptotically stable and the H_∞ -norm requirement is satisfied.

On the other hand, time delays are frequently encountered in real-world application such as communications, engineering, and biological systems. The occurrence of time delays may induce instability, oscillation, and poor performance. Consequently, research on time-delay systems has been a topic of recurring interest over the past decades. Current efforts can be classified into several categories, for example, simple delay and multiple delays [12], delay-independence [23, 24] and delay-dependence [5, 8, 25–30], time-varying delays [31, 32] and constant delays, retarded-type delay and neutral-type delay [30, 33], and mixed delays [34, 35]. However, in some applications, such as these systems connected over a wireless networks/or neural networks, as pointed out in [36], networks usually have a spatial extent due of the presence of a multitude of parallel pathways with a variety of axon sizes and lengths, and therefore the propagation delays can be distributed over a period of time, so it is essential to describe the distributed time delay under the probability framework as possible as. In this paper, the probability distribution of the time-vary delays are described for Itô type discrete-time stochastic distribution by a binary switching sequence satisfying the Bernoulli-distributed model.

Motivated by the aforementioned discussions, in this paper, we model the stochastic nonlinearities, the missing measurements, and the distributed time-vary delays by Bernoulli distributed white sequence with known conditional probability distribution. We aim at designing an estimator such that, for all possible measurements missing and distributed time-vary delays to obtain the estimation error system mean-square stable. The solvability of the addressed estimation problem can be expressed as the feasibility of a set of LMIs. Finally, a numerical simulation example is exploited to show the effectiveness of the results derived. The main contributions of this paper are summarized as the following: (1) a new estimation problem is studied for the stochastic nonlinear systems with both distributed time-vary delays and measurements missing phenomenon; (2) a mean-square stable performance is taken into consideration for the addressed stochastic nonlinear systems with distributed time-vary delays and missing measurements.

The rest of this paper is organized as follows. Section 2 briefly introduces the problem under consideration. In Section 3, a sufficient condition is established such that, for the missing measurements, the randomly distributed time-varying delays and nonlinearities,

the estimation error system is the mean-square stability. A numerical example is given in Section 4. This paper is concluded in Section 5.

Notations. The notation used here is fairly standard except where otherwise stated. \mathbb{R}^n , $\mathbb{R}^{n \times m}$, and \mathbb{I}^+ denote, respectively, the n -dimension Euclidean space, the set of all $n \times m$ real matrices, and the set of nonnegative integers. $(\Omega, \mathcal{F}, \{\mathcal{F}_k\}_{k \in \mathbb{I}^+}, \mathbb{P})$ is complete filtered probability space, Ω is the sample space, \mathcal{F} is the σ -algebra of subsets of the sample space, and \mathbb{P} is the probability measure on \mathcal{F} . $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x . $\text{Prob}\{\cdot\}$ is used for the occurrence probability of the event “ \cdot ”. The superscript “ T ” stands for matrix transposition. $P > 0$ ($P \geq 0$) means that matrix P is real symmetric and positive definite (positive semi-definite). $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of a matrix. I and 0 represent the identity matrix and the zero matrix with appropriate dimensions, respectively. $\text{diag}\{X_1, X_2, \dots, X_n\}$ stands for a block-diagonal matrix with matrices X_1, X_2, \dots, X_n on the diagonal. In symmetric block matrices or long matrix expressions, we use “ $*$ ” to represent a term, that is, induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulation and Preliminaries

Consider the following class of stochastic nonlinear system with distributed time-varying delays:

$$\begin{aligned} x(k+1) &= Ax(k) + \kappa_1(k)B \sum_{m=-\tau(k)}^{-1} x(k+m) + \kappa_2(k)f(x(k)) + E_1x(k)\omega(k), \\ y(k) &= \kappa_3(k)Cx(k) + E_2x(k)\omega(k), \\ z(k) &= H_1x(k), \end{aligned} \quad (2.1)$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $y(k) \in \mathbb{R}^m$ is the measured output vector, $z(k) \in \mathbb{R}^q$ is the signal to be estimated, $\omega(k)$ is a one-dimensional, zero-mean, Gaussian white noise sequence on a probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_k\}_{k \in \mathbb{I}^+}, \mathbb{P})$ with $\mathbb{E}\{\omega^2(k)\} = 1$, A, B, C, E_1, E_2 , and H_1 are known real constant matrices with appropriate dimensions, $\tau(k)$ denoting time-varying delays are positive integers and bounded, namely, $0 < \tau_l \leq \tau(k) \leq \tau_u$, the stochastic variables $\kappa_1(k) \in \mathbb{R}$, $\kappa_2(k) \in \mathbb{R}$, and $\kappa_3(k) \in \mathbb{R}$ are Bernoulli distributed white sequence taking the values of 0 and 1 with

$$\text{Prob}\{\kappa_1(k) = 1\} = \mathbb{E}\{\kappa_1(k)\} := \alpha_1, \quad (2.2)$$

$$\text{Prob}\{\kappa_1(k) = 0\} := 1 - \alpha_1, \quad (2.3)$$

$$\text{Prob}\{\kappa_2(k) = 1\} = \mathbb{E}\{\kappa_2(k)\} := \alpha_2, \quad (2.4)$$

$$\text{Prob}\{\kappa_2(k) = 0\} := 1 - \alpha_2, \quad (2.5)$$

$$\text{Prob}\{\kappa_3(k) = 1\} = \mathbb{E}\{\kappa_3(k)\} := \alpha_3, \quad (2.6)$$

$$\text{Prob}\{\kappa_3(k) = 0\} := 1 - \alpha_3, \quad (2.7)$$

where $\alpha_1 \in [0, 1]$, $\alpha_2 \in [0, 1]$, and $\alpha_3 \in [0, 1]$ are known positive scalars.

Remark 2.1. The nonlinear stochastic $f(x(k))$ is assumed to have the following for all $x(k)$:

$$\begin{aligned}\mathbb{E}\{f(x(k)) \mid x(k)\} &= 0, \\ \mathbb{E}\{f(x(k)) f^T(x(k)) \mid x(k)\} &= 0, \quad k \neq j, \\ \mathbb{E}\{f(x(k)) f^T(x(k)) \mid x(k)\} &\leq \sum_{i=1}^q \Pi_i x^T(k) \Phi_i x(k),\end{aligned}\tag{2.8}$$

where q is a known nonnegative integer, $\Pi_i = \bar{\Pi}_i \bar{\Pi}_i^T$, Π_i , $\bar{\Pi}_i$, and Φ_i ($i = 1, \dots, q$) are known matrices with appropriate dimensions. For convenience, one assumes that $f(x(k))$ is unrelated with $\kappa_1(k)$, $\kappa_2(k)$, $\kappa_3(k)$, and $\omega(k)$.

In this paper, we aim at designing a linear estimator of the following structure:

$$x_f(k+1) = A_f x_f(k) + A_k y(k), \quad \hat{z}(k) = H_2 x_f(k), \quad \hat{z}(0) = 0,\tag{2.9}$$

where $x_f \in \mathbb{R}^n$ is the state estimate, $\hat{z}(k)$ is the estimate output, H_2 is a known real constant matrix with appropriate dimension, and A_f and A_k are estimator parameters to be determined.

By defining $\hat{x}(k) = [x^T(k) \ x_f^T(k)]^T$, we have the following augmented system:

$$\begin{aligned}\hat{x}(k+1) &= \mathcal{A} \hat{x}(k) + \bar{\mathcal{A}} \hat{x}(k) + \mathcal{B} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) + \bar{\mathcal{B}} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \\ &\quad + \mathcal{N} h(k) + \bar{\mathcal{N}} h(k) + \mathcal{E} \hat{x}(k) \omega(k),\end{aligned}\tag{2.10}$$

where

$$\begin{aligned}\mathcal{A} &= \begin{bmatrix} A & 0 \\ \alpha_3 A_k C & A_f \end{bmatrix}, \quad \bar{\mathcal{A}} = \begin{bmatrix} 0 & 0 \\ (\kappa_3(k) - \alpha_3) A_k C & 0 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} \alpha_1 B & 0 \\ 0 & 0 \end{bmatrix}, \\ \bar{\mathcal{B}} &= \begin{bmatrix} (\kappa_1(k) - \alpha_1) B & 0 \\ 0 & 0 \end{bmatrix}, \quad \hat{x}(k+i) = \begin{bmatrix} x(k+i) \\ x_f(k+i) \end{bmatrix}, \quad h(k) = \begin{bmatrix} f(x(k)) \\ 0 \end{bmatrix}, \\ \mathcal{E} &= \begin{bmatrix} E_1 & 0 \\ A_k E_2 & 0 \end{bmatrix}, \quad \widehat{\mathcal{N}} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{N} = \alpha_2 \widehat{\mathcal{N}}, \quad \bar{\mathcal{N}} = (\kappa_2(k) - \alpha_2) \widehat{\mathcal{N}}.\end{aligned}\tag{2.11}$$

Observe the system (2.10) and let $\hat{x}(k; \varphi)$ denote the state trajectory from the initial data $\hat{x}(s) = \varphi(s)$ on $-\xi_M \leq s \leq -\xi_m$. Obviously, $\hat{x}(k; 0) \equiv 0$ is the trivial solution of system (2.10) corresponding to the initial data $\varphi = 0$.

In what follows, we aim to design a linear estimator of the form (2.9) for system (2.1) such that, for all admissible randomly occurring distributed time-varying delays, missing measurements, stochastic nonlinearities, and estimation error system (2.10) is mean-square stable.

3. Main Results

The following lemmas are essential in establishing our main results.

Lemma 3.1 (Schur Complement). *There are constant matrices Υ_1 , Υ_2 , and Υ_3 where $\Upsilon_1 = \Upsilon_1^T$ and $\Upsilon_2 = \Upsilon_2^T > 0$, then $\Upsilon_1 + \Upsilon_3^T \Upsilon_2^{-1} \Upsilon_3 < 0$ if and only if $\begin{bmatrix} \Upsilon_1 & \Upsilon_3^T \\ \Upsilon_3 & -\Upsilon_2 \end{bmatrix} < 0$.*

Lemma 3.2. *Let $\mathcal{W} \in \mathbb{R}^{n \times n}$ be a positive semidefinite matrix, $x_i \in \mathbb{R}^n$ be a vector, and $a_i \geq 0$ ($i = 1, 2, \dots$) be scalars. If the series concerned are convergent, then the following inequality holds [35]*

$$\left(\sum_{i=1}^{+\infty} a_i x_i \right)^T \mathcal{W} \left(\sum_{i=1}^{+\infty} a_i x_i \right) \leq \left(\sum_{i=1}^{+\infty} a_i \right) \sum_{i=1}^{+\infty} a_i x_i^T \mathcal{W} x_i. \quad (3.1)$$

In the following theorem, Lyapunov stability theorem and a LMI-based method are combined together to deal with the stability analysis issue for the estimator design of the discrete-time stochastic nonlinear system with distributed time-varying delays and missing measurements. A sufficient condition is derived that ensures the solvability of the estimation problem.

Theorem 3.3. *Given the estimator parameters A_f and A_k consider the estimation error system (2.10). If there exist positive definite matrices $P = P^T > 0$, $Q = Q^T > 0$, and positive scalars $\varpi_i > 0$ ($i = 1, 2, \dots, q$) such that the following matrix inequalities,*

$$\begin{bmatrix} -P & * & * & * & * & * & * & * \\ 0 & -Q & * & * & * & * & * & * \\ P\mathcal{E} & 0 & -P & * & * & * & * & * \\ \beta_1 Q & 0 & 0 & -Q & * & * & * & * \\ P\mathcal{A} & P\mathcal{B} & 0 & 0 & -P & * & * & * \\ \beta_3 P & 0 & 0 & 0 & 0 & -P & * & * \\ \beta_4 \hat{\Phi} & 0 & 0 & 0 & 0 & 0 & -\Xi & * \\ 0 & \beta_2 P\mathcal{F} & 0 & 0 & 0 & 0 & 0 & -P \end{bmatrix} < 0, \quad (3.2)$$

$$\Delta = \begin{bmatrix} -\varpi_i I & * \\ P \overline{\Pi}_i & -P \end{bmatrix} < 0, \quad i = 1, 2, \dots, q, \quad (3.3)$$

hold, where

$$\begin{aligned} \beta_1 &= \left(\tau_u + \frac{1}{2}(\tau_u - \tau_l)(\tau_u + \tau_l - 1) \right)^{1/2}, & \beta_2 &= (\alpha_1(1 - \alpha_1))^{1/2}, \\ \beta_3 &= (\alpha_3(1 - \alpha_3))^{1/2}, & \beta_4 &= (\alpha_2)^{1/2}, & \hat{\Phi} &= \left[\varpi_1 \overline{\Phi}_1^{1/2}, \dots, \varpi_q \overline{\Phi}_q^{1/2} \right]^T, \\ \overline{\Phi}_i &= \begin{bmatrix} \Phi_i & 0 \\ 0 & 0 \end{bmatrix}, & \overline{\Pi}_i &= \begin{bmatrix} \hat{\pi}_i \\ \hat{\pi}_i \end{bmatrix}, & \Xi &= \text{diag}\{\varpi_1 I, \dots, \varpi_q I\}, & \mathcal{F} &= \begin{bmatrix} B & 0 \\ 0 & 0 \end{bmatrix}, \end{aligned} \quad (3.4)$$

then the estimation error system (2.10) is mean-square stable.

Proof. Define the following Lyapunov functional candidate for system (2.10):

$$\begin{aligned}
V(\hat{x}(k), k) &= \hat{x}^T(k)P\hat{x}(k) + \sum_{i=-\tau(k)}^{-1} \sum_{j=k+i}^{k-1} \hat{x}^T(j)Q\hat{x}(j) \\
&+ \sum_{i=-\tau_u}^{-\tau_1-1} \sum_{j=i+1}^{-1} \sum_{n=k+j}^{k-1} \hat{x}^T(n)Q\hat{x}(n).
\end{aligned} \tag{3.5}$$

By calculating the difference of the Lyapunov functional (3.5), based on Lemma 3.2, one has,

$$\begin{aligned}
&\mathbb{E}\{\Delta V(\hat{x}(k), k)\} \\
&= \mathbb{E}\{V(\hat{x}(k+1), k+1) \mid \hat{x}(k)\} - V\{\hat{x}(k), k\} \\
&= \left[\mathcal{A}\hat{x}(k) + \mathcal{B} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right]^T P \left[\mathcal{A}\hat{x}(k) + \mathcal{B} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right] \\
&+ \mathbb{E} \left\{ \left[\bar{\mathcal{B}} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right]^T P \left[\bar{\mathcal{B}} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right] \right\} + \alpha_3(1-\alpha_3)\hat{x}^T(k)P\hat{x}(k) \\
&+ \hat{x}^T(k)\mathcal{E}^T P \mathcal{E} \hat{x}(k) + \alpha_2 \mathbb{E} \left\{ h^T(k) \widehat{\mathcal{N}}^T P \widehat{\mathcal{N}} h(k) \right\} - \hat{x}^T(k)P\hat{x}(k) \\
&+ \sum_{i=-\tau(k+1)}^{-1} \sum_{j=k+i+1}^k \hat{x}^T(j)Q\hat{x}(j) - \sum_{i=-\tau(k)}^{-1} \sum_{j=k+i}^{k-1} \hat{x}^T(j)Q\hat{x}(j) \\
&+ \sum_{i=-\tau_u}^{-\tau_1-1} \sum_{j=i+1}^{-1} \left[\sum_{n=k+j+1}^k - \sum_{n=k+j}^{k-1} \right] \hat{x}^T(n)Q\hat{x}(n) \\
&= \left[\mathcal{A}\hat{x}(k) + \mathcal{B} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right]^T P \left[\mathcal{A}\hat{x}(k) + \mathcal{B} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right] \\
&+ \mathbb{E} \left\{ \left[\bar{\mathcal{B}} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right]^T P \left[\bar{\mathcal{B}} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right] \right\} + \alpha_3(1-\alpha_3)\hat{x}^T(k)P\hat{x}(k) \\
&+ \hat{x}^T(k)\mathcal{E}^T P \mathcal{E} \hat{x}(k) + \alpha_2 \mathbb{E} \left\{ h^T(k) \widehat{\mathcal{N}}^T P \widehat{\mathcal{N}} h(k) \right\} - \hat{x}^T(k)P\hat{x}(k) \\
&+ \sum_{i=-\tau(k+1)}^{-1} \left[\sum_{j=k+i+1}^{k-1} \hat{x}^T(j)Q\hat{x}(j) + \hat{x}^T(k)Q\hat{x}(k) \right] \\
&- \sum_{i=-\tau(k)}^{-1} \left[\sum_{j=k+i+1}^{k-1} \hat{x}^T(j)Q\hat{x}(j) + \hat{x}^T(k+i)Q\hat{x}(k+i) \right] \\
&+ \sum_{i=-\tau_u}^{-\tau_1-1} \sum_{j=i+1}^{-1} \left[\hat{x}^T(k)Q\hat{x}(k) - \hat{x}^T(k+j)Q\hat{x}(k+j) \right]
\end{aligned}$$

$$\begin{aligned}
&\leq \left[\mathcal{A}\hat{x}(k) + \mathcal{B} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right]^T P \left[\mathcal{A}\hat{x}(k) + \mathcal{B} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right] \\
&\quad + \mathbb{E} \left\{ \left[\bar{\mathcal{B}} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right]^T \left[\bar{\mathcal{B}} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right] \right\} + \alpha_3(1-\alpha_3)\hat{x}^T(k)P\hat{x}(k) \\
&\quad + \hat{x}^T(k)\mathcal{E}^T P \mathcal{E} \hat{x}(k) + \alpha_2 \mathbb{E} \left\{ h^T(k) \widehat{\mathcal{N}}^T P \widehat{\mathcal{N}} h(k) \right\} - \hat{x}^T(k)P\hat{x}(k) \\
&\quad + \sum_{i=-\tau_u}^{-\tau_l-1} \sum_{j=k+i+1}^{k-1} \hat{x}^T(j)Q\hat{x}(j) + \sum_{i=-\tau_l}^{-1} \sum_{j=k+i+1}^{k-1} \hat{x}^T(j)Q\hat{x}(j) \\
&\quad + \tau_u \hat{x}^T(k)Q\hat{x}(k) - \sum_{i=-\tau_l}^{-1} \sum_{j=k+i+1}^{k-1} \hat{x}^T(j)Q\hat{x}(j) - \sum_{i=-\tau(k)}^{-1} \hat{x}^T(k+i)Q\hat{x}(k+i) \\
&\quad + \frac{1}{2}(\tau_u - \tau_l)(\tau_u + \tau_l - 1)\hat{x}^T(k)Q\hat{x}(k) - \sum_{i=-\tau_u}^{-\tau_l-1} \sum_{j=k+i+1}^{k-1} \hat{x}^T(j)Q\hat{x}(j) \\
&= \left[\mathcal{A}\hat{x}(k) + \mathcal{B} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right]^T P \left[\mathcal{A}\hat{x}(k) + \mathcal{B} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right] \\
&\quad + \mathbb{E} \left\{ \left[\bar{\mathcal{B}} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right]^T P \left[\bar{\mathcal{B}} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right] \right\} + \alpha_3(1-\alpha_3)\hat{x}^T(k)P\hat{x}(k) \\
&\quad + \hat{x}^T(k)\mathcal{E}^T P \mathcal{E} \hat{x}(k) + \alpha_2 \mathbb{E} \left\{ h^T(k) \widehat{\mathcal{N}}^T P \widehat{\mathcal{N}} h(k) \right\} - \hat{x}^T(k)P\hat{x}(k) \\
&\quad + \tau_u \hat{x}^T(k)Q\hat{x}(k) - \sum_{i=-\tau(k)}^{-1} \hat{x}^T(k+i)Q\hat{x}(k+i) \\
&\quad + \frac{1}{2}(\tau_u - \tau_l)(\tau_u + \tau_l - 1)\hat{x}^T(k)Q\hat{x}(k) \\
&\leq \left[\mathcal{A}\hat{x}(k) + \mathcal{B} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right]^T P \left[\mathcal{A}\hat{x}(k) + \mathcal{B} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right] \\
&\quad + \mathbb{E} \left\{ \left[\bar{\mathcal{B}} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right]^T P \left[\bar{\mathcal{B}} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right] \right\} + \alpha_3(1-\alpha_3)\hat{x}^T(k)P\hat{x}(k) \\
&\quad + \hat{x}^T(k)\mathcal{E}^T P \mathcal{E} \hat{x}(k) + \alpha_2 \mathbb{E} \left\{ h^T(k) \widehat{\mathcal{N}}^T P \widehat{\mathcal{N}} h(k) \right\} - \hat{x}^T(k)P\hat{x}(k) \\
&\quad + \tau_u \hat{x}^T(k)Q\hat{x}(k) + \frac{1}{2}(\tau_u - \tau_l)(\tau_u + \tau_l - 1)\hat{x}^T(k)Q\hat{x}(k) \\
&\quad - \sum_{i=-\tau(k)}^{-1} \hat{x}^T(k+i)Q\hat{x}(k+i)
\end{aligned}$$

$$\begin{aligned}
&\leq \left[\mathcal{A}\hat{x}(k) + \mathcal{B} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right]^T P \left[\mathcal{A}\hat{x}(k) + \mathcal{B} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right] \\
&+ \mathbb{E} \left\{ \left[\bar{\mathcal{B}} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right]^T P \left[\bar{\mathcal{B}} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right] \right\} + \alpha_3(1-\alpha_3)\hat{x}^T(k)P\hat{x}(k) \\
&+ \hat{x}^T(k)\mathcal{E}^T P \mathcal{E} \hat{x}(k) + \alpha_2 \mathbb{E} \left\{ h^T(k) \widehat{\mathcal{N}}^T P \widehat{\mathcal{N}} h(k) \right\} - \hat{x}^T(k)P\hat{x}(k) \\
&+ \tau_u \hat{x}^T(k)Q\hat{x}(k) + \frac{1}{2}(\tau_u - \tau_l)(\tau_u + \tau_l - 1)\hat{x}^T(k)Q\hat{x}(k) \\
&- \tau_u^{-1} \left(\sum_{i=-\tau(k)}^{-1} \hat{x}^T(k+i) \right) Q \left(\sum_{i=-\tau(k)}^{-1} \hat{x}(k+i) \right).
\end{aligned} \tag{3.6}$$

From (2.8), it can be seen that

$$\mathbb{E} \left\{ h^T(k) \widehat{\mathcal{N}}^T P \widehat{\mathcal{N}} h(k) \right\} \leq \sum_{i=1}^q \left[\hat{x}^T(k) \bar{\Phi}_i \hat{x}(k) \right] \text{tr} \left(\widehat{\mathcal{N}} \Pi_i \widehat{\mathcal{N}}^T P \right), \tag{3.7}$$

where $\Pi_i := \bar{\Pi}_i \bar{\Pi}_i^T$ with $\bar{\Phi}_i$ and $\bar{\Pi}_i$ defined in (3.4).

Furthermore,

$$\begin{aligned}
&\mathbb{E} \left\{ \left[\mathcal{B} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right]^T P \left[\mathcal{B} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m) \right] \right\} \\
&\leq \beta_2^2 \sum_{m=-\tau(k)}^{-1} \hat{x}^T(k+m) \mathcal{F}^T P \mathcal{F} \sum_{m=-\tau(k)}^{-1} \hat{x}(k+m),
\end{aligned} \tag{3.8}$$

where β_2 is defined in (3.4).

From (3.6)–(3.8), one has

$$\mathbb{E} \{ \Delta V(\hat{x}(k), k) \} \leq \mathbb{E} \left\{ \eta^T(k) \Theta \eta(k) \right\}, \tag{3.9}$$

where $\eta(k) = [\hat{x}^T(k), \sum_{i=-\tau(k)}^{-1} \hat{x}^T(k+i)]^T$ and

$$\Theta = \begin{bmatrix} \Theta_1 & \mathcal{A}P\mathcal{B} \\ * & \Theta_2 \end{bmatrix}, \tag{3.10}$$

where $\Theta_1 = -P + \mathcal{E}^T P \mathcal{E} + (\tau_u + (1/2)(\tau_u - \tau_l)(\tau_u + \tau_l - 1))Q + \mathcal{A}^T P \mathcal{A} + \beta_4^2 \sum_{i=1}^q \bar{\Phi}_i \text{tr}(\widehat{\mathcal{N}} \Pi_i \widehat{\mathcal{N}}^T P) + \beta_3^2 P$, $\Theta_2 = -(1/\tau_u)Q + \mathcal{B}^T P \mathcal{B} + \beta_2^2 \mathcal{F}^T P \mathcal{F}$, $\beta_3, \beta_4, \mathcal{F}$ are defined in (3.4).

From Lemma 3.1, (3.10) holds if and only if $\text{tr}(\widehat{\mathcal{N}} \Pi_i \widehat{\mathcal{N}}^T P)$. Furthermore, by Lemma 3.1, one can obtain from (3.2), (3.3) that $\Theta < 0$ and, subsequently,

$$\mathbb{E}\{\Delta V(\hat{x}(k), k)\} < -\lambda_{\min}(\Theta)|\hat{x}(k)|^2. \quad (3.11)$$

Thus, the augmented estimation system (2.10) is mean-square stable. \square

The following theorem is focused on the design of the desired estimation parameters A_f and A_k by using the results in Theorem 3.3.

Theorem 3.4. Consider the augmented estimation system (2.10) with given estimator parameters. If there exist positive-definite matrices $S = S^T > 0$, $R = R^T > 0$, $Q = Q^T > 0$, matrices \tilde{A}_f, \tilde{A}_k , and positive scalars $\varpi_i > 0$, ($i = 1, 2, \dots, q$) such that the following linear matrix inequalities holds

$$\Gamma = \begin{bmatrix} -S & * & * & * & * & * & * & * & * & * & * & * & * \\ -S & -R & * & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & -Q & * & * & * & * & * & * & * & * & * & * \\ SE_1 & E_1 & 0 & -S & * & * & * & * & * & * & * & * & * \\ \phi_1 & \phi_2 & 0 & -S & -R & * & * & * & * & * & * & * & * \\ \beta_1 Q & \beta_1 Q & 0 & 0 & 0 & -Q & * & * & * & * & * & * & * \\ SA & SA & \alpha_1 SB & 0 & 0 & 0 & -S & * & * & * & * & * & * \\ \phi_3 & \phi_4 & \alpha_1 RB & 0 & 0 & 0 & -S & -R & * & * & * & * & * \\ \beta_3 S & \beta_3 S & 0 & 0 & 0 & 0 & 0 & 0 & -S & * & * & * & * \\ \beta_3 S & \beta_3 R & 0 & 0 & 0 & 0 & 0 & 0 & -S & -R & * & * & * \\ \tilde{\Phi} & \tilde{\Phi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Xi & * & * \\ 0 & 0 & \beta_2 SB & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R & * \\ 0 & 0 & \beta_2 RB & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S & -R \end{bmatrix} < 0, \quad (3.12)$$

$$\begin{bmatrix} -\varpi_i I & * & * \\ S\tilde{\pi}_i & -S & * \\ R\tilde{\pi}_i + \tilde{A}_f \tilde{\pi}_i & -S & -R \end{bmatrix} < 0, \quad i = 1, 2, \dots, q, \quad (3.13)$$

$$S - R < 0 \quad (3.14)$$

hold, where α_1 is defined in (2.2), $\beta_1, \beta_2, \beta_3$, and β_4 are defined in (3.4),

$$\begin{aligned} \tilde{\Phi}^T &= \left[\beta_4 [\varpi_1 \Phi_1^{1/2}]^T, \dots, \beta_4 [\varpi_q \Phi_q^{1/2}]^T \right], \\ \phi_1 &= RE_1 + \tilde{A}_f E_2, & \phi_2 &= RE_1 + \tilde{A}_f E_2, \\ \phi_3 &= RA + \alpha_2 \tilde{A}_f C + \tilde{A}_k, & \phi_4 &= RA + \alpha_2 \tilde{A}_f C, \end{aligned} \quad (3.15)$$

then the estimator parameters are designed as

$$A_k = X_{12}^{-1} \tilde{A}_f, \quad A_f = X_{12}^{-1} \tilde{A}_k S^{-1} \left(Y_{12}^T \right)^{-1}, \quad (3.16)$$

where X_{12}, Y_{12} are any square and nonsingular matrices satisfying $X_{12} Y_{12}^T = I - RS^{-1} < 0$, then the estimation error system (2.10) is mean-square stable.

Proof. Recall that our goal is to derive the expression of the estimator parameters from (2.9). To do this, we partition P and P^{-1} as

$$P = \begin{bmatrix} R & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} S^{-1} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix}, \quad (3.17)$$

where the partitioning of P and P^{-1} is compatible with that of \mathcal{A} defined in (2.11), that is, $R \in R^{n \times n}$, $X_{12} \in R^{n \times n}$, $X_{22} \in R^{n \times n}$, $S \in R^{n \times n}$, $Y_{12} \in R^{n \times n}$, and $Y_{22} \in R^{n \times n}$. Define

$$T_1 = \begin{bmatrix} S^{-1} & I \\ Y_{12}^T & 0 \end{bmatrix}, \quad T_2 = \begin{bmatrix} I & R \\ 0 & X_{12}^T \end{bmatrix} \quad (3.18)$$

which imply that $PT_1 = T_2$ and $T_1^T PT_1 = T_1^T T_2$.

By applying the congruence transformations $\text{diag}\{T_1, I, T_1, I, T_1, T_1, I, \dots, I, T_1\}$ and the congruence transformations $\text{diag}\{S, I, I, S, I, I, S, I, S, I, I, \dots, I, S, I\}$ to (3.2), we have (3.12).

Again, performing the congruence transformation $\text{diag}\{I, T_1\}$ to (3.3) lead to (3.19)

$$\begin{bmatrix} -\varpi_i I & * & * \\ \hat{\pi}_i & -S^{-1} & * \\ R\hat{\pi}_i + X_{12}A_k\hat{\pi}_i & -I & -R \end{bmatrix} < 0, \quad i = 1, 2, \dots, q. \quad (3.19)$$

Then, one uses congruence transformation $\text{diag}\{I, S, I\}$ to (3.19) and we have

$$\begin{bmatrix} -\varpi_i I & * & * \\ S\hat{\pi}_i & -S & * \\ R\hat{\pi}_i + X_{12}A_k\hat{\pi}_i & -S & -R \end{bmatrix} < 0, \quad i = 1, 2, \dots, q. \quad (3.20)$$

Furthermore, if (3.12) is feasible, we have $\begin{bmatrix} -S & -S \\ -S & -R \end{bmatrix} < 0$ or $\begin{bmatrix} -S^{-1} & I \\ I & R \end{bmatrix} > 0$.

It follows directly from $XX^{-1} = I$ that $I - RS^{-1} = X_{12}Y_{12}^T < 0$. Hence, one can always find square and nonsingular X_{12} and Y_{12} [37]. Therefore, this completes the proof. \square

4. Numerical Example

In this section, an example is presented to illustrate the usefulness and flexibility of the estimator design method developed in this paper. The system data of (2.1)–(2.9) are the following:

$$\begin{aligned} A &= \begin{bmatrix} 0.15 & 0 \\ 0.2 & 0.1 \end{bmatrix}, & B &= \begin{bmatrix} 0.09 & 0 \\ 0 & 0.09 \end{bmatrix}, & C &= \begin{bmatrix} 1.2 & 0 \\ 0 & 1.2 \end{bmatrix}, \\ E_1 &= \begin{bmatrix} 0.12 & 0 \\ 0 & 0.12 \end{bmatrix}, & E_2 &= \begin{bmatrix} 0.12 & 0 \\ 0 & 0.12 \end{bmatrix}, & H_1 = H_2 &= \begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix}, \end{aligned} \quad (4.1)$$

where $n = q = 2$, $\tau(k) = 1 + (1 + (-1)^k)$, $\tau_l = 1$, $\tau_u = 3$.

$f(x(k))$ describes the stochastic nonlinear function of the states in (2.1), which is bounded as follows:

$$\mathbb{E}\{f(x(k))f(x(k))^T \mid x(k)\} = \begin{bmatrix} 0.22 \\ 0.22 \end{bmatrix} \begin{bmatrix} 0.22 \\ 0.22 \end{bmatrix}^T x^T(k) \begin{bmatrix} 0.11 & 0 \\ 0 & 0.11 \end{bmatrix} x(k). \quad (4.2)$$

Let $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, and $\alpha_3 = 0.9$. Using Matlab LMI Toolbox to solve the LMIs in (3.12)–(3.14), one has

$$\begin{aligned} S &= \begin{bmatrix} 0.6726 & -0.0035 \\ -0.0035 & 0.6563 \end{bmatrix}, & R &= \begin{bmatrix} 1.8796 & -0.0041 \\ -0.0041 & 1.8411 \end{bmatrix}, \\ Q &= \begin{bmatrix} 0.0668 & -0.0013 \\ -0.0013 & 0.0693 \end{bmatrix}, & \varpi_1 &= 1.0776, \quad \varpi_2 = 1.3335. \end{aligned} \quad (4.3)$$

Thus, we can calculate the estimator parameters as follows:

$$A_f = \begin{bmatrix} 0.1325 & 0.0486 \\ 0.0462 & -0.1465 \end{bmatrix}, \quad A_k = \begin{bmatrix} -0.9144 & -0.1677 \\ -0.1684 & -0.8160 \end{bmatrix}. \quad (4.4)$$

Remark 4.1. Seldom of the estimation literature explicitly introduce the effects of the estimators by the digits in the graphs, for example [18]. In this paper, some digits are marked in Figures 1–4. Figures 1–2 show the actual measurements and ideal measurements. Figures 3–4 plot the estimation errors. From these digits in the graphs, it can be seen that the designed estimator performs well.

5. Conclusions

In this paper, we research the estimation problem for a class of stochastic nonlinear systems with both the probabilistic distributed time-varying delays and missing measurements. The distributed time-varying delays and missing measurements are assumed to occur in random ways, and the occurring probabilities are governed by Bernoulli stochastic variables. A linear estimator is designed such that, for the admissible random distributed delays, the

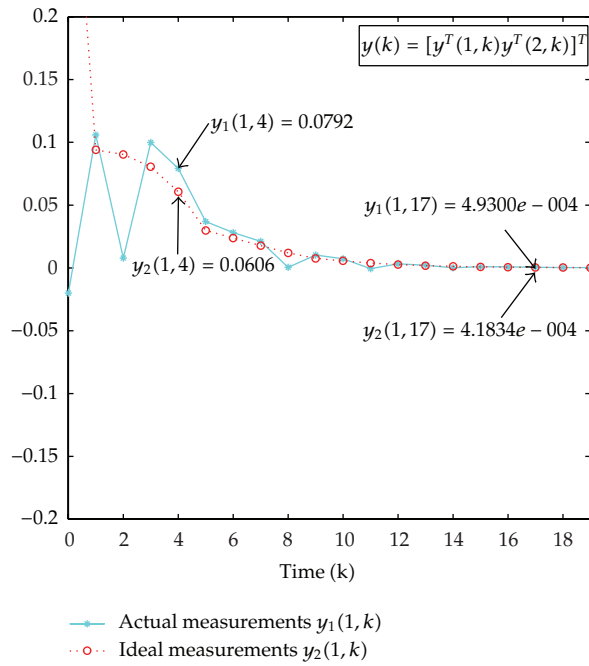


Figure 1: Actual Measurements $y_1(1, k)$ and ideal Measurements $y_2(1, k)$.

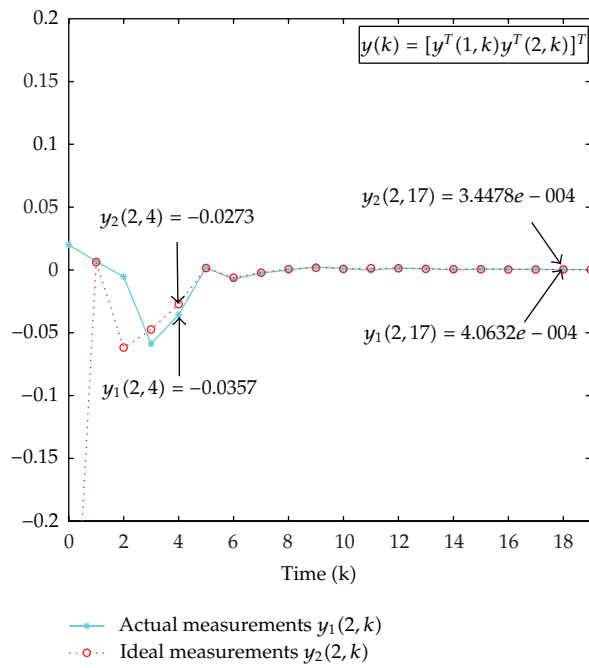


Figure 2: Actual Measurements $y_1(2, k)$ and ideal Measurements $y_2(2, k)$.

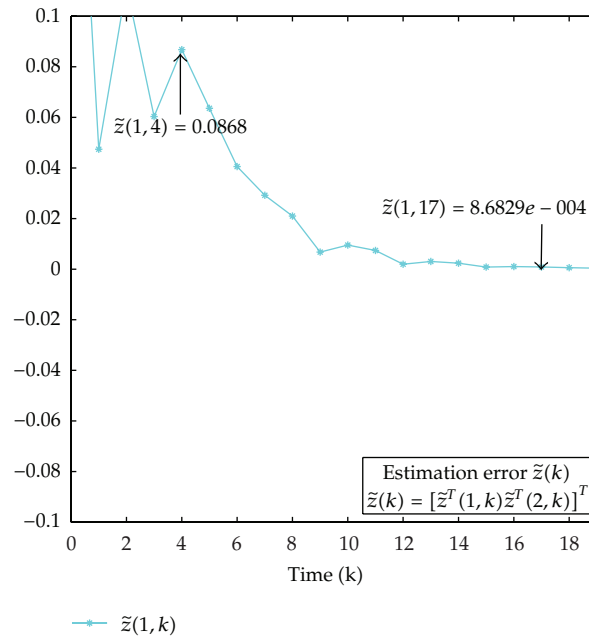


Figure 3: Estimation Errors $\tilde{z}(1, k)$.

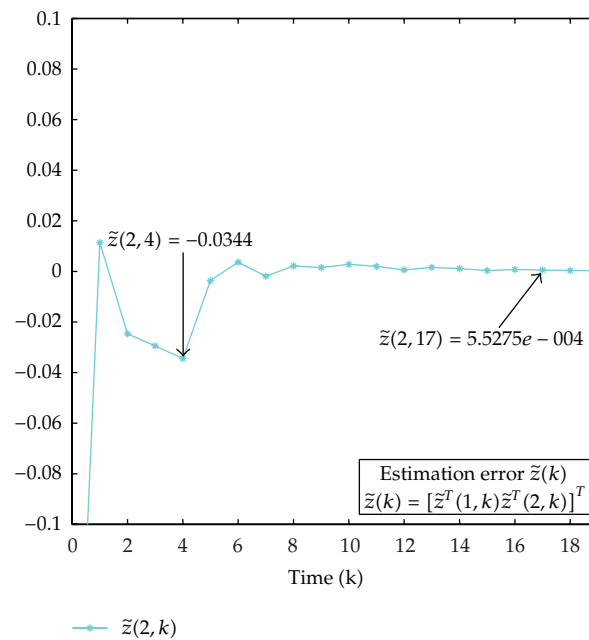


Figure 4: Estimation Errors $\tilde{z}(2, k)$.

stochastic disturbances, and the stochastic nonlinearities, the error dynamics of the estimation process is mean-square stable. At last, an illustrative example has been exploited to show the effectiveness of the proposed approach. In the future, we plan to consider the estimation problem with Markovian switching in the finite-horizon case, and the nonlinearities are in more general forms.

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