# GONI: PRIMES REPRESENTED BY BINARY QUADRATIC FORMS

**Pete L. Clark**
*Department of Mathematics, University of Georgia, Athens, Georgia*
pete@math.uga.edu

**Jacob Hicks**
*Department of Mathematics, University of Georgia, Athens, Georgia*
jhicks@math.uga.edu

**Hans Parshall**
*Department of Mathematics, University of Georgia, Athens, Georgia*
hans@math.uga.edu

**Katherine Thompson**
*Department of Mathematics, University of Georgia, Athens, Georgia*
thompson@math.uga.edu

*Received: 8/4/12, Revised: 2/28/13, Accepted: 5/2/13, Published: 6/14/13*

**Abstract**

We list 2779 regular primitive positive definite integral binary quadratic forms, and show that, conditional on the Generalized Riemann Hypothesis, this is the complete list of *regular*, positive definite binary integral quadratic forms (up to $\mathrm{SL}_2(\mathbb{Z})$-equivalence). For each of these 2779 forms we determine the primes that they represent by elementary combinatorial methods, avoiding Gauss's genus theory. The key intermediate result is a *Small Multiple Theorem* for representations of primes by integral binary forms.

## 1. Introduction

An **imaginary quadratic discriminant** is a negative integer $\Delta$ which is 0 or 1 modulo 4. For a given imaginary quadratic disciminant $\Delta$, let $C(\Delta)$ be the set of $\mathrm{SL}_2(\mathbb{Z})$-equivalence classes of primitive positive definite integral binary quadratic forms of discriminant $\Delta$. Then $C(\Delta)$ is a finite set [3, Thm. 2.13] which, when endowed with Gauss's composition law, becomes a finite abelian group, the **class group of discriminant** $\Delta$ [3, Thm. 3.9].

Thus a form $q$ of discriminant $\Delta$ determines an element $[q] \in C(\Delta)$. A quadratic form $q$ is **ambiguous** if $[q]^2 = 1$. For a $q = \langle A, B, C\rangle$, the form $\overline{q} = \langle A, -B, C\rangle$ represents the inverse of $[q]$ in $C(\Delta)$ [3, Thm. 3.9]. Note that $q$ and $\overline{q}$ are $\mathrm{GL}_2(\mathbb{Z})$-equivalent: $\overline{q}(x, y) = q(x, -y)$, so $q$ and $\overline{q}$ represent the same integers.

A discriminant $\Delta$ is **idoneal** if every $q \in C(\Delta)$ is ambiguous; this holds if and only if $C(\Delta) \cong (\mathbb{Z}/2\mathbb{Z})^r$ for some $r \in \mathbb{N}$. A quadratic form is **idoneal** if its discriminant is idoneal. A discriminant $\Delta$ is **bi-idoneal** if $C(\Delta) \cong (\mathbb{Z}/4\mathbb{Z}) \oplus (\mathbb{Z}/2\mathbb{Z})^r$ for some $r \in \mathbb{N}$. A quadratic form $q$ is **bi-idoneal** if $\Delta$ is bi-idoneal and $q$ is *not* ambiguous.

A **full congruence class of primes** is the set of all primes $p \nmid 2\Delta$ with $p \equiv n \pmod{N}$ for fixed coprime positive integers $n$ and $N$. We say $q$ is **regular** if the set of primes $p \nmid 2\Delta$ represented by $q$ is a union of full congruence classes.

Recall Fermat's Two Squares Theorem: an odd prime $p$ is of the form $x^2 + y^2$ if and only if $p \equiv 1 \pmod{4}$. In our terminology then the form $q(x, y) = x^2 + y^2$ is regular. Indeed, much classical work on quadratic forms can be phrased as showing that certain specific binary quadratic forms represent full congruence classes of primes, or are regular. Among primitive, positive definite, integral binary quadratic forms, how many are regular? How many represent full congruence classes of primes? Remarkably, this problem has recently been solved (conditionally on GRH) but the answer does not appear explicitly in the literature. Here it is:

**Theorem 1.** *Let $q$ be a primitive, positive definite integral binary quadratic form.*

*a) The following are equivalent:*
*(i) $q$ is regular.*
*(ii) $q$ represents a full congruence class of primes.*
*(iii) $q$ is either idoneal or bi-idoneal.*

*b) There are at least 425 and at most 432 imaginary quadratic discriminants which are either idoneal or bi-idoneal. These 425 known discriminants give rise to precisely 2779 $\mathrm{SL}_2(\mathbb{Z})$-equivalence classes of regular forms: see Table 1.*

*c) The list of idoneal and bi-idoneal discriminants of part b) is complete among all imaginary quadratic discriminants $\Delta$ with $|\Delta| \leq 80604484$. Assuming the Riemann Hypothesis for Dedekind zeta functions of imaginary quadratic fields, there are precisely 425 imaginary discriminants which are idoneal or bi-idoneal.*

For these 2779 regular forms, it is natural to ask for explicit congruence conditions, as in Fermat's Two Squares Theorem. The following result accomplishes this.

**Theorem 2.** *Let $q = \langle A, B, C \rangle$ be one of the 2779 primitive, positive definite integral binary quadratic forms in Table 1, and let $\Delta = B^2 - 4AC$ be the discriminant of $q$. For a prime $p \nmid 2\Delta$, the following are equivalent:*

*a) The form $q$ integrally represents $p$: there are $x, y \in \mathbb{Z}$ with $q(x, y) = p$.*

*b) All of the following conditions hold:*
*(i) $(\frac{\Delta}{p}) = 1$.*

*(ii) For each odd prime* $m \mid \Delta$*, if* $m \nmid A$*, then* $(\frac{p}{m}) = (\frac{A}{m})$*, and if* $m \nmid C$*, then* $(\frac{p}{m}) = (\frac{C}{m})$.

*(iii) If* $16 \mid \Delta$ *and* $2 \nmid A$*, then* $p \equiv A \pmod 4$*. If* $16 \mid \Delta$ *and* $2 \nmid C$*, then* $p \equiv C \pmod 4$.

*(iv) If* $32 \mid \Delta$ *and* $2 \nmid A$*, then* $p \equiv A \pmod 8$*. If* $32 \mid \Delta$ *and* $2 \nmid C$*, then* $p \equiv C \pmod 8$.

We will prove Theorem 1 by deducing it from Gauss's genus theory together with results of Meyer, Weinberger, Louboutin, Kaplan-Williams and Voight. We do this mostly for completeness and perspective. Our main goal is quite different: we will give a new proof of Theorem 2 using none of Gauss's genus theory but instead using elementary ideas from the **Geometry of Numbers**. Our methods build on the classical proof of the Two Squares Theorem via Minkowski's Convex Body Theorem and its recent generalization to the 65 principal idoneal forms $x^2 + Dy^2$ of T. Hagedorn [5], although we find it simpler to use (sharp) bounds on minima of binary quadratic forms going back to Lagrange and Legendre.

We may compare the two methods as follows: let $q$ be a binary form of discriminant $\Delta$, and let $p \nmid 2\Delta$ be a prime. To analyze the question of whether $q$ represents $p$, genus theory begins with the observation that $(\frac{\Delta}{p}) = 1$ if and only if some $q' \in C(\Delta)$ represents $p$ and attempts to rule out the representation of $p$ by all forms $q' \neq q$. Our method begins with a *small multiple theorem*: if $(\frac{\Delta}{p}) = 1$, then $q$ represents some multiple $kp$ of $p$ with $k$ bounded in terms of $\Delta$ and via a combination of *elimination* and *reduction* attempts to show that we may take $k = 1$. Our method is more computational – at present it is more a technique than a theory – and the reasons for its success in all 2779 cases are rather mysterious! However, our technique can be used in settings where genus theory does not apply: in [2] and [8] some of us use these ideas to establish universality of most (but not all) of the 112 positive definite quaternary universal forms of square discriminant. In [1], the first author extends the method to a technique for proving representation theorems for quadratic forms in $2d$ variables over a normed Dedekind domain.

## 2. Proof of Theorem 1

### 2.1. Part a)

(i) $\implies$ (ii): By [3, Thm. 9.12], $q$ represents infinitely many prime numbers. Having established this, the implication is immediate.

(ii) $\implies$ (iii): Suppose that there are coprime integers $n$ and $N$ such that for all primes $p$, if $p \nmid 2\Delta$ and $p \equiv n \pmod N$, then $q$ represents $p$. By [9, Thm. 2], if $q$ is ambiguous then $\Delta$ is idoneal hence so is $q$; whereas if $q$ is not ambiguous then $\Delta$ is bi-idoneal and hence – since $q$ is not ambiguous – so is $q$.

(iii) $\Longrightarrow$ (i): Let $G(\Delta) = C(\Delta)/C(\Delta)^2$, and let $r : C(\Delta) \to G(\Delta)$ be the quotient map. The fibers of $r$ are called **genera**; they are cosets of $C(\Delta)^2$, the **principal genus**. Let $c = \#C(\Delta)/\#G(\Delta)$. Thus $\Delta$ is idoneal if and only if $c = 1$ and bi-idoneal if and only if $c = 2$. For $q \in C(\Delta)$, we define $g(q)$ to be the set of $n \in (\mathbb{Z}/\Delta\mathbb{Z})^\times$ which are represented by $q$. We will need the following tenets of genus theory:

- For all $q, q' \in C(\Delta)$, $g(q) = g(q') \iff r(q) = r(q')$ [3, pp. 53-54].
- If $q \in C(\Delta)^2$, then $g(q)$ is a subgroup, $H$, of $(\mathbb{Z}/\Delta\mathbb{Z})^\times$ [3, Lem. 2.24, Thm. 3.15].
- For all $q \in C(\Delta)$, $g(q)$ is a coset of $H$ in $(\mathbb{Z}/\Delta\mathbb{Z})^\times$ [3, Lem. 2.24, Thm. 3.15].
- Let $n$ be a positive integer which is relatively prime to $2\Delta$. Then there is $q \in C(\Delta)$ representing $n$ if and only if $\left(\frac{\Delta}{n}\right) = 1$ [3, Thm. 2.16].

In particular, let $p \nmid 2\Delta$ be an odd prime. Then if $\left(\frac{\Delta}{p}\right) = -1$, no $q \in C(\Delta)$ represents $p$, whereas if $\left(\frac{\Delta}{p}\right) = 1$, then some $q \in C(\Delta)$ represents $p$, and if $q, q' \in C(\Delta)$ both represent $p$, then $r(q) = r(q')$.

Suppose $\Delta$ is idoneal, let $q \in C(\Delta)$, and let $p \nmid 2\Delta$ be a prime. If $q$ represents $p$ then $p \in g(q)$; conversely, if $p \in g(q)$ then $\left(\frac{\Delta}{p}\right) = 1$, so some $q' \in C(\Delta)$ represents $p$ and any such $q'$ must lie in $g(q)$. But since $\Delta$ is idoneal, $c = 1$, and $q$ is the only form in $r(q)$. Thus $q$ represents $p$ if and only if $p \in g(q)$, so $q$ is regular.

Suppose $\Delta$ is bi-idoneal, let $q \in C(\Delta)$ be a nonambiguous form, and let $p \nmid 2\Delta$ be a prime. As above, if $q$ represents $p$ then $p \in g(q)$; conversely, if $p \in g(q)$ then some $q' \in r(q)$ repesents $p$. But since $c = 2$, $r(q) = \{[q], [\overline{q}]\} = \{[q], [q]^{-1}\}$, and $q$ and $\overline{q}$ represent the same primes. Thus $q$ represents $p$ if and only if $p \in g(q)$, so $q$ is regular.

**Remark.** The implication (ii) $\Longrightarrow$ (iii) for *fundamental* discriminants was first proven by Kusaba [10], using methods of class field theory. In [9] the general case is proved, and their proof uses Gauss's genus theory together with a theorem of Meyer [12]: if $n$ and $N$ are coprime positive integers, $p \equiv n \pmod{N}$, $p \nmid 2\Delta$ is a prime number and $q \in C(\Delta)$ represents $p$, then $q$ represents infinitely many prime numbers $q \equiv n \pmod{N}$. See [6] for a proof of Meyer's theorem and a second proof of (ii) $\Longrightarrow$ (iii), both using class field theory.

### 2.2. Part b)

That the total number of idoneal and bi-idoneal discriminants lies between 425 and 432 is [14, Thm. 8.2].

### 2.3. Part c)

This is [14, Prop. 5.1] and [14, Thm. 8.6]. The latter result builds on work of Weinberger [15] and Louboutin [11].

## 3. A Small Multiple Theorem

Let $q = \langle A, B, C\rangle$ be a real binary quadratic form with discriminant $\Delta \neq 0$. Recall:
- If $\Delta > 0$, then $q$ is **indefinite**: it assumes both positive and negative values.
- If $\Delta < 0$ and $A, C > 0$, then $q$ is **positive definite**: it assumes only positive values except at $(x, y) = (0, 0)$.
- If $\Delta < 0$ and $A, C < 0$, then $q$ is **negative definite**: it assumes only negative values except at $(x, y) = (0, 0)$. Since $q$ is negative definite if and only if $-q$ is positive definite, negative definite forms do not require separate consideration.

**Theorem 3.** *Let $q = \langle A, B, C\rangle$ be a binary form over $\mathbb{R}$ with discriminant $\Delta$.*
*a) If $\Delta < 0$, there are integers $x$ and $y$, not both zero, such that $|q(x, y)| \leq \sqrt{\frac{|\Delta|}{3}}$.*
*b) If $\Delta > 0$, there are integers $x$ and $y$, not both zero, such that $|q(x, y)| \leq \sqrt{\frac{\Delta}{5}}$.*

*Proof.* The core of the proof is the following "reduction lemma": if $x_0, y_0$ are coprime integers with $q(x_0, y_0) = M \neq 0$, then there are $b, c \in \mathbb{R}$ such that $q$ is $\mathrm{SL}_2(\mathbb{Z})$-equivalent to $Mx^2 + bxy + cy^2$ with $-|M| < b \leq |M|$. For the details, see e.g. [7, Thm. 453, Thm. 454]. □

A **lattice** $\Lambda \subset \mathbb{R}^N$ is the set of all $\mathbb{Z}$-linear combinations of an $\mathbb{R}$-basis $\mathfrak{b} = \{v_1, \ldots, v_N\}$ for $\mathbb{R}^N$. If $M_{\mathfrak{b}} \in M_N(\mathbb{R})$ is the matrix with columns $v_1, \ldots, v_N$, then $\Lambda = M_{\mathfrak{b}}\mathbb{Z}^N$.

**Proposition 1.** *Let $q = \langle A, B, C\rangle$ be an integral form of discriminant $\Delta$. Let $p$ be an odd prime with $(\frac{\Delta}{p}) = 1$. Then there is an index $p$ sublattice $\Lambda_p \subset \mathbb{Z}^2$ such that for all $(x, y) \in \Lambda_p$, $q(x, y) \equiv 0 \pmod{p}$.*

*Proof.* If $p \mid A$, take $M_p = \begin{bmatrix} 1 & 0 \\ 0 & p \end{bmatrix}$ and $\Lambda_p = M_p\mathbb{Z}^2$. If $p \nmid A$, by the quadratic formula in $\mathbb{Z}/p\mathbb{Z}$, there is $r \in \mathbb{Z}$ with $Ar^2 + Br + C \equiv 0 \pmod{p}$; set $M_p = \begin{bmatrix} p & r \\ 0 & 1 \end{bmatrix}$ and $\Lambda_p = M_p\mathbb{Z}^2$. In either case, $q(x, y) \equiv 0 \pmod{p}$ for all $(x, y) \in \Lambda_p$. □

**Theorem 4.** *Let $q = \langle A, B, C\rangle$ be an integral form of discriminant $\Delta$. Let $p$ be an odd prime with $(\frac{\Delta}{p}) = 1$.*
*a) If $q$ is positive definite, there are $x, y, k \in \mathbb{Z}$ with $q(x, y) = kp$ and $1 \leq k \leq \sqrt{\frac{|\Delta|}{3}}$.*
*b) If $q$ is indefinite, there are $x, y, k \in \mathbb{Z}$ with $q(x, y) = kp$ and $1 \leq |k| \leq \sqrt{\frac{\Delta}{5}}$.*

*Proof.* By Proposition 1, there is an index $p$ sublattice $\Lambda_p = M_p\mathbb{Z}^2 \subset \mathbb{Z}^2$ with $q(x, y) \equiv 0 \pmod{p}$ for all $(x, y) \in \Lambda_p$. Thus the quadratic form $q'(x, y) = q(M_p(x, y))$ has discriminant $(\det M_p)^2\Delta = p^2\Delta$ and is such that $q'(x, y) \equiv 0 \pmod{p}$ for all $(x, y) \in \mathbb{Z}^2$. Apply Theorem 3 to $q'$: if $q$ is positive definite, there are integers $x$ and $y$, not both zero, such that $|q(M_p(x,y)| = |q'(x,y)| \leq \left(\sqrt{\frac{|\Delta|}{3}}\right) p$. Thus $q(x,y) = kp$ with $1 \leq |k| \leq \sqrt{\frac{|\Delta|}{3}}$; since $q$ is positive definite, $k > 0$. If $\Delta > 0$, there are integers $x$ and $y$, not both zero, such that $|q(M_p(x,y)| = |q'(x,y)| \leq \left(\sqrt{\frac{\Delta}{5}}\right) p$, so $q(x,y) = kp$ with $1 \leq |k| \leq \sqrt{\frac{\Delta}{5}}$. □

**Remark.** Taking $q = \langle 1,1,1\rangle$ (resp. $\langle 1,1,-1\rangle$) shows that the bound in Theorem 4a) (resp. Theorem 4b) is sharp.

**Remark.** Let $q = \langle A,B,C\rangle$ be positive definite with $|\Delta| < 12$. Then $\sqrt{\frac{|\Delta|}{3}} < 2$, and Theorem 4 takes the form: every odd prime $p$ with $\left(\frac{\Delta}{p}\right) = 1$ is $\mathbb{Z}$-represented by $q$. It is easy to see that these are the only odd primes $p \nmid 2\Delta$ which are represented by $q$ (c.f. Proposition 2), so this proves Theorem 2 for these forms, namely for $\langle 1,1,1\rangle$, $\langle 1,0,1\rangle$, $\langle 1,1,2\rangle$, $\langle 1,0,2\rangle$, and $\langle 1,1,3\rangle$.

## 4. 2779 Regular Forms

In this section we will use Theorem 4 to prove Theorem 2.

Henceforth "forms" are primitive, positive definite integral binary quadratic forms.

### 4.1. Necessity

**Proposition 2.** *Let $q = \langle A,B,C\rangle$ be a form with discriminant $\Delta$. Let $p$ be an odd prime not dividing $\Delta$. Suppose there exist $x, y \in \mathbb{Z}$ with $q(x,y) = p$. Then $p$ satisfies conditions (i) - (iv) from Theorem 2.*

*Proof.* Via the discriminant-preserving transformation $\langle A,B,C\rangle \mapsto \langle C,B,A\rangle$ we may assume in $m \nmid A$ in part (ii) and $2 \nmid A$ in parts (iii) and (iv); otherwise, $q$ would not be primitive.
(i) If both $x$ and $y$ were divisible by $p$, this would imply $p^2 \mid q(x,y) = p$, a contradiction. If $p \nmid y$, then we have $A(xy^{-1})^2 + B(xy^{-1}) + C \equiv 0 \pmod{p}$. Let $r \in \mathbb{Z}$ with $r \equiv xy^{-1} \pmod{p}$. Then

$$(2Ar + B)^2 = 4A(Ar^2 + Br + C) + B^2 - 4AC \equiv \Delta \pmod{p}$$

As $p \nmid \Delta$, we conclude $(\frac{\Delta}{p}) = 1$. The case $p \nmid x$ follows similarly.
(ii) Let $m$ be an odd prime such that $m \mid \Delta$ and $m \nmid A$. Via a change of variables we can diagonalize $q$ over $\mathbb{Z}/m\mathbb{Z}$ as $\langle A, 0, C - B^2(4A)^{-1}\rangle$, so there are $w, z \in \mathbb{Z}$ with

$$p = q(x,y) \equiv Aw^2 + (C - B^2(4A)^{-1})z^2 \pmod{m}.$$

Multiplying by $4A$ gives $4Ap \equiv 4A^2w^2 \pmod{m}$. Hence $p \equiv Aw^2 \pmod{m}$. It follows that $(\frac{p}{m}) = (\frac{A}{m})$.
(iii) Suppose $2 \nmid A$ and $\Delta \equiv 0 \pmod{16}$. We have $B^2 \equiv 4AC \pmod{16}$, so $B = 2B_0$ for some $B_0 \in \mathbb{Z}$. Then $4(B_0^2 - AC) \equiv 0 \pmod{16}$, so $B_0^2 - AC \equiv 0 \pmod{4}$.
Case 1: $B_0$ is odd. Then $A \equiv C \equiv \pm 1 \pmod{4}$. Now, $Ax^2 + 2B_0xy + Cy^2 = p$, so $x^2 + y^2 \equiv p \equiv 1 \pmod{2}$, and $x \not\equiv y \pmod{2}$. If $y \equiv 0 \pmod{2}$, $p \equiv A \pmod{4}$ as claimed. Similarly if $x \equiv 0 \pmod{2}$, $p \equiv C \pmod{4}$. But since $A \equiv C \pmod{4}$, $p \equiv A \pmod{4}$ as claimed.
Case 2: $B_0$ is even. Then $AC \equiv 0 \pmod{4}$. As $2 \nmid A$, $C \equiv 0 \pmod{4}$. Hence, $Ax^2 \equiv p \pmod{4}$, and so $p \equiv A \pmod{4}$ as claimed.
(iv) Suppose $2 \nmid A$ and $\Delta \equiv 0 \pmod{32}$. Put $B = 2B_0$, so $B_0^2 - AC \equiv 0 \pmod{8}$.
Case 1: $B_0$ is odd, Then $A \equiv C \pmod{2}$ and in fact $A \equiv C \pmod{8}$. Thus $x^2 + y^2 \equiv p \equiv 1 \pmod{2}$, so $x \not\equiv y \pmod{2}$. If $y \equiv 0 \pmod{2}$, set $y = 2y_0$. Then $Ax^2 + 4y_0(B_0x + Cy_0) = p$. If $y_0$ is even, then $Ax^2 \equiv A \equiv p \pmod{8}$. If instead $y_0$ is odd, then since $B_0$, $x$, and $C$ are odd, $B_0x + Cy_0$ is even and $Ax^2 \equiv A \equiv p \pmod{8}$. Similarly if $x \equiv 0 \pmod{2}$, then $p \equiv C \equiv A \pmod{8}$.
Case 2: $B_0$ is even. Put $B_0 = 2B_1$ and $C = 4C_0$, so $B_1^2 \equiv AC_0 \pmod{2}$ and

$$p = Ax^2 + Bxy + Cy^2 = Ax^2 + 4y(B_1x + C_0y).$$

Thus $x$ is odd and $x^2 \equiv 1 \pmod{8}$. If $y$ is even, then $p \equiv Ax^2 \equiv A \pmod{8}$. If $y$ is odd then either $B_1 \equiv C_0 \equiv 0 \pmod{2}$ so $p \equiv Ax^2 \equiv A \pmod{8}$ or $B_1 \equiv C_0 \equiv 1 \pmod{2}$, so $B_1x + C_0y$ is even and once again $p \equiv Ax^2 \equiv A \pmod{8}$. □

### 4.2. Sufficiency

Our proof that (b) implies (a) in Theorem 2 is handled individually for each of the 2779 forms. For each form, we apply a three step process. First, we use Theorem 4 to demonstrate that our form represents a small multiple of a prime. In the second step, we *eliminate* certain multiples from consideration. In the final step, we *reduce* the remaining multiples to find a representation of $p$.

**Example.** Consider $q = \langle 3, 3, 5 \rangle$ with $\Delta = -51$. Let $p$ be an odd prime not dividing $\Delta$ that satisfies conditions (i) - (iv) of Theorem 2.

`Step 1.` From condition (i) of Theorem 2, $(\frac{\Delta}{p}) = 1$. Apply Theorem 4: there are $x, y, k \in \mathbb{Z}$ with $q(x, y) = kp$ and $1 \leq k \leq \sqrt{\frac{51}{3}} = 4.123\ldots$.

`Step 2 (Elimination).` We will show that the cases $k = 2$ and $k = 3$ cannot occur.
• Suppose $q(x, y) = 2p$. Then $x$ and $y$ are both even, so $q(x, y) = 2p \equiv 0 \pmod{4}$, contradicting the fact that $p$ is odd.
• Suppose $q(x, y) = 3p$. Then $q(x, y) \equiv 5y^2 \equiv 0 \pmod{3}$, so $3 \mid y$. Hence,

$q(x,y) \equiv 3x^2 \equiv 3p \pmod 9$, so $\left(\frac{p}{3}\right) = 1$. As $3 \mid \Delta$, from condition (ii) of Theorem 2, $(\frac{p}{3}) = (\frac{5}{3}) = -1$: contradiction.

`Step 3 (Reduction).` Note that we cannot hope to eliminate the possibility of $k = 4$: indeed, we want to show that there are $x, y \in \mathbb{Z}$ such that $q(x,y) = p$, and then necessarily $q(2x, 2y) = 4p$. (A similar argument will be needed for any value of $k$ which is a perfect square). We must instead argue that a representation of $4p$ by $q$ implies a representation of $p$ by $q$. In this case, this is easy: suppose $q(x,y) = 4p$. Then as above $x$ and $y$ are both even, so $q(\frac{x}{2}, \frac{y}{2}) = p$.

In Lemmas 1 and 2, we collect a number of congruence restrictions that apply assuming a form $q$ represents $kp$. In particular, for our 2779 forms, we use Lemma 1 in the elimination step and Lemma 2 in the reduction step.

**Lemma 1 (Elimination).** *Let $q = \langle A, B, C\rangle$ be a form of discriminant $\Delta$. Let $p \nmid 2\Delta$ be a prime. Suppose there are $x, y, k \in \mathbb{Z}$, $k \geq 1$, with $q(x,y) = kp$.*

*a) Let $a \in \mathbb{Z}$, $a > 1$. Suppose $2^{a+2} \mid \Delta$ and $2^a \mid B$. If $p \equiv A \pmod{2^a}$, then $k$ is a square modulo $2^a$.*

*b) If $k$ is even, $A, C$ are odd, $B \equiv 0 \pmod 4$ and $A + C \not\equiv 2 \pmod 4$, then $4 \mid k$.*

*c) Let $m$ be an odd prime dividing $\Delta$. If $(\frac{p}{m}) = (\frac{A}{m})$, then $k$ is a square modulo $m$.*

*d) Let $m$ be an odd prime dividing $k$. If $(\frac{\Delta}{m}) = -1$ or $m^2 \mid \Delta$, then $m^2 \mid k$.*

*e) Let $m$ be an odd prime dividing $\gcd(\Delta, k)$ such that $m^2 \nmid k$. If $(\frac{p}{m}) = (\frac{A}{m})$ then $(\frac{k/m}{m}) = (\frac{-\Delta/m}{m})$.*

*Proof.* a) Since $\Delta \equiv B^2 \equiv 0 \pmod{2^{a+2}}$, and $A$ is odd, $2^a \mid C$. Then $kp \equiv Ax^2 \equiv px^2 \pmod{2^a}$, and since $p$ is odd, this implies $k \equiv x^2 \pmod{2^a}$.

b) We have $q(x,y) \equiv Ax^2 + Cy^2 \equiv A(x^2 - y^2) \equiv kp \pmod 4$. Since $k$ is even, $x \equiv y \pmod 2$ and thus $kp \equiv A(x^2 - y^2) \equiv 0 \pmod 4$. Since $p$ is odd, $4 \mid k$.

c) Via a change of variables we can diagonalize $q$ over $\mathbb{Z}/m\mathbb{Z}$ as $\langle A, 0, C - B^2(4A)^{-1}\rangle$, so there are $w, z \in \mathbb{Z}$ with $kp = q(x,y) \equiv Aw^2 + (C - B^2(4A)^{-1})z^2 \pmod m$. Thus, $4Akp \equiv 4A^2w^2 \pmod m$, implying $kp \equiv Aw^2 \pmod m$. As $(\frac{p}{m}) = (\frac{A}{m}) \neq 0$, $k$ is a square modulo $m$.

d) Suppose first that $(\frac{\Delta}{m}) = -1$. We have $q(x,y) \equiv 0 \pmod m$. If $m \nmid y$, then $q(xy^{-1}, 1) \equiv 0 \pmod m$, so $\Delta$ is a square modulo $m$: contradiction. So $m \mid y$. Then $Ax^2 \equiv 0 \pmod m$, and $m \nmid A$, since otherwise $\Delta \equiv B^2 \pmod m$. Hence $m \mid x$. Then $m^2 \mid q(x,y) = kp$, and since $(\frac{\Delta}{p}) = 1$, we have $p \neq m$ and $m^2 \mid k$.

Next suppose $m^2 \mid \Delta$. If $m \mid \gcd(A, C)$, since $m \mid \Delta$ we would also have $m \mid B$, contradicting the primitivity of $q$. We may assume without loss of generality that $m \nmid A$. As $B^2 - 4AC \equiv 0 \pmod m$, $C \equiv B^2(4A)^{-1} \pmod m$. Hence,

$Ax^2 + Bxy + B^2(4A)^{-1}y^2 \equiv 0 \pmod{m}$, so by multiplying through by $4A$,

$$4A^2x^2 + 4ABxy + B^2y^2 \equiv (2Ax + By)^2 \equiv 0 \pmod{m}.$$

Since $m$ is prime, $2Ax+By \equiv 0 \pmod{m}$, so $4A^2x^2+4ABxy+B^2y^2 \equiv 0 \pmod{m^2}$. As $B^2 - 4AC \equiv 0 \pmod{m^2}$, we have $B^2(4A)^{-1} \equiv C \pmod{m^2}$. Then

$$4Akp \equiv 4A^2x^2 + 4ABxy + B^2y^2 \equiv 0 \pmod{m^2}.$$

Since $p \nmid \Delta$, $m \neq p$. Then $m$ does not divide $4Ap$, so $m^2 \mid k$.

e) Since $m \mid \Delta$ and $p \nmid \Delta$, $m \neq p$. We may write $\Delta = m\Delta_0$ and $k = mk_0$ with $\Delta_0, k_0 \in \mathbb{Z}$ and $m \nmid k_0$. Then $Ax^2 + Bxy + Cy^2 \equiv mk_0p \pmod{m^2}$. As in part d),

$$Ax^2 + Bxy + (B^2(4A)^{-1})y^2 \equiv 0 \pmod{m^2}.$$

Subtracting gives $(C - B^2(4A^{-1}))y^2 \equiv mk_0p \pmod{m^2}$. Since $\gcd(m, k_0p) = 1$, it follows that $m \nmid y$. Multiplying through by $4A$, we get

$$-m\Delta_0y^2 \equiv (4AC - B^2)y^2 \equiv 4Amk_0p \pmod{m^2}.$$

Then $(4Ak_0p + \Delta_0y^2)m \equiv 0 \pmod{m^2}$, so $4Apk_0 \equiv -\Delta_0y^2 \pmod{m}$. It follows that $(\frac{-\Delta_0}{m}) = (\frac{-\Delta_0y^2}{m}) = (\frac{4Apk_0}{m}) \equiv (\frac{A}{m})(\frac{p}{m})(\frac{k_0}{m}) = (\frac{k_0}{m})$. □

**Lemma 2 (Reduction).** *Let $q = \langle A, B, C\rangle$ have discriminant $\Delta$. Let $p$ be an odd prime not dividing $\Delta$. Suppose there exist $x, y, k \in \mathbb{Z}$ with $q(x, y) = kp$ and $k \geq 1$.*

*a) Let $a \in \mathbb{Z}$ with $a \geq 1$. If $p \equiv A \pmod{2^a}$, then $q(x, y) \equiv Ak \pmod{2^ak}$.*

*b) Let $a \in \mathbb{Z}$ with $a \geq 0$, and let $m \mid \Delta$ be an odd prime. If $m^{2a} \mid k$, $m^{2a+1} \nmid k$, and $(\frac{p}{m}) = (\frac{A}{m})$, then we have $(\frac{q(x,y)/m^{2a}}{m}) = (\frac{Ak/m^{2a}}{m})$.*

*Proof.* For a), write $p = 2^a\ell + A$. Then $q(x, y) \equiv k(2^a\ell + A) \equiv Ak \pmod{2^ak}$. For b), write $k = m^{2a}k_0$. Then $(\frac{q(x,y)/m^{2a}}{m}) = (\frac{k_0p}{m}) = (\frac{Ak_0}{m})$. □

### 4.3. Proof of Theorem 2

(a) $\implies$ (b): This is Proposition 2.

(b) $\implies$ (a): Let $q = \langle A, B, C\rangle$ be one of the 2779 regular forms, and let $p \nmid 2\Delta$ be a prime satisfying conditions (i) - (iv) from Theorem 2.

`Step 1.` Using condition (i), Theorem 4 implies there exist $x, y, k \in \mathbb{Z}$ such that $q(x, y) = kp$ with $1 \leq k \leq \sqrt{\frac{|\Delta|}{3}}$.

`Step 2 (Elimination).` For each $k \in \{2, \ldots, \lfloor\sqrt{\frac{|\Delta|}{3}}\rfloor\}$, assume $q(x, y) = kp$. If $k$ does not satisfy the conditions imposed on it by Lemma 1, we have a contradiction.

We similarly have a contradiction if $k$ does not satisfy the conditions imposed on it by applying Lemma 1 to the equivalent forms $q(y,x) = \langle C, B, A\rangle$ and $q(x+y, x+2y) = \langle A+B+C, 2A+3B+4C, A+2B+4C\rangle$ representing $kp$. We eliminate these $k$ from consideration.

`Step 3 (Reduction).` For each $k \in \{2, \ldots, \lfloor\sqrt{\frac{|\Delta|}{3}}\rfloor\}$ that was not eliminated in Step 2, assume $q(x,y) = kp$. Using a computer, we have verified that this assumption leads to a representation of $p$ by $q$ in every case. Our algorithm is as follows. First, we construct the finite set of matrices

$$\mathcal{M} = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(\mathbb{Z}) \;\middle|\; a \geq 0,\; q(a,c) = kA \text{ and } q(b,d) = kC \right\}$$

by enumerating the representations of $kA$ and $kC$ by $q$. Given $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathcal{M}$,

$$q(M(x,y)) = kAx^2 + (2abA + (ad+bc)B + 2cdC)xy + kCy^2.$$

In particular, $q(M(x,y)) = kq(x,y)$ whenever $2abA + (ad+bc)B + 2cdC = kB$. By iterating over $\mathcal{M}$ and checking this condition, we verify that there exists some $M \in \mathcal{M}$ such that $q(M(x,y)) = kq(x,y)$. Fixing such an $M$, we further check whether for each $(x,y) \in \mathbb{Z}^2$ with $q(x,y) \equiv 0 \pmod{k}$ that also satisfies the congruence restrictions imposed by Lemma 2, the pair $(x_0, y_0) = M(x,y)$ satisfies $x_0 \equiv y_0 \equiv 0 \pmod{k}$. It suffices to check this condition modulo $k\Delta$ by an exhaustive search. In every case we've considered, this search successfully produces such an $M \in \mathcal{M}$. Once such an $M$ has been found, we can set $x_0 = kw$ and $y_0 = kz$. Then $q(M(x,y)) = q(kw, kz) = k^2p$, so $q(w,z) = p$. Therefore, we've shown that $q$ represents $p$.

**Example.** Consider $q = \langle 2, 1, 7\rangle$ with $\Delta = -55$. Let $p$ be an odd prime not dividing $\Delta$ that satisfies conditions (i) - (iv) of Theorem 2.

`Step 1.` From condition (i) of Theorem 2, $(\frac{\Delta}{p}) = 1$. Thus, applying Theorem 4 yields $x, y, k \in \mathbb{Z}$ with $q(x,y) = kp$ and $1 \leq k \leq \sqrt{\frac{55}{3}} = 4.28\ldots$ .

`Step 2 (Elimination).` By Lemma 1(c), $k$ is a square modulo 5. As $(\frac{2}{5}) = (\frac{3}{5}) = -1$, $k \in \{1, 4\}$.

`Step 3 (Reduction).` Suppose $q(x,y) = 4p$. One might try to argue, as in the example in Section 4.2, that both $x$ and $y$ are even. However, this need not be the case: e.g. $q$ represents 7 and $q(3,1) = 4 \cdot 7$. Applying the algorithm described above we obtain

$$\mathcal{M} = \left\{ \begin{bmatrix} 1 & -3 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}, \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -3 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \right\}.$$

Set $M = \begin{bmatrix} 1 & -3 \\ -1 & -1 \end{bmatrix}$. Set $(x_0, y_0) = M(x, y) = (x - 3y, -x - y)$ and note $q(x_0, y_0) = 4q(x, y) = 16p$. If we knew $x_0 \equiv y_0 \equiv 0 \pmod 4$, then we could divide through by 4 to obtain an integer representation of $p$. Certainly we need only consider $(x, y) \in \mathbb{Z}^2$ with $q(x, y) \equiv 0 \pmod 4$. Further, since we're assuming $(\frac{p}{5}) = (\frac{2}{5}) = -1$ and $(\frac{p}{11}) = (\frac{2}{11}) = -1$, condition (ii) of Theorem 2 implies we need only consider $(x, y) \in \mathbb{Z}^2$ with $(\frac{q(x,y)}{5}) = (\frac{4p}{5}) = -1$ and $(\frac{q(x,y)}{11}) = (\frac{4p}{11}) = -1$. By an exhaustive search modulo 220, we verify the only such $(x, y) \in \mathbb{Z}^2$ yield $x_0 \equiv y_0 \equiv 0 \pmod 4$. Setting $x_0 = 4w$ and $y_0 = 4z$, we have $q(x_0, y_0) = 32w^2 + 16wz + 224z^2 = 16p$. Dividing through by 16, we see $q(w, z) = 2w^2 + wz + 7z^2 = p$. Therefore, we've shown that $q$ represents $p$.

**Acknowledgements.** This work was done in the context of a VIGRE Research Group at the University of Georgia during the 2011-2012 academic year. The group was led by the first author, with participants the other three authors together with Christopher Drupieski (postdoc), Brian Bonsignore, Harrison Chapman, Lauren Huckaba, David Krumm, Allan Lacy Mora, Nham Ngo, Alex Rice, James Stankewicz, Lee Troupe, Nathan Walters (doctoral students) and Jun Zhang (master's student).

## References

[1] P.L. Clark, *Geometry of numbers explained*, in preparation.

[2] P.L. Clark, J. Hicks, K. Thompson and N. Walters, *GoNII: Universal quaternary quadratic forms.* Integers 12 (2012), A50, 16 pp.

[3] D.A. Cox, *Primes of the Form* $x^2 + ny^2$, John Wiley & Sons Inc., 1989.

[4] C.F. Gauss, *Disquisitiones Arithmeticae (English Edition)*, trans. A.A. Clarke, Springer-Verlag, 1986.

[5] T.R. Hagedorn, *Primes of the Form* $x^2 + ny^2$ *and the Geometry of (Convenient) Numbers*, preprint.

[6] F. Halter-Koch, *Representation of prime powers in arithmetical progressions by binary quadratic forms.* Les XXIIèmes Journées Arithmetiques (Lille, 2001). J. Théor. Nombres Bordeaux 15 (2003), no. 1, 141-149.

[7] G.H. Hardy and E.M. Wright, *An Introduction to the Theory of Numbers*. Sixth edition. Revised by D. R. Heath-Brown and J. H. Silverman. Oxford, 2008.

[8] J. Hicks and K. Thompson, *GoNIII: More universal quaternary quadratic forms*, in preparation.

[9] P. Kaplan and K.S. Williams, *Representation of primes in arithmetic progression by binary quadratic forms*. J. Number Theory (1993), 61-67.

[10] T. Kusaba, *Remarque sur la distribution des nombres premiers*. C. R. Acad. Sci. Paris Sér. A-B 265 (1967), A405-A407.

[11] S. Louboutin, *Minorations (sous l'hypothèse de Riemann généralisée) des nombres de classes des corps quadratiques imaginaires. Application*. C. R. Acad. Sci. Paris Sér. I Math. 310 (1990), no. 12, 795-800.

[12] A. Meyer, *Über einen satz von Dirichlet*. J. Reine Angew. Math. 103 (1888), 98–117.

[13] W. A. Stein et al., *Sage Mathematics Software (Version 4.7.1)*, The Sage Development Team, 2011, `http://www.sagemath.org`

[14] J. Voight, *Quadratic forms that represent almost the same primes*. Math. Comp. 76 (2007), 1589-1617.

[15] P.J. Weinberger, *Exponents of the class groups of complex quadratic fields*. Acta Arith. 22 (1973), 117-124.

## Appendix

In Table 1, we list the reduced representative for each of the 2779 $\mathrm{SL}_2(\mathbb{Z})$ equivalence classes of regular forms. The discriminants were calculated by Voight in [14]. We redid this calculation, and in so doing found a minor error of tabulation which Voight confirmed. The forms were generated using Sage.

Table 1: Representatives for 2779 $\mathrm{SL}_2(\mathbb{Z})$-equivalence Classes of Regular Forms

| $\|\Delta\|$ | $\langle A,B,C\rangle$ | $\|\Delta\|$ | $\langle A,B,C\rangle$ | $\|\Delta\|$ | $\langle A,B,C\rangle$ | $\|\Delta\|$ | $\langle A,B,C\rangle$ |
|---|---|---|---|---|---|---|---|
| 3 | $\langle 1,1,1\rangle$ | 4 | $\langle 1,0,1\rangle$ | 7 | $\langle 1,1,2\rangle$ | 8 | $\langle 1,0,2\rangle$ |
| 11 | $\langle 1,1,3\rangle$ | 12 | $\langle 1,0,3\rangle$ | 15 | $\langle 1,1,4\rangle$ | 15 | $\langle 2,1,2\rangle$ |
| 16 | $\langle 1,0,4\rangle$ | 19 | $\langle 1,1,5\rangle$ | 20 | $\langle 1,0,5\rangle$ | 20 | $\langle 2,2,3\rangle$ |
| 24 | $\langle 1,0,6\rangle$ | 24 | $\langle 2,0,3\rangle$ | 27 | $\langle 1,1,7\rangle$ | 28 | $\langle 1,0,7\rangle$ |
| 32 | $\langle 1,0,8\rangle$ | 32 | $\langle 3,2,3\rangle$ | 35 | $\langle 1,1,9\rangle$ | 35 | $\langle 3,1,3\rangle$ |
| 36 | $\langle 1,0,9\rangle$ | 36 | $\langle 2,2,5\rangle$ | 39 | $\langle 2,\pm1,5\rangle$ | 40 | $\langle 1,0,10\rangle$ |
| 40 | $\langle 2,0,5\rangle$ | 43 | $\langle 1,1,11\rangle$ | 48 | $\langle 1,0,12\rangle$ | 48 | $\langle 3,0,4\rangle$ |
| 51 | $\langle 1,1,13\rangle$ | 51 | $\langle 3,3,5\rangle$ | 52 | $\langle 1,0,13\rangle$ | 52 | $\langle 2,2,7\rangle$ |
| 55 | $\langle 2,\pm1,7\rangle$ | 56 | $\langle 3,\pm2,5\rangle$ | 60 | $\langle 1,0,15\rangle$ | 60 | $\langle 3,0,5\rangle$ |
| 63 | $\langle 2,\pm1,8\rangle$ | 64 | $\langle 1,0,16\rangle$ | 64 | $\langle 4,4,5\rangle$ | 67 | $\langle 1,1,17\rangle$ |
| 68 | $\langle 3,\pm2,6\rangle$ | 72 | $\langle 1,0,18\rangle$ | 72 | $\langle 2,0,9\rangle$ | 75 | $\langle 1,1,19\rangle$ |
| 75 | $\langle 3,3,7\rangle$ | 80 | $\langle 3,\pm2,7\rangle$ | 84 | $\langle 1,0,21\rangle$ | 84 | $\langle 2,2,11\rangle$ |
| 84 | $\langle 3,0,7\rangle$ | 84 | $\langle 5,4,5\rangle$ | 88 | $\langle 1,0,22\rangle$ | 88 | $\langle 2,0,11\rangle$ |
| 91 | $\langle 1,1,23\rangle$ | 91 | $\langle 5,3,5\rangle$ | 96 | $\langle 1,0,24\rangle$ | 96 | $\langle 3,0,8\rangle$ |
| 96 | $\langle 4,4,7\rangle$ | 96 | $\langle 5,2,5\rangle$ | 99 | $\langle 1,1,25\rangle$ | 99 | $\langle 5,1,5\rangle$ |
| 100 | $\langle 1,0,25\rangle$ | 100 | $\langle 2,2,13\rangle$ | 112 | $\langle 1,0,28\rangle$ | 112 | $\langle 4,0,7\rangle$ |
| 115 | $\langle 1,1,29\rangle$ | 115 | $\langle 5,5,7\rangle$ | 120 | $\langle 1,0,30\rangle$ | 120 | $\langle 2,0,15\rangle$ |
| 120 | $\langle 3,0,10\rangle$ | 120 | $\langle 5,0,6\rangle$ | 123 | $\langle 1,1,31\rangle$ | 123 | $\langle 3,3,11\rangle$ |
| 128 | $\langle 3,\pm2,11\rangle$ | 132 | $\langle 1,0,33\rangle$ | 132 | $\langle 2,2,17\rangle$ | 132 | $\langle 3,0,11\rangle$ |
| 132 | $\langle 6,6,7\rangle$ | 136 | $\langle 5,\pm2,7\rangle$ | 144 | $\langle 5,\pm4,8\rangle$ | 147 | $\langle 1,1,37\rangle$ |

Table 1: Representatives for 2779 $SL_2(\mathbb{Z})$-equivalence Classes of Regular Forms

| $\lvert\Delta\rvert$ | $\langle A, B, C\rangle$ | $\lvert\Delta\rvert$ | $\langle A, B, C\rangle$ | $\lvert\Delta\rvert$ | $\langle A, B, C\rangle$ | $\lvert\Delta\rvert$ | $\langle A, B, C\rangle$ |
|---|---|---|---|---|---|---|---|
| 147 | $\langle 3,3,13\rangle$ | 148 | $\langle 1,0,37\rangle$ | 148 | $\langle 2,2,19\rangle$ | 155 | $\langle 3,\pm1,13\rangle$ |
| 156 | $\langle 5,\pm2,8\rangle$ | 160 | $\langle 1,0,40\rangle$ | 160 | $\langle 4,4,11\rangle$ | 160 | $\langle 5,0,8\rangle$ |
| 160 | $\langle 7,6,7\rangle$ | 163 | $\langle 1,1,41\rangle$ | 168 | $\langle 1,0,42\rangle$ | 168 | $\langle 2,0,21\rangle$ |
| 168 | $\langle 3,0,14\rangle$ | 168 | $\langle 6,0,7\rangle$ | 171 | $\langle 5,\pm3,9\rangle$ | 180 | $\langle 1,0,45\rangle$ |
| 180 | $\langle 2,2,23\rangle$ | 180 | $\langle 5,0,9\rangle$ | 180 | $\langle 7,4,7\rangle$ | 184 | $\langle 5,\pm4,10\rangle$ |
| 187 | $\langle 1,1,47\rangle$ | 187 | $\langle 7,3,7\rangle$ | 192 | $\langle 1,0,48\rangle$ | 192 | $\langle 3,0,16\rangle$ |
| 192 | $\langle 4,4,13\rangle$ | 192 | $\langle 7,2,7\rangle$ | 195 | $\langle 1,1,49\rangle$ | 195 | $\langle 3,3,17\rangle$ |
| 195 | $\langle 5,5,11\rangle$ | 195 | $\langle 7,1,7\rangle$ | 196 | $\langle 5,\pm2,10\rangle$ | 203 | $\langle 3,\pm1,17\rangle$ |
| 208 | $\langle 7,\pm4,8\rangle$ | 219 | $\langle 5,\pm1,11\rangle$ | 220 | $\langle 7,\pm2,8\rangle$ | 224 | $\langle 3,\pm2,19\rangle$ |
| 224 | $\langle 5,\pm4,12\rangle$ | 228 | $\langle 1,0,57\rangle$ | 228 | $\langle 2,2,29\rangle$ | 228 | $\langle 3,0,19\rangle$ |
| 228 | $\langle 6,6,11\rangle$ | 232 | $\langle 1,0,58\rangle$ | 232 | $\langle 2,0,29\rangle$ | 235 | $\langle 1,1,59\rangle$ |
| 235 | $\langle 5,5,13\rangle$ | 240 | $\langle 1,0,60\rangle$ | 240 | $\langle 3,0,20\rangle$ | 240 | $\langle 4,0,15\rangle$ |
| 240 | $\langle 5,0,12\rangle$ | 252 | $\langle 8,\pm6,9\rangle$ | 256 | $\langle 5,\pm2,13\rangle$ | 259 | $\langle 5,\pm1,13\rangle$ |
| 260 | $\langle 3,\pm2,22\rangle$ | 260 | $\langle 6,\pm2,11\rangle$ | 264 | $\langle 5,\pm4,14\rangle$ | 264 | $\langle 7,\pm4,10\rangle$ |
| 267 | $\langle 1,1,67\rangle$ | 267 | $\langle 3,3,23\rangle$ | 275 | $\langle 3,\pm1,23\rangle$ | 276 | $\langle 5,\pm2,14\rangle$ |
| 276 | $\langle 7,\pm2,10\rangle$ | 280 | $\langle 1,0,70\rangle$ | 280 | $\langle 2,0,35\rangle$ | 280 | $\langle 5,0,14\rangle$ |
| 280 | $\langle 7,0,10\rangle$ | 288 | $\langle 1,0,72\rangle$ | 288 | $\langle 4,4,19\rangle$ | 288 | $\langle 8,0,9\rangle$ |
| 288 | $\langle 8,8,11\rangle$ | 291 | $\langle 5,\pm3,15\rangle$ | 292 | $\langle 7,\pm4,11\rangle$ | 308 | $\langle 3,\pm2,26\rangle$ |
| 308 | $\langle 6,\pm2,13\rangle$ | 312 | $\langle 1,0,78\rangle$ | 312 | $\langle 2,0,39\rangle$ | 312 | $\langle 3,0,26\rangle$ |
| 312 | $\langle 6,0,13\rangle$ | 315 | $\langle 1,1,79\rangle$ | 315 | $\langle 5,5,17\rangle$ | 315 | $\langle 7,7,13\rangle$ |
| 315 | $\langle 9,9,11\rangle$ | 320 | $\langle 3,\pm2,27\rangle$ | 320 | $\langle 7,\pm4,12\rangle$ | 323 | $\langle 3,\pm1,27\rangle$ |
| 328 | $\langle 7,\pm6,13\rangle$ | 336 | $\langle 5,\pm2,17\rangle$ | 336 | $\langle 8,\pm4,11\rangle$ | 340 | $\langle 1,0,85\rangle$ |
| 340 | $\langle 2,2,43\rangle$ | 340 | $\langle 5,0,17\rangle$ | 340 | $\langle 10,10,11\rangle$ | 352 | $\langle 1,0,88\rangle$ |
| 352 | $\langle 4,4,23\rangle$ | 352 | $\langle 8,0,11\rangle$ | 352 | $\langle 8,8,13\rangle$ | 355 | $\langle 7,\pm3,13\rangle$ |
| 360 | $\langle 7,\pm2,13\rangle$ | 360 | $\langle 9,\pm6,11\rangle$ | 363 | $\langle 7,\pm1,13\rangle$ | 372 | $\langle 1,0,93\rangle$ |
| 372 | $\langle 2,2,47\rangle$ | 372 | $\langle 3,0,31\rangle$ | 372 | $\langle 6,6,17\rangle$ | 384 | $\langle 5,\pm4,20\rangle$ |
| 384 | $\langle 7,\pm6,15\rangle$ | 387 | $\langle 9,\pm3,11\rangle$ | 388 | $\langle 7,\pm2,14\rangle$ | 400 | $\langle 8,\pm4,13\rangle$ |
| 403 | $\langle 1,1,101\rangle$ | 403 | $\langle 11,9,11\rangle$ | 408 | $\langle 1,0,102\rangle$ | 408 | $\langle 2,0,51\rangle$ |
| 408 | $\langle 3,0,34\rangle$ | 408 | $\langle 6,0,17\rangle$ | 420 | $\langle 1,0,105\rangle$ | 420 | $\langle 2,2,53\rangle$ |
| 420 | $\langle 3,0,35\rangle$ | 420 | $\langle 5,0,21\rangle$ | 420 | $\langle 6,6,19\rangle$ | 420 | $\langle 7,0,15\rangle$ |
| 420 | $\langle 10,10,13\rangle$ | 420 | $\langle 11,8,11\rangle$ | 427 | $\langle 1,1,107\rangle$ | 427 | $\langle 7,7,17\rangle$ |
| 435 | $\langle 1,1,109\rangle$ | 435 | $\langle 3,3,37\rangle$ | 435 | $\langle 5,5,23\rangle$ | 435 | $\langle 11,7,11\rangle$ |
| 448 | $\langle 1,0,112\rangle$ | 448 | $\langle 4,4,29\rangle$ | 448 | $\langle 7,0,16\rangle$ | 448 | $\langle 11,6,11\rangle$ |
| 456 | $\langle 5,\pm2,23\rangle$ | 456 | $\langle 10,\pm8,13\rangle$ | 468 | $\langle 7,\pm6,18\rangle$ | 468 | $\langle 9,\pm6,14\rangle$ |
| 475 | $\langle 7,\pm1,17\rangle$ | 480 | $\langle 1,0,120\rangle$ | 480 | $\langle 3,0,40\rangle$ | 480 | $\langle 4,4,31\rangle$ |
| 480 | $\langle 5,0,24\rangle$ | 480 | $\langle 8,0,15\rangle$ | 480 | $\langle 8,8,17\rangle$ | 480 | $\langle 11,2,11\rangle$ |
| 480 | $\langle 12,12,13\rangle$ | 483 | $\langle 1,1,121\rangle$ | 483 | $\langle 3,3,41\rangle$ | 483 | $\langle 7,7,19\rangle$ |
| 483 | $\langle 11,1,11\rangle$ | 504 | $\langle 5,\pm4,26\rangle$ | 504 | $\langle 10,\pm4,13\rangle$ | 507 | $\langle 7,\pm5,19\rangle$ |
| 520 | $\langle 1,0,130\rangle$ | 520 | $\langle 2,0,65\rangle$ | 520 | $\langle 5,0,26\rangle$ | 520 | $\langle 10,0,13\rangle$ |
| 528 | $\langle 7,\pm2,19\rangle$ | 528 | $\langle 8,\pm4,17\rangle$ | 532 | $\langle 1,0,133\rangle$ | 532 | $\langle 2,2,67\rangle$ |
| 532 | $\langle 7,0,19\rangle$ | 532 | $\langle 13,12,13\rangle$ | 544 | $\langle 5,\pm4,28\rangle$ | 544 | $\langle 7,\pm4,20\rangle$ |
| 552 | $\langle 7,\pm6,21\rangle$ | 552 | $\langle 11,\pm8,14\rangle$ | 555 | $\langle 1,1,139\rangle$ | 555 | $\langle 3,3,47\rangle$ |
| 555 | $\langle 5,5,29\rangle$ | 555 | $\langle 13,11,13\rangle$ | 564 | $\langle 5,\pm4,29\rangle$ | 564 | $\langle 10,\pm6,15\rangle$ |
| 568 | $\langle 11,\pm2,13\rangle$ | 576 | $\langle 5,\pm2,29\rangle$ | 576 | $\langle 9,\pm6,17\rangle$ | 580 | $\langle 7,\pm6,22\rangle$ |
| 580 | $\langle 11,\pm6,14\rangle$ | 592 | $\langle 8,\pm4,19\rangle$ | 595 | $\langle 1,1,149\rangle$ | 595 | $\langle 5,5,31\rangle$ |
| 595 | $\langle 7,7,23\rangle$ | 595 | $\langle 13,9,13\rangle$ | 600 | $\langle 7,\pm4,22\rangle$ | 600 | $\langle 11,\pm4,14\rangle$ |
| 603 | $\langle 9,\pm3,17\rangle$ | 612 | $\langle 7,\pm2,22\rangle$ | 612 | $\langle 11,\pm2,14\rangle$ | 616 | $\langle 5,\pm2,31\rangle$ |
| 616 | $\langle 10,\pm8,17\rangle$ | 624 | $\langle 5,\pm4,32\rangle$ | 624 | $\langle 11,\pm6,15\rangle$ | 627 | $\langle 1,1,157\rangle$ |
| 627 | $\langle 3,3,53\rangle$ | 627 | $\langle 11,11,17\rangle$ | 627 | $\langle 13,7,13\rangle$ | 640 | $\langle 7,\pm2,23\rangle$ |
| 640 | $\langle 11,\pm8,16\rangle$ | 651 | $\langle 5,\pm3,33\rangle$ | 651 | $\langle 11,\pm3,15\rangle$ | 660 | $\langle 1,0,165\rangle$ |
| 660 | $\langle 2,2,83\rangle$ | 660 | $\langle 3,0,55\rangle$ | 660 | $\langle 5,0,33\rangle$ | 660 | $\langle 6,6,29\rangle$ |
| 660 | $\langle 10,10,19\rangle$ | 660 | $\langle 11,0,15\rangle$ | 660 | $\langle 13,4,13\rangle$ | 667 | $\langle 11,\pm9,17\rangle$ |
| 672 | $\langle 1,0,168\rangle$ | 672 | $\langle 3,0,56\rangle$ | 672 | $\langle 4,4,43\rangle$ | 672 | $\langle 7,0,24\rangle$ |
| 672 | $\langle 8,0,21\rangle$ | 672 | $\langle 8,8,23\rangle$ | 672 | $\langle 12,12,17\rangle$ | 672 | $\langle 13,2,13\rangle$ |
| 708 | $\langle 1,0,177\rangle$ | 708 | $\langle 2,2,89\rangle$ | 708 | $\langle 3,0,59\rangle$ | 708 | $\langle 6,6,31\rangle$ |
| 715 | $\langle 1,1,179\rangle$ | 715 | $\langle 5,5,37\rangle$ | 715 | $\langle 11,11,19\rangle$ | 715 | $\langle 13,13,17\rangle$ |
| 720 | $\langle 7,\pm6,27\rangle$ | 720 | $\langle 8,\pm4,23\rangle$ | 723 | $\langle 11,\pm5,17\rangle$ | 736 | $\langle 5,\pm2,37\rangle$ |
| 736 | $\langle 11,\pm10,19\rangle$ | 760 | $\langle 1,0,190\rangle$ | 760 | $\langle 2,0,95\rangle$ | 760 | $\langle 5,0,38\rangle$ |
| 760 | $\langle 10,0,19\rangle$ | 763 | $\langle 13,\pm11,17\rangle$ | 768 | $\langle 7,\pm4,28\rangle$ | 768 | $\langle 13,\pm8,16\rangle$ |
| 772 | $\langle 11,\pm8,19\rangle$ | 792 | $\langle 9,\pm6,23\rangle$ | 792 | $\langle 13,\pm12,18\rangle$ | 795 | $\langle 1,1,199\rangle$ |
| 795 | $\langle 3,3,67\rangle$ | 795 | $\langle 5,5,41\rangle$ | 795 | $\langle 15,15,17\rangle$ | 819 | $\langle 5,\pm1,41\rangle$ |
| 819 | $\langle 9,\pm3,23\rangle$ | 820 | $\langle 11,\pm4,19\rangle$ | 820 | $\langle 13,\pm8,17\rangle$ | 832 | $\langle 7,\pm6,31\rangle$ |
| 832 | $\langle 11,\pm2,19\rangle$ | 840 | $\langle 1,0,210\rangle$ | 840 | $\langle 2,0,105\rangle$ | 840 | $\langle 3,0,70\rangle$ |
| 840 | $\langle 5,0,42\rangle$ | 840 | $\langle 6,0,35\rangle$ | 840 | $\langle 7,0,30\rangle$ | 840 | $\langle 10,0,21\rangle$ |

Table 1: Representatives for 2779 $SL_2(\mathbb{Z})$-equivalence Classes of Regular Forms

| $\lvert\Delta\rvert$ | $\langle A, B, C\rangle$ | $\lvert\Delta\rvert$ | $\langle A, B, C\rangle$ | $\lvert\Delta\rvert$ | $\langle A, B, C\rangle$ | $\lvert\Delta\rvert$ | $\langle A, B, C\rangle$ |
|---|---|---|---|---|---|---|---|
| 840 | ⟨14, 0, 15⟩ | 852 | ⟨7, ±4, 31⟩ | 852 | ⟨14, ±10, 17⟩ | 868 | ⟨11, ±10, 22⟩ |
| 868 | ⟨13, ±4, 17⟩ | 880 | ⟨7, ±4, 32⟩ | 880 | ⟨13, ±2, 17⟩ | 900 | ⟨9, ±6, 26⟩ |
| 900 | ⟨13, ±6, 18⟩ | 912 | ⟨8, ±4, 29⟩ | 912 | ⟨11, ±10, 23⟩ | 915 | ⟨7, ±3, 33⟩ |
| 915 | ⟨11, ±3, 21⟩ | 928 | ⟨1, 0, 232⟩ | 928 | ⟨4, 4, 59⟩ | 928 | ⟨8, 0, 29⟩ |
| 928 | ⟨8, 8, 31⟩ | 952 | ⟨11, ±4, 22⟩ | 952 | ⟨13, ±6, 19⟩ | 955 | ⟨7, ±5, 35⟩ |
| 960 | ⟨1, 0, 240⟩ | 960 | ⟨3, 0, 80⟩ | 960 | ⟨4, 4, 61⟩ | 960 | ⟨5, 0, 48⟩ |
| 960 | ⟨12, 12, 23⟩ | 960 | ⟨15, 0, 16⟩ | 960 | ⟨16, 16, 19⟩ | 960 | ⟨17, 14, 17⟩ |
| 987 | ⟨11, ±5, 23⟩ | 987 | ⟨13, ±1, 19⟩ | 1003 | ⟨11, ±3, 23⟩ | 1008 | ⟨9, ±6, 29⟩ |
| 1008 | ⟨11, ±2, 23⟩ | 1012 | ⟨1, 0, 253⟩ | 1012 | ⟨2, 2, 127⟩ | 1012 | ⟨11, 0, 23⟩ |
| 1012 | ⟨17, 12, 17⟩ | 1027 | ⟨7, ±3, 37⟩ | 1032 | ⟨7, ±2, 37⟩ | 1032 | ⟨14, ±12, 21⟩ |
| 1035 | ⟨7, ±1, 37⟩ | 1035 | ⟨9, ±3, 29⟩ | 1056 | ⟨5, ±2, 53⟩ | 1056 | ⟨7, ±6, 39⟩ |
| 1056 | ⟨13, ±6, 21⟩ | 1056 | ⟨15, ±12, 20⟩ | 1060 | ⟨7, ±2, 38⟩ | 1060 | ⟨14, ±2, 19⟩ |
| 1092 | ⟨1, 0, 273⟩ | 1092 | ⟨2, 2, 137⟩ | 1092 | ⟨3, 0, 91⟩ | 1092 | ⟨6, 6, 47⟩ |
| 1092 | ⟨7, 0, 39⟩ | 1092 | ⟨13, 0, 21⟩ | 1092 | ⟨14, 14, 23⟩ | 1092 | ⟨17, 8, 17⟩ |
| 1120 | ⟨1, 0, 280⟩ | 1120 | ⟨4, 4, 71⟩ | 1120 | ⟨5, 0, 56⟩ | 1120 | ⟨7, 0, 40⟩ |
| 1120 | ⟨8, 0, 35⟩ | 1120 | ⟨8, 8, 37⟩ | 1120 | ⟨17, 6, 17⟩ | 1120 | ⟨19, 18, 19⟩ |
| 1128 | ⟨11, ±4, 26⟩ | 1128 | ⟨13, ±4, 22⟩ | 1131 | ⟨5, ±3, 57⟩ | 1131 | ⟨15, ±3, 19⟩ |
| 1140 | ⟨7, ±6, 42⟩ | 1140 | ⟨11, ±2, 26⟩ | 1140 | ⟨13, ±2, 22⟩ | 1140 | ⟨14, ±6, 21⟩ |
| 1152 | ⟨11, ±6, 27⟩ | 1152 | ⟨16, ±8, 19⟩ | 1155 | ⟨1, 1, 289⟩ | 1155 | ⟨3, 3, 97⟩ |
| 1155 | ⟨5, 5, 59⟩ | 1155 | ⟨7, 7, 43⟩ | 1155 | ⟨11, 11, 29⟩ | 1155 | ⟨15, 15, 23⟩ |
| 1155 | ⟨17, 1, 17⟩ | 1155 | ⟨19, 17, 19⟩ | 1204 | ⟨5, ±4, 61⟩ | 1204 | ⟨10, ±6, 31⟩ |
| 1227 | ⟨11, ±7, 29⟩ | 1240 | ⟨11, ±6, 29⟩ | 1240 | ⟨17, ±16, 22⟩ | 1243 | ⟨17, ±7, 19⟩ |
| 1248 | ⟨1, 0, 312⟩ | 1248 | ⟨3, 0, 104⟩ | 1248 | ⟨4, 4, 79⟩ | 1248 | ⟨8, 0, 39⟩ |
| 1248 | ⟨8, 8, 41⟩ | 1248 | ⟨12, 12, 29⟩ | 1248 | ⟨13, 0, 24⟩ | 1248 | ⟨19, 14, 19⟩ |
| 1275 | ⟨11, ±1, 29⟩ | 1275 | ⟨13, ±5, 25⟩ | 1288 | ⟨13, ±8, 26⟩ | 1288 | ⟨17, ±2, 19⟩ |
| 1312 | ⟨7, ±2, 47⟩ | 1312 | ⟨13, ±12, 28⟩ | 1320 | ⟨1, 0, 330⟩ | 1320 | ⟨2, 0, 165⟩ |
| 1320 | ⟨3, 0, 110⟩ | 1320 | ⟨5, 0, 66⟩ | 1320 | ⟨6, 0, 55⟩ | 1320 | ⟨10, 0, 33⟩ |
| 1320 | ⟨11, 0, 30⟩ | 1320 | ⟨15, 0, 22⟩ | 1332 | ⟨9, ±6, 38⟩ | 1332 | ⟨18, ±6, 19⟩ |
| 1344 | ⟨5, ±4, 68⟩ | 1344 | ⟨11, ±8, 32⟩ | 1344 | ⟨15, ±6, 23⟩ | 1344 | ⟨17, ±4, 20⟩ |
| 1360 | ⟨8, ±4, 43⟩ | 1360 | ⟨11, ±2, 31⟩ | 1380 | ⟨1, 0, 345⟩ | 1380 | ⟨2, 2, 173⟩ |
| 1380 | ⟨3, 0, 115⟩ | 1380 | ⟨5, 0, 69⟩ | 1380 | ⟨6, 6, 59⟩ | 1380 | ⟨10, 10, 37⟩ |
| 1380 | ⟨15, 0, 23⟩ | 1380 | ⟨19, 8, 19⟩ | 1387 | ⟨13, ±11, 29⟩ | 1395 | ⟨13, ±3, 27⟩ |
| 1395 | ⟨17, ±13, 23⟩ | 1408 | ⟨13, ±10, 29⟩ | 1408 | ⟨16, ±8, 23⟩ | 1411 | ⟨5, ±3, 71⟩ |
| 1428 | ⟨1, 0, 357⟩ | 1428 | ⟨2, 2, 179⟩ | 1428 | ⟨3, 0, 119⟩ | 1428 | ⟨6, 6, 61⟩ |
| 1428 | ⟨7, 0, 51⟩ | 1428 | ⟨14, 14, 29⟩ | 1428 | ⟨17, 0, 21⟩ | 1428 | ⟨19, 4, 19⟩ |
| 1435 | ⟨1, 1, 359⟩ | 1435 | ⟨5, 5, 73⟩ | 1435 | ⟨7, 7, 53⟩ | 1435 | ⟨19, 3, 19⟩ |
| 1440 | ⟨7, ±4, 52⟩ | 1440 | ⟨9, ±6, 41⟩ | 1440 | ⟨11, ±10, 35⟩ | 1440 | ⟨13, ±4, 28⟩ |
| 1443 | ⟨11, ±3, 33⟩ | 1443 | ⟨17, ±11, 23⟩ | 1467 | ⟨9, ±3, 41⟩ | 1488 | ⟨8, ±4, 47⟩ |
| 1488 | ⟨17, ±12, 24⟩ | 1507 | ⟨13, ±1, 29⟩ | 1540 | ⟨1, 0, 385⟩ | 1540 | ⟨2, 2, 193⟩ |
| 1540 | ⟨5, 0, 77⟩ | 1540 | ⟨7, 0, 55⟩ | 1540 | ⟨10, 10, 41⟩ | 1540 | ⟨11, 0, 35⟩ |
| 1540 | ⟨14, 14, 31⟩ | 1540 | ⟨22, 22, 23⟩ | 1555 | ⟨17, ±3, 23⟩ | 1560 | ⟨7, ±6, 57⟩ |
| 1560 | ⟨14, ±8, 29⟩ | 1560 | ⟨17, ±2, 23⟩ | 1560 | ⟨19, ±6, 21⟩ | 1600 | ⟨13, ±8, 32⟩ |
| 1600 | ⟨17, ±10, 25⟩ | 1632 | ⟨1, 0, 408⟩ | 1632 | ⟨3, 0, 136⟩ | 1632 | ⟨4, 4, 103⟩ |
| 1632 | ⟨8, 0, 51⟩ | 1632 | ⟨8, 8, 53⟩ | 1632 | ⟨12, 12, 37⟩ | 1632 | ⟨17, 0, 24⟩ |
| 1632 | ⟨23, 22, 23⟩ | 1635 | ⟨11, ±9, 39⟩ | 1635 | ⟨13, ±9, 33⟩ | 1659 | ⟨5, ±1, 83⟩ |
| 1659 | ⟨15, ±9, 29⟩ | 1672 | ⟨7, ±6, 61⟩ | 1672 | ⟨14, ±8, 31⟩ | 1680 | ⟨8, ±4, 53⟩ |
| 1680 | ⟨11, ±6, 39⟩ | 1680 | ⟨13, ±6, 33⟩ | 1680 | ⟨19, ±12, 24⟩ | 1683 | ⟨7, ±5, 61⟩ |
| 1683 | ⟨9, ±3, 47⟩ | 1716 | ⟨5, ±2, 86⟩ | 1716 | ⟨10, ±2, 43⟩ | 1716 | ⟨15, ±12, 31⟩ |
| 1716 | ⟨17, ±16, 29⟩ | 1752 | ⟨13, ±4, 34⟩ | 1752 | ⟨17, ±4, 26⟩ | 1768 | ⟨11, ±6, 41⟩ |
| 1768 | ⟨22, ±16, 23⟩ | 1771 | ⟨5, ±3, 89⟩ | 1771 | ⟨13, ±7, 35⟩ | 1780 | ⟨13, ±12, 37⟩ |
| 1780 | ⟨19, ±14, 26⟩ | 1792 | ⟨11, ±10, 43⟩ | 1792 | ⟨16, ±8, 29⟩ | 1824 | ⟨5, ±4, 92⟩ |
| 1824 | ⟨13, ±10, 37⟩ | 1824 | ⟨15, ±6, 31⟩ | 1824 | ⟨20, ±4, 23⟩ | 1827 | ⟨17, ±3, 27⟩ |
| 1827 | ⟨19, ±15, 27⟩ | 1848 | ⟨1, 0, 462⟩ | 1848 | ⟨2, 0, 231⟩ | 1848 | ⟨3, 0, 154⟩ |
| 1848 | ⟨6, 0, 77⟩ | 1848 | ⟨7, 0, 66⟩ | 1848 | ⟨11, 0, 42⟩ | 1848 | ⟨14, 0, 33⟩ |
| 1848 | ⟨21, 0, 22⟩ | 1860 | ⟨7, ±4, 67⟩ | 1860 | ⟨13, ±8, 37⟩ | 1860 | ⟨14, ±10, 35⟩ |
| 1860 | ⟨21, ±18, 26⟩ | 1920 | ⟨11, ±4, 44⟩ | 1920 | ⟨13, ±2, 37⟩ | 1920 | ⟨16, ±8, 31⟩ |
| 1920 | ⟨17, ±16, 32⟩ | 1947 | ⟨13, ±9, 39⟩ | 1947 | ⟨17, ±5, 29⟩ | 1992 | ⟨13, ±6, 39⟩ |
| 1992 | ⟨23, ±20, 26⟩ | 1995 | ⟨1, 1, 499⟩ | 1995 | ⟨3, 3, 167⟩ | 1995 | ⟨5, 5, 101⟩ |
| 1995 | ⟨7, 7, 73⟩ | 1995 | ⟨15, 15, 37⟩ | 1995 | ⟨19, 19, 31⟩ | 1995 | ⟨21, 21, 29⟩ |
| 1995 | ⟨23, 11, 23⟩ | 2016 | ⟨5, ±2, 101⟩ | 2016 | ⟨13, ±8, 40⟩ | 2016 | ⟨19, ±6, 27⟩ |
| 2016 | ⟨20, ±12, 27⟩ | 2020 | ⟨11, ±2, 46⟩ | 2020 | ⟨22, ±2, 23⟩ | 2035 | ⟨7, ±3, 73⟩ |
| 2035 | ⟨19, ±13, 29⟩ | 2040 | ⟨7, ±2, 73⟩ | 2040 | ⟨13, ±12, 42⟩ | 2040 | ⟨14, ±12, 39⟩ |
| 2040 | ⟨21, ±12, 26⟩ | 2067 | ⟨11, ±1, 47⟩ | 2067 | ⟨19, ±17, 31⟩ | 2080 | ⟨1, 0, 520⟩ |
| 2080 | ⟨4, 4, 131⟩ | 2080 | ⟨5, 0, 104⟩ | 2080 | ⟨8, 0, 65⟩ | 2080 | ⟨8, 8, 67⟩ |

Table 1: Representatives for 2779 $\mathrm{SL}_2(\mathbb{Z})$-equivalence Classes of Regular Forms

| $\|\Delta\|$ | $\langle A, B, C\rangle$ | $\|\Delta\|$ | $\langle A, B, C\rangle$ | $\|\Delta\|$ | $\langle A, B, C\rangle$ | $\|\Delta\|$ | $\langle A, B, C\rangle$ |
|---|---|---|---|---|---|---|---|
| 2080 | ⟨13, 0, 40⟩ | 2080 | ⟨20, 20, 31⟩ | 2080 | ⟨23, 6, 23⟩ | 2088 | ⟨9, ±6, 59⟩ |
| 2088 | ⟨18, ±12, 31⟩ | 2100 | ⟨11, ±10, 50⟩ | 2100 | ⟨17, ±12, 33⟩ | 2100 | ⟨19, ±16, 31⟩ |
| 2100 | ⟨22, ±10, 25⟩ | 2112 | ⟨7, ±4, 76⟩ | 2112 | ⟨17, ±8, 32⟩ | 2112 | ⟨19, ±4, 28⟩ |
| 2112 | ⟨21, ±18, 29⟩ | 2115 | ⟨9, ±3, 59⟩ | 2115 | ⟨13, ±11, 43⟩ | 2128 | ⟨8, ±4, 67⟩ |
| 2128 | ⟨13, ±2, 41⟩ | 2139 | ⟨5, ±1, 107⟩ | 2139 | ⟨15, ±9, 37⟩ | 2163 | ⟨11, ±9, 51⟩ |
| 2163 | ⟨17, ±9, 33⟩ | 2208 | ⟨7, ±2, 79⟩ | 2208 | ⟨11, ±6, 51⟩ | 2208 | ⟨17, ±6, 33⟩ |
| 2208 | ⟨21, ±12, 28⟩ | 2212 | ⟨17, ±10, 34⟩ | 2212 | ⟨19, ±12, 31⟩ | 2244 | ⟨5, ±4, 113⟩ |
| 2244 | ⟨10, ±6, 57⟩ | 2244 | ⟨15, ±6, 38⟩ | 2244 | ⟨19, ±6, 30⟩ | 2272 | ⟨11, ±4, 52⟩ |
| 2272 | ⟨13, ±4, 44⟩ | 2275 | ⟨19, ±9, 31⟩ | 2275 | ⟨23, ±5, 25⟩ | 2280 | ⟨7, ±4, 82⟩ |
| 2280 | ⟨14, ±4, 41⟩ | 2280 | ⟨17, ±10, 35⟩ | 2280 | ⟨21, ±18, 31⟩ | 2340 | ⟨11, ±6, 54⟩ |
| 2340 | ⟨19, ±4, 31⟩ | 2340 | ⟨22, ±6, 27⟩ | 2340 | ⟨23, ±12, 27⟩ | 2368 | ⟨19, ±8, 32⟩ |
| 2368 | ⟨23, ±22, 31⟩ | 2392 | ⟨7, ±4, 86⟩ | 2392 | ⟨14, ±4, 43⟩ | 2400 | ⟨7, ±6, 87⟩ |
| 2400 | ⟨11, ±8, 56⟩ | 2400 | ⟨21, ±6, 29⟩ | 2400 | ⟨25, ±20, 28⟩ | 2436 | ⟨5, ±2, 122⟩ |
| 2436 | ⟨10, ±2, 61⟩ | 2436 | ⟨15, ±12, 43⟩ | 2436 | ⟨23, ±18, 30⟩ | 2451 | ⟨5, ±3, 123⟩ |
| 2451 | ⟨15, ±3, 41⟩ | 2464 | ⟨5, ±4, 124⟩ | 2464 | ⟨17, ±16, 40⟩ | 2464 | ⟨19, ±14, 35⟩ |
| 2464 | ⟨20, ±4, 31⟩ | 2475 | ⟨23, ±3, 27⟩ | 2475 | ⟨25, ±15, 27⟩ | 2496 | ⟨5, ±2, 125⟩ |
| 2496 | ⟨11, ±10, 59⟩ | 2496 | ⟨15, ±12, 44⟩ | 2496 | ⟨20, ±12, 33⟩ | 2520 | ⟨9, ±6, 71⟩ |
| 2520 | ⟨17, ±8, 38⟩ | 2520 | ⟨18, ±12, 37⟩ | 2520 | ⟨19, ±8, 34⟩ | 2580 | ⟨11, ±4, 59⟩ |
| 2580 | ⟨17, ±2, 38⟩ | 2580 | ⟨19, ±2, 34⟩ | 2580 | ⟨22, ±18, 33⟩ | 2632 | ⟨19, ±16, 38⟩ |
| 2632 | ⟨23, ±6, 29⟩ | 2640 | ⟨8, ±4, 83⟩ | 2640 | ⟨13, ±8, 52⟩ | 2640 | ⟨19, ±18, 39⟩ |
| 2640 | ⟨24, ±12, 29⟩ | 2667 | ⟨17, ±11, 41⟩ | 2667 | ⟨23, ±1, 29⟩ | 2688 | ⟨13, ±4, 52⟩ |
| 2688 | ⟨16, ±8, 43⟩ | 2688 | ⟨17, ±10, 41⟩ | 2688 | ⟨23, ±16, 32⟩ | 2715 | ⟨7, ±1, 97⟩ |
| 2715 | ⟨21, ±15, 35⟩ | 2755 | ⟨13, ±1, 53⟩ | 2755 | ⟨17, ±13, 43⟩ | 2760 | ⟨11, ±10, 65⟩ |
| 2760 | ⟨13, ±10, 55⟩ | 2760 | ⟨22, ±12, 33⟩ | 2760 | ⟨26, ±16, 29⟩ | 2772 | ⟨13, ±6, 54⟩ |
| 2772 | ⟨17, ±4, 41⟩ | 2772 | ⟨26, ±6, 27⟩ | 2772 | ⟨27, ±24, 31⟩ | 2788 | ⟨19, ±10, 38⟩ |
| 2788 | ⟨23, ±8, 31⟩ | 2832 | ⟨8, ±4, 89⟩ | 2832 | ⟨24, ±12, 31⟩ | 2880 | ⟨7, ±2, 103⟩ |
| 2880 | ⟨23, ±8, 32⟩ | 2880 | ⟨27, ±24, 32⟩ | 2880 | ⟨27, ±12, 28⟩ | 2907 | ⟨27, ±21, 31⟩ |
| 2907 | ⟨27, ±15, 29⟩ | 2968 | ⟨13, ±10, 59⟩ | 2968 | ⟨26, ±16, 31⟩ | 3003 | ⟨1, 1, 751⟩ |
| 3003 | ⟨3, 3, 251⟩ | 3003 | ⟨7, 7, 109⟩ | 3003 | ⟨11, 11, 71⟩ | 3003 | ⟨13, 13, 61⟩ |
| 3003 | ⟨21, 21, 41⟩ | 3003 | ⟨29, 19, 29⟩ | 3003 | ⟨31, 29, 31⟩ | 3040 | ⟨1, 0, 760⟩ |
| 3040 | ⟨4, 4, 191⟩ | 3040 | ⟨5, 0, 152⟩ | 3040 | ⟨8, 0, 95⟩ | 3040 | ⟨8, 8, 97⟩ |
| 3040 | ⟨19, 0, 40⟩ | 3040 | ⟨20, 20, 43⟩ | 3040 | ⟨29, 18, 29⟩ | 3060 | ⟨9, ±6, 86⟩ |
| 3060 | ⟨11, ±8, 71⟩ | 3060 | ⟨18, ±6, 43⟩ | 3060 | ⟨22, ±14, 37⟩ | 3108 | ⟨11, ±4, 71⟩ |
| 3108 | ⟨13, ±8, 61⟩ | 3108 | ⟨22, ±18, 39⟩ | 3108 | ⟨26, ±18, 33⟩ | 3168 | ⟨9, ±6, 89⟩ |
| 3168 | ⟨13, ±2, 61⟩ | 3168 | ⟨19, ±10, 43⟩ | 3168 | ⟨23, ±12, 36⟩ | 3172 | ⟨19, ±18, 46⟩ |
| 3172 | ⟨23, ±18, 38⟩ | 3192 | ⟨11, ±8, 74⟩ | 3192 | ⟨17, ±2, 47⟩ | 3192 | ⟨22, ±8, 37⟩ |
| 3192 | ⟨31, ±30, 33⟩ | 3220 | ⟨11, ±6, 74⟩ | 3220 | ⟨13, ±2, 62⟩ | 3220 | ⟨22, ±6, 37⟩ |
| 3220 | ⟨26, ±2, 31⟩ | 3243 | ⟨17, ±15, 51⟩ | 3243 | ⟨19, ±5, 43⟩ | 3315 | ⟨1, 1, 829⟩ |
| 3315 | ⟨3, 3, 277⟩ | 3315 | ⟨5, 5, 167⟩ | 3315 | ⟨13, 13, 67⟩ | 3315 | ⟨15, 15, 59⟩ |
| 3315 | ⟨17, 17, 53⟩ | 3315 | ⟨29, 7, 29⟩ | 3315 | ⟨31, 23, 31⟩ | 3355 | ⟨13, ±5, 65⟩ |
| 3355 | ⟨23, ±7, 37⟩ | 3360 | ⟨1, 0, 840⟩ | 3360 | ⟨3, 0, 280⟩ | 3360 | ⟨4, 4, 211⟩ |
| 3360 | ⟨5, 0, 168⟩ | 3360 | ⟨7, 0, 120⟩ | 3360 | ⟨8, 0, 105⟩ | 3360 | ⟨8, 8, 107⟩ |
| 3360 | ⟨12, 12, 73⟩ | 3360 | ⟨15, 0, 56⟩ | 3360 | ⟨20, 20, 47⟩ | 3360 | ⟨21, 0, 40⟩ |
| 3360 | ⟨24, 0, 35⟩ | 3360 | ⟨24, 24, 41⟩ | 3360 | ⟨28, 28, 37⟩ | 3360 | ⟨29, 2, 29⟩ |
| 3360 | ⟨31, 22, 31⟩ | 3432 | ⟨17, ±6, 51⟩ | 3432 | ⟨19, ±8, 46⟩ | 3432 | ⟨23, ±8, 38⟩ |
| 3432 | ⟨31, ±28, 34⟩ | 3480 | ⟨13, ±2, 67⟩ | 3480 | ⟨19, ±4, 46⟩ | 3480 | ⟨23, ±4, 38⟩ |
| 3480 | ⟨26, ±24, 39⟩ | 3507 | ⟨13, ±9, 69⟩ | 3507 | ⟨23, ±9, 39⟩ | 3520 | ⟨7, ±6, 127⟩ |
| 3520 | ⟨13, ±4, 68⟩ | 3520 | ⟨17, ±4, 52⟩ | 3520 | ⟨28, ±20, 35⟩ | 3588 | ⟨11, ±8, 83⟩ |
| 3588 | ⟨17, ±4, 53⟩ | 3588 | ⟨22, ±14, 43⟩ | 3588 | ⟨33, ±30, 34⟩ | 3627 | ⟨9, ±3, 101⟩ |
| 3627 | ⟨11, ±5, 83⟩ | 3640 | ⟨11, ±10, 85⟩ | 3640 | ⟨17, ±10, 55⟩ | 3640 | ⟨22, ±12, 43⟩ |
| 3640 | ⟨31, ±24, 34⟩ | 3648 | ⟨11, ±2, 83⟩ | 3648 | ⟨23, ±20, 44⟩ | 3648 | ⟨29, ±8, 32⟩ |
| 3648 | ⟨32, ±24, 33⟩ | 3712 | ⟨16, ±8, 59⟩ | 3712 | ⟨31, ±16, 32⟩ | 3795 | ⟨13, ±1, 73⟩ |
| 3795 | ⟨17, ±9, 57⟩ | 3795 | ⟨19, ±9, 51⟩ | 3795 | ⟨29, ±27, 39⟩ | 3808 | ⟨11, ±8, 88⟩ |
| 3808 | ⟨13, ±12, 76⟩ | 3808 | ⟨19, ±12, 52⟩ | 3808 | ⟨29, ±22, 37⟩ | 3828 | ⟨7, ±6, 138⟩ |
| 3828 | ⟨14, ±6, 69⟩ | 3828 | ⟨21, ±6, 46⟩ | 3828 | ⟨23, ±6, 42⟩ | 3840 | ⟨16, ±8, 61⟩ |
| 3840 | ⟨17, ±6, 57⟩ | 3840 | ⟨19, ±6, 51⟩ | 3840 | ⟨23, ±22, 47⟩ | 3843 | ⟨9, ±3, 107⟩ |
| 3843 | ⟨17, ±13, 59⟩ | 4020 | ⟨13, ±6, 78⟩ | 4020 | ⟨17, ±14, 62⟩ | 4020 | ⟨26, ±6, 39⟩ |
| 4020 | ⟨31, ±14, 34⟩ | 4032 | ⟨9, ±6, 113⟩ | 4032 | ⟨11, ±4, 92⟩ | 4032 | ⟨23, ±4, 44⟩ |
| 4032 | ⟨29, ±12, 36⟩ | 4048 | ⟨8, ±4, 127⟩ | 4048 | ⟨17, ±10, 61⟩ | 4123 | ⟨17, ±5, 61⟩ |
| 4123 | ⟨29, ±13, 37⟩ | 4128 | ⟨7, ±4, 148⟩ | 4128 | ⟨21, ±18, 53⟩ | 4128 | ⟨23, ±14, 47⟩ |
| 4128 | ⟨28, ±4, 37⟩ | 4180 | ⟨17, ±6, 62⟩ | 4180 | ⟨23, ±12, 47⟩ | 4180 | ⟨29, ±24, 41⟩ |
| 4180 | ⟨31, ±6, 34⟩ | 4260 | ⟨13, ±2, 82⟩ | 4260 | ⟨23, ±8, 47⟩ | 4260 | ⟨26, ±2, 41⟩ |
| 4260 | ⟨31, ±24, 39⟩ | 4323 | ⟨19, ±3, 57⟩ | 4323 | ⟨23, ±1, 47⟩ | 4368 | ⟨8, ±4, 137⟩ |
| 4368 | ⟨17, ±16, 68⟩ | 4368 | ⟨23, ±18, 51⟩ | 4368 | ⟨24, ±12, 47⟩ | 4420 | ⟨7, ±2, 158⟩ |

Table 1: Representatives for 2779 $\mathrm{SL}_2(\mathbb{Z})$-equivalence Classes of Regular Forms

| $\lvert\Delta\rvert$ | $\langle A, B, C\rangle$ | $\lvert\Delta\rvert$ | $\langle A, B, C\rangle$ | $\lvert\Delta\rvert$ | $\langle A, B, C\rangle$ | $\lvert\Delta\rvert$ | $\langle A, B, C\rangle$ |
|---|---|---|---|---|---|---|---|
| 4420 | $\langle 14, \pm 2, 79\rangle$ | 4420 | $\langle 19, \pm 8, 59\rangle$ | 4420 | $\langle 35, \pm 30, 38\rangle$ | 4440 | $\langle 11, \pm 2, 101\rangle$ |
| 4440 | $\langle 19, \pm 14, 61\rangle$ | 4440 | $\langle 22, \pm 20, 55\rangle$ | 4440 | $\langle 33, \pm 24, 38\rangle$ | 4452 | $\langle 11, \pm 6, 102\rangle$ |
| 4452 | $\langle 17, \pm 6, 66\rangle$ | 4452 | $\langle 22, \pm 6, 51\rangle$ | 4452 | $\langle 33, \pm 6, 34\rangle$ | 4480 | $\langle 16, \pm 8, 71\rangle$ |
| 4480 | $\langle 17, \pm 12, 68\rangle$ | 4480 | $\langle 19, \pm 2, 59\rangle$ | 4480 | $\langle 32, \pm 16, 37\rangle$ | 4488 | $\langle 13, \pm 6, 87\rangle$ |
| 4488 | $\langle 26, \pm 20, 47\rangle$ | 4488 | $\langle 29, \pm 6, 39\rangle$ | 4488 | $\langle 31, \pm 10, 37\rangle$ | 4512 | $\langle 11, \pm 8, 104\rangle$ |
| 4512 | $\langle 13, \pm 8, 88\rangle$ | 4512 | $\langle 31, \pm 18, 39\rangle$ | 4512 | $\langle 33, \pm 30, 41\rangle$ | 4515 | $\langle 13, \pm 3, 87\rangle$ |
| 4515 | $\langle 19, \pm 11, 61\rangle$ | 4515 | $\langle 23, \pm 19, 53\rangle$ | 4515 | $\langle 29, \pm 3, 39\rangle$ | 4680 | $\langle 9, \pm 6, 131\rangle$ |
| 4680 | $\langle 18, \pm 12, 67\rangle$ | 4680 | $\langle 23, \pm 14, 53\rangle$ | 4680 | $\langle 31, \pm 30, 45\rangle$ | 4740 | $\langle 11, \pm 10, 110\rangle$ |
| 4740 | $\langle 22, \pm 10, 55\rangle$ | 4740 | $\langle 29, \pm 4, 41\rangle$ | 4740 | $\langle 33, \pm 12, 37\rangle$ | 4788 | $\langle 9, \pm 6, 134\rangle$ |
| 4788 | $\langle 13, \pm 10, 94\rangle$ | 4788 | $\langle 18, \pm 6, 67\rangle$ | 4788 | $\langle 26, \pm 10, 47\rangle$ | 4960 | $\langle 11, \pm 10, 115\rangle$ |
| 4960 | $\langle 17, \pm 2, 73\rangle$ | 4960 | $\langle 23, \pm 10, 55\rangle$ | 4960 | $\langle 29, \pm 12, 44\rangle$ | 4992 | $\langle 16, \pm 8, 79\rangle$ |
| 4992 | $\langle 19, \pm 10, 67\rangle$ | 4992 | $\langle 29, \pm 24, 48\rangle$ | 4992 | $\langle 32, \pm 16, 41\rangle$ | 5083 | $\langle 19, \pm 3, 67\rangle$ |
| 5083 | $\langle 31, \pm 1, 41\rangle$ | 5115 | $\langle 7, \pm 3, 183\rangle$ | 5115 | $\langle 17, \pm 11, 77\rangle$ | 5115 | $\langle 21, \pm 3, 61\rangle$ |
| 5115 | $\langle 35, \pm 25, 41\rangle$ | 5152 | $\langle 13, \pm 10, 101\rangle$ | 5152 | $\langle 17, \pm 4, 76\rangle$ | 5152 | $\langle 19, \pm 4, 68\rangle$ |
| 5152 | $\langle 31, \pm 26, 47\rangle$ | 5160 | $\langle 13, \pm 12, 102\rangle$ | 5160 | $\langle 17, \pm 12, 78\rangle$ | 5160 | $\langle 26, \pm 12, 51\rangle$ |
| 5160 | $\langle 34, \pm 12, 39\rangle$ | 5187 | $\langle 11, \pm 7, 119\rangle$ | 5187 | $\langle 17, \pm 7, 77\rangle$ | 5187 | $\langle 29, \pm 27, 51\rangle$ |
| 5187 | $\langle 33, \pm 15, 41\rangle$ | 5208 | $\langle 19, \pm 6, 69\rangle$ | 5208 | $\langle 23, \pm 6, 57\rangle$ | 5208 | $\langle 37, \pm 34, 43\rangle$ |
| 5208 | $\langle 38, \pm 32, 41\rangle$ | 5280 | $\langle 1, 0, 1320\rangle$ | 5280 | $\langle 3, 0, 440\rangle$ | 5280 | $\langle 4, 4, 331\rangle$ |
| 5280 | $\langle 5, 0, 264\rangle$ | 5280 | $\langle 8, 0, 165\rangle$ | 5280 | $\langle 8, 8, 167\rangle$ | 5280 | $\langle 11, 0, 120\rangle$ |
| 5280 | $\langle 12, 12, 113\rangle$ | 5280 | $\langle 15, 0, 88\rangle$ | 5280 | $\langle 20, 20, 71\rangle$ | 5280 | $\langle 24, 0, 55\rangle$ |
| 5280 | $\langle 24, 24, 61\rangle$ | 5280 | $\langle 33, 0, 40\rangle$ | 5280 | $\langle 37, 14, 37\rangle$ | 5280 | $\langle 40, 40, 43\rangle$ |
| 5280 | $\langle 41, 38, 41\rangle$ | 5355 | $\langle 9, \pm 3, 149\rangle$ | 5355 | $\langle 13, \pm 1, 103\rangle$ | 5355 | $\langle 23, \pm 21, 63\rangle$ |
| 5355 | $\langle 31, \pm 15, 45\rangle$ | 5412 | $\langle 13, \pm 10, 106\rangle$ | 5412 | $\langle 23, \pm 4, 59\rangle$ | 5412 | $\langle 26, \pm 10, 53\rangle$ |
| 5412 | $\langle 39, \pm 36, 43\rangle$ | 5440 | $\langle 11, \pm 4, 124\rangle$ | 5440 | $\langle 31, \pm 4, 44\rangle$ | 5440 | $\langle 32, \pm 24, 47\rangle$ |
| 5440 | $\langle 32, \pm 8, 43\rangle$ | 5460 | $\langle 1, 0, 1365\rangle$ | 5460 | $\langle 2, 2, 683\rangle$ | 5460 | $\langle 3, 0, 455\rangle$ |
| 5460 | $\langle 5, 0, 273\rangle$ | 5460 | $\langle 6, 6, 229\rangle$ | 5460 | $\langle 7, 0, 195\rangle$ | 5460 | $\langle 10, 10, 139\rangle$ |
| 5460 | $\langle 13, 0, 105\rangle$ | 5460 | $\langle 14, 14, 101\rangle$ | 5460 | $\langle 15, 0, 91\rangle$ | 5460 | $\langle 21, 0, 65\rangle$ |
| 5460 | $\langle 26, 26, 59\rangle$ | 5460 | $\langle 30, 30, 53\rangle$ | 5460 | $\langle 35, 0, 39\rangle$ | 5460 | $\langle 37, 4, 37\rangle$ |
| 5460 | $\langle 42, 42, 43\rangle$ | 5467 | $\langle 19, \pm 9, 73\rangle$ | 5467 | $\langle 31, \pm 19, 47\rangle$ | 5520 | $\langle 8, \pm 4, 173\rangle$ |
| 5520 | $\langle 19, \pm 16, 76\rangle$ | 5520 | $\langle 24, \pm 12, 59\rangle$ | 5520 | $\langle 37, \pm 20, 40\rangle$ | 5712 | $\langle 8, \pm 4, 179\rangle$ |
| 5712 | $\langle 19, \pm 8, 76\rangle$ | 5712 | $\langle 24, \pm 12, 61\rangle$ | 5712 | $\langle 29, \pm 28, 56\rangle$ | 5952 | $\langle 17, \pm 10, 89\rangle$ |
| 5952 | $\langle 29, \pm 14, 53\rangle$ | 5952 | $\langle 32, \pm 24, 51\rangle$ | 5952 | $\langle 32, \pm 8, 47\rangle$ | 6160 | $\langle 8, \pm 4, 193\rangle$ |
| 6160 | $\langle 23, \pm 2, 67\rangle$ | 6160 | $\langle 31, \pm 28, 56\rangle$ | 6160 | $\langle 40, \pm 20, 41\rangle$ | 6195 | $\langle 11, \pm 3, 141\rangle$ |
| 6195 | $\langle 31, \pm 25, 55\rangle$ | 6195 | $\langle 33, \pm 3, 47\rangle$ | 6195 | $\langle 37, \pm 13, 43\rangle$ | 6240 | $\langle 7, \pm 2, 223\rangle$ |
| 6240 | $\langle 17, \pm 4, 92\rangle$ | 6240 | $\langle 19, \pm 12, 84\rangle$ | 6240 | $\langle 21, \pm 12, 76\rangle$ | 6240 | $\langle 23, \pm 4, 68\rangle$ |
| 6240 | $\langle 28, \pm 12, 57\rangle$ | 6240 | $\langle 29, \pm 16, 56\rangle$ | 6240 | $\langle 35, \pm 30, 51\rangle$ | 6307 | $\langle 19, \pm 1, 83\rangle$ |
| 6307 | $\langle 23, \pm 15, 71\rangle$ | 6420 | $\langle 11, \pm 2, 146\rangle$ | 6420 | $\langle 22, \pm 2, 73\rangle$ | 6420 | $\langle 31, \pm 20, 55\rangle$ |
| 6420 | $\langle 33, \pm 24, 53\rangle$ | 6435 | $\langle 9, \pm 3, 179\rangle$ | 6435 | $\langle 17, \pm 5, 95\rangle$ | 6435 | $\langle 19, \pm 5, 85\rangle$ |
| 6435 | $\langle 37, \pm 15, 45\rangle$ | 6528 | $\langle 16, \pm 8, 103\rangle$ | 6528 | $\langle 23, \pm 2, 71\rangle$ | 6528 | $\langle 32, \pm 16, 53\rangle$ |
| 6528 | $\langle 37, \pm 24, 48\rangle$ | 6580 | $\langle 11, \pm 8, 151\rangle$ | 6580 | $\langle 17, \pm 4, 97\rangle$ | 6580 | $\langle 22, \pm 14, 77\rangle$ |
| 6580 | $\langle 34, \pm 30, 55\rangle$ | 6612 | $\langle 17, \pm 16, 101\rangle$ | 6612 | $\langle 23, \pm 14, 74\rangle$ | 6612 | $\langle 34, \pm 18, 51\rangle$ |
| 6612 | $\langle 37, \pm 14, 46\rangle$ | 6688 | $\langle 7, \pm 2, 239\rangle$ | 6688 | $\langle 28, \pm 12, 61\rangle$ | 6688 | $\langle 31, \pm 16, 56\rangle$ |
| 6688 | $\langle 37, \pm 34, 53\rangle$ | 6708 | $\langle 23, \pm 10, 74\rangle$ | 6708 | $\langle 29, \pm 22, 62\rangle$ | 6708 | $\langle 31, \pm 22, 58\rangle$ |
| 6708 | $\langle 37, \pm 10, 46\rangle$ | 6720 | $\langle 11, \pm 10, 155\rangle$ | 6720 | $\langle 13, \pm 12, 132\rangle$ | 6720 | $\langle 19, \pm 14, 91\rangle$ |
| 6720 | $\langle 31, \pm 10, 55\rangle$ | 6720 | $\langle 32, \pm 24, 57\rangle$ | 6720 | $\langle 32, \pm 8, 53\rangle$ | 6720 | $\langle 33, \pm 12, 52\rangle$ |
| 6720 | $\langle 39, \pm 12, 44\rangle$ | 6820 | $\langle 19, \pm 18, 94\rangle$ | 6820 | $\langle 29, \pm 16, 61\rangle$ | 6820 | $\langle 37, \pm 32, 53\rangle$ |
| 6820 | $\langle 38, \pm 18, 47\rangle$ | 6840 | $\langle 9, \pm 6, 191\rangle$ | 6840 | $\langle 18, \pm 12, 97\rangle$ | 6840 | $\langle 29, \pm 2, 59\rangle$ |
| 6840 | $\langle 43, \pm 30, 45\rangle$ | 7008 | $\langle 13, \pm 8, 136\rangle$ | 7008 | $\langle 17, \pm 8, 104\rangle$ | 7008 | $\langle 39, \pm 18, 47\rangle$ |
| 7008 | $\langle 43, \pm 42, 51\rangle$ | 7035 | $\langle 11, \pm 7, 161\rangle$ | 7035 | $\langle 23, \pm 7, 77\rangle$ | 7035 | $\langle 31, \pm 23, 61\rangle$ |
| 7035 | $\langle 33, \pm 15, 55\rangle$ | 7072 | $\langle 11, \pm 10, 163\rangle$ | 7072 | $\langle 23, \pm 14, 79\rangle$ | 7072 | $\langle 29, \pm 2, 61\rangle$ |
| 7072 | $\langle 41, \pm 12, 44\rangle$ | 7140 | $\langle 13, \pm 6, 138\rangle$ | 7140 | $\langle 19, \pm 2, 94\rangle$ | 7140 | $\langle 23, \pm 6, 78\rangle$ |
| 7140 | $\langle 26, \pm 6, 69\rangle$ | 7140 | $\langle 29, \pm 20, 65\rangle$ | 7140 | $\langle 37, \pm 36, 57\rangle$ | 7140 | $\langle 38, \pm 2, 47\rangle$ |
| 7140 | $\langle 39, \pm 6, 46\rangle$ | 7315 | $\langle 13, \pm 11, 143\rangle$ | 7315 | $\langle 29, \pm 15, 65\rangle$ | 7315 | $\langle 31, \pm 1, 59\rangle$ |
| 7315 | $\langle 37, \pm 23, 53\rangle$ | 7392 | $\langle 1, 0, 1848\rangle$ | 7392 | $\langle 3, 0, 616\rangle$ | 7392 | $\langle 4, 4, 463\rangle$ |
| 7392 | $\langle 7, 0, 264\rangle$ | 7392 | $\langle 8, 0, 231\rangle$ | 7392 | $\langle 8, 8, 233\rangle$ | 7392 | $\langle 11, 0, 168\rangle$ |
| 7392 | $\langle 12, 12, 157\rangle$ | 7392 | $\langle 21, 0, 88\rangle$ | 7392 | $\langle 24, 0, 77\rangle$ | 7392 | $\langle 24, 24, 83\rangle$ |
| 7392 | $\langle 28, 28, 73\rangle$ | 7392 | $\langle 33, 0, 56\rangle$ | 7392 | $\langle 43, 2, 43\rangle$ | 7392 | $\langle 44, 44, 53\rangle$ |
| 7392 | $\langle 47, 38, 47\rangle$ | 7395 | $\langle 7, \pm 5, 265\rangle$ | 7395 | $\langle 21, \pm 9, 89\rangle$ | 7395 | $\langle 31, \pm 13, 61\rangle$ |
| 7395 | $\langle 35, \pm 5, 53\rangle$ | 7480 | $\langle 19, \pm 14, 101\rangle$ | 7480 | $\langle 23, \pm 8, 82\rangle$ | 7480 | $\langle 38, \pm 24, 53\rangle$ |
| 7480 | $\langle 41, \pm 8, 46\rangle$ | 7540 | $\langle 17, \pm 12, 113\rangle$ | 7540 | $\langle 23, \pm 2, 82\rangle$ | 7540 | $\langle 34, \pm 22, 59\rangle$ |
| 7540 | $\langle 41, \pm 2, 46\rangle$ | 7755 | $\langle 7, \pm 1, 277\rangle$ | 7755 | $\langle 19, \pm 15, 105\rangle$ | 7755 | $\langle 21, \pm 15, 95\rangle$ |
| 7755 | $\langle 35, \pm 15, 57\rangle$ | 7968 | $\langle 13, \pm 12, 156\rangle$ | 7968 | $\langle 23, \pm 6, 87\rangle$ | 7968 | $\langle 29, \pm 6, 69\rangle$ |
| 7968 | $\langle 39, \pm 12, 52\rangle$ | 7995 | $\langle 19, \pm 17, 109\rangle$ | 7995 | $\langle 23, \pm 3, 87\rangle$ | 7995 | $\langle 29, \pm 3, 69\rangle$ |
| 7995 | $\langle 37, \pm 21, 57\rangle$ | 8008 | $\langle 17, \pm 4, 118\rangle$ | 8008 | $\langle 29, \pm 24, 74\rangle$ | 8008 | $\langle 34, \pm 4, 59\rangle$ |

Table 1: Representatives for 2779 $\mathrm{SL}_2(\mathbb{Z})$-equivalence Classes of Regular Forms

| $\|\Delta\|$ | $\langle A, B, C\rangle$ | $\|\Delta\|$ | $\langle A, B, C\rangle$ | $\|\Delta\|$ | $\langle A, B, C\rangle$ | $\|\Delta\|$ | $\langle A, B, C\rangle$ |
|---|---|---|---|---|---|---|---|
| 8008 | $\langle 37, \pm 24, 58\rangle$ | 8052 | $\langle 19, \pm 2, 106\rangle$ | 8052 | $\langle 31, \pm 16, 67\rangle$ | 8052 | $\langle 38, \pm 2, 53\rangle$ |
| 8052 | $\langle 41, \pm 36, 57\rangle$ | 8160 | $\langle 7, \pm 4, 292\rangle$ | 8160 | $\langle 13, \pm 2, 157\rangle$ | 8160 | $\langle 21, \pm 18, 101\rangle$ |
| 8160 | $\langle 28, \pm 4, 73\rangle$ | 8160 | $\langle 35, \pm 10, 59\rangle$ | 8160 | $\langle 39, \pm 24, 56\rangle$ | 8160 | $\langle 41, \pm 32, 56\rangle$ |
| 8160 | $\langle 43, \pm 28, 52\rangle$ | 8320 | $\langle 16, \pm 8, 131\rangle$ | 8320 | $\langle 23, \pm 12, 92\rangle$ | 8320 | $\langle 31, \pm 22, 71\rangle$ |
| 8320 | $\langle 32, \pm 16, 67\rangle$ | 8352 | $\langle 9, \pm 6, 233\rangle$ | 8352 | $\langle 31, \pm 24, 72\rangle$ | 8352 | $\langle 36, \pm 12, 59\rangle$ |
| 8352 | $\langle 37, \pm 26, 61\rangle$ | 8512 | $\langle 13, \pm 4, 164\rangle$ | 8512 | $\langle 32, \pm 24, 71\rangle$ | 8512 | $\langle 32, \pm 8, 67\rangle$ |
| 8512 | $\langle 41, \pm 4, 52\rangle$ | 8547 | $\langle 17, \pm 15, 129\rangle$ | 8547 | $\langle 23, \pm 3, 93\rangle$ | 8547 | $\langle 31, \pm 3, 69\rangle$ |
| 8547 | $\langle 43, \pm 15, 51\rangle$ | 8580 | $\langle 7, \pm 4, 307\rangle$ | 8580 | $\langle 14, \pm 10, 155\rangle$ | 8580 | $\langle 21, \pm 18, 106\rangle$ |
| 8580 | $\langle 29, \pm 2, 74\rangle$ | 8580 | $\langle 31, \pm 10, 70\rangle$ | 8580 | $\langle 35, \pm 10, 62\rangle$ | 8580 | $\langle 37, \pm 2, 58\rangle$ |
| 8580 | $\langle 42, \pm 18, 53\rangle$ | 8680 | $\langle 13, \pm 2, 167\rangle$ | 8680 | $\langle 26, \pm 24, 89\rangle$ | 8680 | $\langle 29, \pm 22, 79\rangle$ |
| 8680 | $\langle 43, \pm 36, 58\rangle$ | 8715 | $\langle 19, \pm 5, 115\rangle$ | 8715 | $\langle 23, \pm 5, 95\rangle$ | 8715 | $\langle 41, \pm 31, 59\rangle$ |
| 8715 | $\langle 43, \pm 33, 57\rangle$ | 8835 | $\langle 11, \pm 3, 201\rangle$ | 8835 | $\langle 33, \pm 3, 67\rangle$ | 8835 | $\langle 41, \pm 29, 59\rangle$ |
| 8835 | $\langle 43, \pm 25, 55\rangle$ | 8932 | $\langle 13, \pm 8, 173\rangle$ | 8932 | $\langle 19, \pm 6, 118\rangle$ | 8932 | $\langle 26, \pm 18, 89\rangle$ |
| 8932 | $\langle 38, \pm 6, 59\rangle$ | 9108 | $\langle 9, \pm 6, 254\rangle$ | 9108 | $\langle 17, \pm 2, 134\rangle$ | 9108 | $\langle 18, \pm 6, 127\rangle$ |
| 9108 | $\langle 34, \pm 2, 67\rangle$ | 9120 | $\langle 7, \pm 6, 327\rangle$ | 9120 | $\langle 17, \pm 14, 137\rangle$ | 9120 | $\langle 21, \pm 6, 109\rangle$ |
| 9120 | $\langle 28, \pm 20, 85\rangle$ | 9120 | $\langle 31, \pm 26, 79\rangle$ | 9120 | $\langle 35, \pm 20, 68\rangle$ | 9120 | $\langle 41, \pm 8, 56\rangle$ |
| 9120 | $\langle 51, \pm 48, 56\rangle$ | 9240 | $\langle 13, \pm 4, 178\rangle$ | 9240 | $\langle 17, \pm 12, 138\rangle$ | 9240 | $\langle 23, \pm 12, 102\rangle$ |
| 9240 | $\langle 26, \pm 4, 89\rangle$ | 9240 | $\langle 34, \pm 12, 69\rangle$ | 9240 | $\langle 37, \pm 26, 67\rangle$ | 9240 | $\langle 39, \pm 30, 65\rangle$ |
| 9240 | $\langle 46, \pm 12, 51\rangle$ | 9568 | $\langle 7, \pm 6, 343\rangle$ | 9568 | $\langle 28, \pm 20, 89\rangle$ | 9568 | $\langle 43, \pm 8, 56\rangle$ |
| 9568 | $\langle 53, \pm 48, 56\rangle$ | 9867 | $\langle 29, \pm 15, 87\rangle$ | 9867 | $\langle 37, \pm 7, 67\rangle$ | 9867 | $\langle 43, \pm 25, 61\rangle$ |
| 9867 | $\langle 47, \pm 35, 59\rangle$ | 10080 | $\langle 9, \pm 6, 281\rangle$ | 10080 | $\langle 17, \pm 16, 152\rangle$ | 10080 | $\langle 19, \pm 16, 136\rangle$ |
| 10080 | $\langle 36, \pm 12, 71\rangle$ | 10080 | $\langle 37, \pm 24, 72\rangle$ | 10080 | $\langle 43, \pm 38, 67\rangle$ | 10080 | $\langle 45, \pm 30, 61\rangle$ |
| 10080 | $\langle 47, \pm 42, 63\rangle$ | 10528 | $\langle 19, \pm 6, 139\rangle$ | 10528 | $\langle 23, \pm 12, 116\rangle$ | 10528 | $\langle 29, \pm 12, 92\rangle$ |
| 10528 | $\langle 41, \pm 38, 73\rangle$ | 10560 | $\langle 13, \pm 10, 205\rangle$ | 10560 | $\langle 19, \pm 2, 139\rangle$ | 10560 | $\langle 29, \pm 24, 96\rangle$ |
| 10560 | $\langle 32, \pm 24, 87\rangle$ | 10560 | $\langle 32, \pm 8, 83\rangle$ | 10560 | $\langle 39, \pm 36, 76\rangle$ | 10560 | $\langle 41, \pm 10, 65\rangle$ |
| 10560 | $\langle 52, \pm 36, 57\rangle$ | 10920 | $\langle 11, \pm 6, 249\rangle$ | 10920 | $\langle 19, \pm 10, 145\rangle$ | 10920 | $\langle 22, \pm 16, 127\rangle$ |
| 10920 | $\langle 29, \pm 10, 95\rangle$ | 10920 | $\langle 33, \pm 6, 83\rangle$ | 10920 | $\langle 38, \pm 28, 77\rangle$ | 10920 | $\langle 55, \pm 50, 61\rangle$ |
| 10920 | $\langle 57, \pm 48, 58\rangle$ | 10948 | $\langle 37, \pm 2, 74\rangle$ | 10948 | $\langle 41, \pm 32, 73\rangle$ | 10948 | $\langle 43, \pm 24, 67\rangle$ |
| 10948 | $\langle 47, \pm 12, 59\rangle$ | 11040 | $\langle 11, \pm 2, 251\rangle$ | 11040 | $\langle 13, \pm 6, 213\rangle$ | 11040 | $\langle 29, \pm 26, 101\rangle$ |
| 11040 | $\langle 33, \pm 24, 88\rangle$ | 11040 | $\langle 39, \pm 6, 71\rangle$ | 11040 | $\langle 43, \pm 22, 67\rangle$ | 11040 | $\langle 44, \pm 20, 65\rangle$ |
| 11040 | $\langle 52, \pm 20, 55\rangle$ | 11067 | $\langle 13, \pm 3, 213\rangle$ | 11067 | $\langle 37, \pm 25, 79\rangle$ | 11067 | $\langle 39, \pm 3, 71\rangle$ |
| 11067 | $\langle 47, \pm 5, 59\rangle$ | 11328 | $\langle 31, \pm 24, 96\rangle$ | 11328 | $\langle 32, \pm 24, 93\rangle$ | 11328 | $\langle 32, \pm 8, 89\rangle$ |
| 11328 | $\langle 43, \pm 14, 67\rangle$ | 11715 | $\langle 17, \pm 7, 173\rangle$ | 11715 | $\langle 29, \pm 1, 101\rangle$ | 11715 | $\langle 43, \pm 29, 73\rangle$ |
| 11715 | $\langle 51, \pm 27, 61\rangle$ | 11872 | $\langle 13, \pm 6, 229\rangle$ | 11872 | $\langle 31, \pm 30, 103\rangle$ | 11872 | $\langle 41, \pm 10, 73\rangle$ |
| 11872 | $\langle 52, \pm 20, 59\rangle$ | 12160 | $\langle 16, \pm 8, 191\rangle$ | 12160 | $\langle 29, \pm 22, 109\rangle$ | 12160 | $\langle 32, \pm 16, 97\rangle$ |
| 12160 | $\langle 43, \pm 40, 80\rangle$ | 12180 | $\langle 13, \pm 12, 237\rangle$ | 12180 | $\langle 17, \pm 14, 182\rangle$ | 12180 | $\langle 26, \pm 14, 119\rangle$ |
| 12180 | $\langle 34, \pm 14, 91\rangle$ | 12180 | $\langle 37, \pm 20, 85\rangle$ | 12180 | $\langle 39, \pm 12, 79\rangle$ | 12180 | $\langle 51, \pm 48, 71\rangle$ |
| 12180 | $\langle 53, \pm 40, 65\rangle$ | 12768 | $\langle 11, \pm 6, 291\rangle$ | 12768 | $\langle 17, \pm 4, 188\rangle$ | 12768 | $\langle 31, \pm 2, 103\rangle$ |
| 12768 | $\langle 33, \pm 6, 97\rangle$ | 12768 | $\langle 37, \pm 16, 88\rangle$ | 12768 | $\langle 44, \pm 28, 77\rangle$ | 12768 | $\langle 47, \pm 4, 68\rangle$ |
| 12768 | $\langle 51, \pm 30, 67\rangle$ | 12915 | $\langle 9, \pm 3, 359\rangle$ | 12915 | $\langle 19, \pm 9, 171\rangle$ | 12915 | $\langle 45, \pm 15, 73\rangle$ |
| 12915 | $\langle 53, \pm 21, 63\rangle$ | 13195 | $\langle 11, \pm 7, 301\rangle$ | 13195 | $\langle 43, \pm 7, 77\rangle$ | 13195 | $\langle 47, \pm 23, 73\rangle$ |
| 13195 | $\langle 55, \pm 15, 61\rangle$ | 13440 | $\langle 16, \pm 8, 211\rangle$ | 13440 | $\langle 29, \pm 4, 116\rangle$ | 13440 | $\langle 31, \pm 18, 111\rangle$ |
| 13440 | $\langle 32, \pm 16, 107\rangle$ | 13440 | $\langle 37, \pm 18, 93\rangle$ | 13440 | $\langle 41, \pm 34, 89\rangle$ | 13440 | $\langle 47, \pm 40, 80\rangle$ |
| 13440 | $\langle 48, \pm 24, 73\rangle$ | 13728 | $\langle 17, \pm 12, 204\rangle$ | 13728 | $\langle 19, \pm 16, 184\rangle$ | 13728 | $\langle 23, \pm 16, 152\rangle$ |
| 13728 | $\langle 31, \pm 6, 111\rangle$ | 13728 | $\langle 37, \pm 6, 93\rangle$ | 13728 | $\langle 51, \pm 12, 68\rangle$ | 13728 | $\langle 53, \pm 30, 69\rangle$ |
| 13728 | $\langle 57, \pm 54, 73\rangle$ | 13860 | $\langle 9, \pm 6, 386\rangle$ | 13860 | $\langle 18, \pm 6, 193\rangle$ | 13860 | $\langle 23, \pm 20, 155\rangle$ |
| 13860 | $\langle 31, \pm 20, 115\rangle$ | 13860 | $\langle 41, \pm 30, 90\rangle$ | 13860 | $\langle 45, \pm 30, 82\rangle$ | 13860 | $\langle 46, \pm 26, 79\rangle$ |
| 13860 | $\langle 62, \pm 42, 63\rangle$ | 13920 | $\langle 13, \pm 4, 268\rangle$ | 13920 | $\langle 19, \pm 8, 184\rangle$ | 13920 | $\langle 23, \pm 8, 152\rangle$ |
| 13920 | $\langle 39, \pm 30, 95\rangle$ | 13920 | $\langle 41, \pm 26, 89\rangle$ | 13920 | $\langle 52, \pm 4, 67\rangle$ | 13920 | $\langle 57, \pm 30, 65\rangle$ |
| 13920 | $\langle 61, \pm 54, 69\rangle$ | 14280 | $\langle 11, \pm 8, 326\rangle$ | 14280 | $\langle 22, \pm 8, 163\rangle$ | 14280 | $\langle 23, \pm 16, 158\rangle$ |
| 14280 | $\langle 33, \pm 30, 115\rangle$ | 14280 | $\langle 46, \pm 16, 79\rangle$ | 14280 | $\langle 47, \pm 14, 77\rangle$ | 14280 | $\langle 55, \pm 30, 69\rangle$ |
| 14280 | $\langle 59, \pm 36, 66\rangle$ | 14560 | $\langle 11, \pm 2, 331\rangle$ | 14560 | $\langle 17, \pm 14, 217\rangle$ | 14560 | $\langle 31, \pm 14, 119\rangle$ |
| 14560 | $\langle 41, \pm 6, 89\rangle$ | 14560 | $\langle 43, \pm 24, 88\rangle$ | 14560 | $\langle 44, \pm 20, 85\rangle$ | 14560 | $\langle 53, \pm 42, 77\rangle$ |
| 14560 | $\langle 55, \pm 20, 68\rangle$ | 14763 | $\langle 23, \pm 7, 161\rangle$ | 14763 | $\langle 47, \pm 29, 83\rangle$ | 14763 | $\langle 53, \pm 17, 71\rangle$ |
| 14763 | $\langle 59, \pm 39, 69\rangle$ | 14820 | $\langle 17, \pm 2, 218\rangle$ | 14820 | $\langle 29, \pm 12, 129\rangle$ | 14820 | $\langle 34, \pm 2, 109\rangle$ |
| 14820 | $\langle 43, \pm 12, 87\rangle$ | 14820 | $\langle 47, \pm 28, 83\rangle$ | 14820 | $\langle 51, \pm 36, 79\rangle$ | 14820 | $\langle 58, \pm 46, 73\rangle$ |
| 14820 | $\langle 59, \pm 44, 71\rangle$ | 16192 | $\langle 17, \pm 14, 241\rangle$ | 16192 | $\langle 32, \pm 24, 131\rangle$ | 16192 | $\langle 32, \pm 8, 127\rangle$ |
| 16192 | $\langle 61, \pm 20, 68\rangle$ | 16555 | $\langle 29, \pm 27, 149\rangle$ | 16555 | $\langle 37, \pm 13, 113\rangle$ | 16555 | $\langle 41, \pm 3, 101\rangle$ |
| 16555 | $\langle 47, \pm 41, 97\rangle$ | 17220 | $\langle 17, \pm 16, 257\rangle$ | 17220 | $\langle 29, \pm 8, 149\rangle$ | 17220 | $\langle 31, \pm 4, 139\rangle$ |
| 17220 | $\langle 34, \pm 18, 129\rangle$ | 17220 | $\langle 43, \pm 18, 102\rangle$ | 17220 | $\langle 51, \pm 18, 86\rangle$ | 17220 | $\langle 58, \pm 50, 85\rangle$ |
| 17220 | $\langle 62, \pm 58, 83\rangle$ | 17472 | $\langle 17, \pm 2, 257\rangle$ | 17472 | $\langle 23, \pm 10, 191\rangle$ | 17472 | $\langle 32, \pm 24, 141\rangle$ |
| 17472 | $\langle 32, \pm 8, 137\rangle$ | 17472 | $\langle 47, \pm 24, 96\rangle$ | 17472 | $\langle 51, \pm 36, 92\rangle$ | 17472 | $\langle 59, \pm 46, 83\rangle$ |
| 17472 | $\langle 68, \pm 36, 69\rangle$ | 17760 | $\langle 11, \pm 4, 404\rangle$ | 17760 | $\langle 19, \pm 10, 235\rangle$ | 17760 | $\langle 33, \pm 18, 137\rangle$ |
| 17760 | $\langle 44, \pm 4, 101\rangle$ | 17760 | $\langle 47, \pm 10, 95\rangle$ | 17760 | $\langle 55, \pm 40, 88\rangle$ | 17760 | $\langle 57, \pm 48, 88\rangle$ |

Table 1: Representatives for 2779 $SL_2(\mathbb{Z})$-equivalence Classes of Regular Forms

| $\|\Delta\|$ | $\langle A, B, C\rangle$ | $\|\Delta\|$ | $\langle A, B, C\rangle$ | $\|\Delta\|$ | $\langle A, B, C\rangle$ | $\|\Delta\|$ | $\langle A, B, C\rangle$ |
|---|---|---|---|---|---|---|---|
| 17760 | $\langle 61, \pm 28, 76\rangle$ | 17952 | $\langle 13, \pm 12, 348\rangle$ | 17952 | $\langle 29, \pm 12, 156\rangle$ | 17952 | $\langle 31, \pm 20, 148\rangle$ |
| 17952 | $\langle 37, \pm 20, 124\rangle$ | 17952 | $\langle 39, \pm 12, 116\rangle$ | 17952 | $\langle 47, \pm 40, 104\rangle$ | 17952 | $\langle 52, \pm 12, 87\rangle$ |
| 17952 | $\langle 53, \pm 42, 93\rangle$ | 18720 | $\langle 9, \pm 6, 521\rangle$ | 18720 | $\langle 23, \pm 18, 207\rangle$ | 18720 | $\langle 31, \pm 2, 151\rangle$ |
| 18720 | $\langle 36, \pm 12, 131\rangle$ | 18720 | $\langle 45, \pm 30, 109\rangle$ | 18720 | $\langle 53, \pm 28, 92\rangle$ | 18720 | $\langle 67, \pm 24, 72\rangle$ |
| 18720 | $\langle 72, \pm 48, 73\rangle$ | 19320 | $\langle 17, \pm 14, 287\rangle$ | 19320 | $\langle 29, \pm 20, 170\rangle$ | 19320 | $\langle 34, \pm 20, 145\rangle$ |
| 19320 | $\langle 41, \pm 14, 119\rangle$ | 19320 | $\langle 51, \pm 48, 106\rangle$ | 19320 | $\langle 53, \pm 48, 102\rangle$ | 19320 | $\langle 58, \pm 20, 85\rangle$ |
| 19320 | $\langle 73, \pm 68, 82\rangle$ | 19380 | $\langle 13, \pm 4, 373\rangle$ | 19380 | $\langle 23, \pm 20, 215\rangle$ | 19380 | $\langle 26, \pm 22, 191\rangle$ |
| 19380 | $\langle 39, \pm 30, 130\rangle$ | 19380 | $\langle 43, \pm 20, 115\rangle$ | 19380 | $\langle 46, \pm 26, 109\rangle$ | 19380 | $\langle 65, \pm 30, 78\rangle$ |
| 19380 | $\langle 69, \pm 66, 86\rangle$ | 19635 | $\langle 19, \pm 7, 259\rangle$ | 19635 | $\langle 31, \pm 9, 159\rangle$ | 19635 | $\langle 37, \pm 7, 133\rangle$ |
| 19635 | $\langle 41, \pm 39, 129\rangle$ | 19635 | $\langle 43, \pm 39, 123\rangle$ | 19635 | $\langle 53, \pm 9, 93\rangle$ | 19635 | $\langle 57, \pm 45, 95\rangle$ |
| 19635 | $\langle 59, \pm 37, 89\rangle$ | 20020 | $\langle 19, \pm 14, 266\rangle$ | 20020 | $\langle 23, \pm 6, 218\rangle$ | 20020 | $\langle 37, \pm 16, 137\rangle$ |
| 20020 | $\langle 38, \pm 14, 133\rangle$ | 20020 | $\langle 46, \pm 6, 109\rangle$ | 20020 | $\langle 47, \pm 40, 115\rangle$ | 20020 | $\langle 61, \pm 54, 94\rangle$ |
| 20020 | $\langle 74, \pm 58, 79\rangle$ | 20640 | $\langle 13, \pm 2, 397\rangle$ | 20640 | $\langle 17, \pm 10, 305\rangle$ | 20640 | $\langle 39, \pm 24, 136\rangle$ |
| 20640 | $\langle 51, \pm 24, 104\rangle$ | 20640 | $\langle 52, \pm 28, 103\rangle$ | 20640 | $\langle 61, \pm 10, 85\rangle$ | 20640 | $\langle 65, \pm 50, 89\rangle$ |
| 20640 | $\langle 68, \pm 44, 83\rangle$ | 20832 | $\langle 19, \pm 12, 276\rangle$ | 20832 | $\langle 23, \pm 12, 228\rangle$ | 20832 | $\langle 37, \pm 6, 141\rangle$ |
| 20832 | $\langle 41, \pm 18, 129\rangle$ | 20832 | $\langle 43, \pm 18, 123\rangle$ | 20832 | $\langle 47, \pm 6, 111\rangle$ | 20832 | $\langle 57, \pm 12, 92\rangle$ |
| 20832 | $\langle 69, \pm 12, 76\rangle$ | 21120 | $\langle 16, \pm 8, 331\rangle$ | 21120 | $\langle 32, \pm 16, 167\rangle$ | 21120 | $\langle 37, \pm 28, 148\rangle$ |
| 21120 | $\langle 41, \pm 6, 129\rangle$ | 21120 | $\langle 43, \pm 6, 123\rangle$ | 21120 | $\langle 48, \pm 24, 113\rangle$ | 21120 | $\langle 61, \pm 48, 96\rangle$ |
| 21120 | $\langle 71, \pm 40, 80\rangle$ | 21840 | $\langle 8, \pm 4, 683\rangle$ | 21840 | $\langle 24, \pm 12, 229\rangle$ | 21840 | $\langle 37, \pm 8, 148\rangle$ |
| 21840 | $\langle 40, \pm 20, 139\rangle$ | 21840 | $\langle 43, \pm 2, 127\rangle$ | 21840 | $\langle 53, \pm 46, 113\rangle$ | 21840 | $\langle 56, \pm 28, 101\rangle$ |
| 21840 | $\langle 59, \pm 52, 104\rangle$ | 22080 | $\langle 19, \pm 6, 291\rangle$ | 22080 | $\langle 32, \pm 24, 177\rangle$ | 22080 | $\langle 32, \pm 8, 173\rangle$ |
| 22080 | $\langle 37, \pm 34, 157\rangle$ | 22080 | $\langle 57, \pm 6, 97\rangle$ | 22080 | $\langle 59, \pm 24, 96\rangle$ | 22080 | $\langle 71, \pm 70, 95\rangle$ |
| 22080 | $\langle 76, \pm 44, 79\rangle$ | 22848 | $\langle 19, \pm 16, 304\rangle$ | 22848 | $\langle 29, \pm 2, 197\rangle$ | 22848 | $\langle 32, \pm 24, 183\rangle$ |
| 22848 | $\langle 32, \pm 8, 179\rangle$ | 22848 | $\langle 57, \pm 54, 113\rangle$ | 22848 | $\langle 61, \pm 24, 96\rangle$ | 22848 | $\langle 73, \pm 72, 96\rangle$ |
| 22848 | $\langle 76, \pm 60, 87\rangle$ | 24640 | $\langle 23, \pm 4, 268\rangle$ | 24640 | $\langle 31, \pm 6, 199\rangle$ | 24640 | $\langle 32, \pm 24, 197\rangle$ |
| 24640 | $\langle 32, \pm 8, 193\rangle$ | 24640 | $\langle 41, \pm 40, 160\rangle$ | 24640 | $\langle 59, \pm 50, 115\rangle$ | 24640 | $\langle 61, \pm 2, 101\rangle$ |
| 24640 | $\langle 67, \pm 4, 92\rangle$ | 27360 | $\langle 9, \pm 6, 761\rangle$ | 27360 | $\langle 29, \pm 4, 236\rangle$ | 27360 | $\langle 36, \pm 12, 191\rangle$ |
| 27360 | $\langle 43, \pm 26, 163\rangle$ | 27360 | $\langle 45, \pm 30, 157\rangle$ | 27360 | $\langle 59, \pm 4, 116\rangle$ | 27360 | $\langle 72, \pm 48, 103\rangle$ |
| 27360 | $\langle 72, \pm 24, 97\rangle$ | 29568 | $\langle 16, \pm 8, 463\rangle$ | 29568 | $\langle 32, \pm 16, 233\rangle$ | 29568 | $\langle 43, \pm 4, 172\rangle$ |
| 29568 | $\langle 47, \pm 18, 159\rangle$ | 29568 | $\langle 48, \pm 24, 157\rangle$ | 29568 | $\langle 53, \pm 18, 141\rangle$ | 29568 | $\langle 73, \pm 56, 112\rangle$ |
| 29568 | $\langle 83, \pm 48, 96\rangle$ | 29920 | $\langle 19, \pm 10, 395\rangle$ | 29920 | $\langle 23, \pm 16, 328\rangle$ | 29920 | $\langle 41, \pm 16, 184\rangle$ |
| 29920 | $\langle 53, \pm 48, 152\rangle$ | 29920 | $\langle 67, \pm 30, 115\rangle$ | 29920 | $\langle 76, \pm 28, 101\rangle$ | 29920 | $\langle 79, \pm 10, 95\rangle$ |
| 29920 | $\langle 92, \pm 76, 97\rangle$ | 31395 | $\langle 17, \pm 15, 465\rangle$ | 31395 | $\langle 31, \pm 15, 255\rangle$ | 31395 | $\langle 43, \pm 9, 183\rangle$ |
| 31395 | $\langle 47, \pm 1, 167\rangle$ | 31395 | $\langle 51, \pm 15, 155\rangle$ | 31395 | $\langle 61, \pm 9, 129\rangle$ | 31395 | $\langle 71, \pm 49, 119\rangle$ |
| 31395 | $\langle 85, \pm 15, 93\rangle$ | 32032 | $\langle 17, \pm 8, 472\rangle$ | 32032 | $\langle 29, \pm 10, 277\rangle$ | 32032 | $\langle 37, \pm 26, 221\rangle$ |
| 32032 | $\langle 59, \pm 8, 136\rangle$ | 32032 | $\langle 68, \pm 60, 131\rangle$ | 32032 | $\langle 71, \pm 42, 119\rangle$ | 32032 | $\langle 79, \pm 68, 116\rangle$ |
| 32032 | $\langle 89, \pm 50, 97\rangle$ | 33915 | $\langle 11, \pm 3, 771\rangle$ | 33915 | $\langle 33, \pm 3, 257\rangle$ | 33915 | $\langle 41, \pm 19, 209\rangle$ |
| 33915 | $\langle 55, \pm 25, 157\rangle$ | 33915 | $\langle 61, \pm 1, 139\rangle$ | 33915 | $\langle 67, \pm 11, 127\rangle$ | 33915 | $\langle 77, \pm 63, 123\rangle$ |
| 33915 | $\langle 79, \pm 23, 109\rangle$ | 34720 | $\langle 13, \pm 4, 668\rangle$ | 34720 | $\langle 29, \pm 14, 301\rangle$ | 34720 | $\langle 43, \pm 14, 203\rangle$ |
| 34720 | $\langle 52, \pm 4, 167\rangle$ | 34720 | $\langle 65, \pm 30, 137\rangle$ | 34720 | $\langle 79, \pm 44, 116\rangle$ | 34720 | $\langle 89, \pm 48, 104\rangle$ |
| 34720 | $\langle 91, \pm 56, 104\rangle$ | 36960 | $\langle 13, \pm 8, 712\rangle$ | 36960 | $\langle 17, \pm 10, 545\rangle$ | 36960 | $\langle 23, \pm 22, 407\rangle$ |
| 36960 | $\langle 37, \pm 22, 253\rangle$ | 36960 | $\langle 39, \pm 18, 239\rangle$ | 36960 | $\langle 51, \pm 24, 184\rangle$ | 36960 | $\langle 52, \pm 44, 187\rangle$ |
| 36960 | $\langle 65, \pm 60, 156\rangle$ | 36960 | $\langle 67, \pm 52, 148\rangle$ | 36960 | $\langle 68, \pm 44, 143\rangle$ | 36960 | $\langle 69, \pm 24, 136\rangle$ |
| 36960 | $\langle 85, \pm 10, 109\rangle$ | 36960 | $\langle 89, \pm 8, 104\rangle$ | 36960 | $\langle 91, \pm 70, 115\rangle$ | 36960 | $\langle 92, \pm 68, 113\rangle$ |
| 36960 | $\langle 104, \pm 96, 111\rangle$ | 40755 | $\langle 23, \pm 1, 443\rangle$ | 40755 | $\langle 31, \pm 17, 331\rangle$ | 40755 | $\langle 41, \pm 9, 249\rangle$ |
| 40755 | $\langle 43, \pm 3, 237\rangle$ | 40755 | $\langle 69, \pm 45, 155\rangle$ | 40755 | $\langle 79, \pm 3, 129\rangle$ | 40755 | $\langle 83, \pm 9, 123\rangle$ |
| 40755 | $\langle 93, \pm 45, 115\rangle$ | 43680 | $\langle 11, \pm 10, 995\rangle$ | 43680 | $\langle 19, \pm 18, 579\rangle$ | 43680 | $\langle 29, \pm 20, 380\rangle$ |
| 43680 | $\langle 33, \pm 12, 332\rangle$ | 43680 | $\langle 44, \pm 12, 249\rangle$ | 43680 | $\langle 55, \pm 10, 199\rangle$ | 43680 | $\langle 57, \pm 18, 193\rangle$ |
| 43680 | $\langle 61, \pm 22, 181\rangle$ | 43680 | $\langle 67, \pm 2, 163\rangle$ | 43680 | $\langle 76, \pm 20, 145\rangle$ | 43680 | $\langle 77, \pm 56, 152\rangle$ |
| 43680 | $\langle 83, \pm 12, 132\rangle$ | 43680 | $\langle 87, \pm 78, 143\rangle$ | 43680 | $\langle 88, \pm 56, 133\rangle$ | 43680 | $\langle 88, \pm 32, 127\rangle$ |
| 43680 | $\langle 95, \pm 20, 116\rangle$ | 57120 | $\langle 11, \pm 6, 1299\rangle$ | 57120 | $\langle 23, \pm 14, 623\rangle$ | 57120 | $\langle 33, \pm 6, 433\rangle$ |
| 57120 | $\langle 44, \pm 28, 329\rangle$ | 57120 | $\langle 47, \pm 28, 308\rangle$ | 57120 | $\langle 55, \pm 50, 271\rangle$ | 57120 | $\langle 59, \pm 46, 251\rangle$ |
| 57120 | $\langle 69, \pm 60, 220\rangle$ | 57120 | $\langle 77, \pm 28, 188\rangle$ | 57120 | $\langle 79, \pm 32, 184\rangle$ | 57120 | $\langle 88, \pm 72, 177\rangle$ |
| 57120 | $\langle 88, \pm 16, 163\rangle$ | 57120 | $\langle 89, \pm 14, 161\rangle$ | 57120 | $\langle 92, \pm 60, 165\rangle$ | 57120 | $\langle 109, \pm 66, 141\rangle$ |
| 57120 | $\langle 115, \pm 60, 132\rangle$ | 77280 | $\langle 17, \pm 6, 1137\rangle$ | 77280 | $\langle 29, \pm 18, 669\rangle$ | 77280 | $\langle 41, \pm 28, 476\rangle$ |
| 77280 | $\langle 51, \pm 6, 379\rangle$ | 77280 | $\langle 53, \pm 10, 365\rangle$ | 77280 | $\langle 68, \pm 28, 287\rangle$ | 77280 | $\langle 73, \pm 10, 265\rangle$ |
| 77280 | $\langle 85, \pm 40, 232\rangle$ | 77280 | $\langle 87, \pm 18, 223\rangle$ | 77280 | $\langle 107, \pm 98, 203\rangle$ | 77280 | $\langle 109, \pm 108, 204\rangle$ |
| 77280 | $\langle 116, \pm 76, 179\rangle$ | 77280 | $\langle 119, \pm 28, 164\rangle$ | 77280 | $\langle 123, \pm 54, 163\rangle$ | 77280 | $\langle 136, \pm 96, 159\rangle$ |
| 77280 | $\langle 136, \pm 40, 145\rangle$ | 87360 | $\langle 32, \pm 24, 687\rangle$ | 87360 | $\langle 32, \pm 8, 683\rangle$ | 87360 | $\langle 37, \pm 16, 592\rangle$ |
| 87360 | $\langle 43, \pm 4, 508\rangle$ | 87360 | $\langle 53, \pm 14, 413\rangle$ | 87360 | $\langle 59, \pm 14, 371\rangle$ | 87360 | $\langle 96, \pm 72, 241\rangle$ |
| 87360 | $\langle 96, \pm 24, 229\rangle$ | 87360 | $\langle 101, \pm 56, 224\rangle$ | 87360 | $\langle 111, \pm 90, 215\rangle$ | 87360 | $\langle 113, \pm 92, 212\rangle$ |
| 87360 | $\langle 127, \pm 4, 172\rangle$ | 87360 | $\langle 129, \pm 90, 185\rangle$ | 87360 | $\langle 139, \pm 40, 160\rangle$ | 87360 | $\langle 148, \pm 132, 177\rangle$ |
| 87360 | $\langle 159, \pm 120, 160\rangle$ | | | | | | |