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Blocking Sets and Derivable Partial Spreads

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Abstract. We prove that a $GF(q)$-linear Rédei blocking set of size $q^t + q^{t-1} + \cdots + q + 1$ of $PG(2, q^t)$ defines a derivable partial spread of $PG(2t-1, q)$. Using such a relationship, we are able to prove that there are at least two inequivalent Rédei minimal blocking sets of size $q^t + q^{t-1} + \cdots + q + 1$ in $PG(2, q^t)$, if $t \geq 4$.

Keywords: spread, translation plane, blocking set

1. Introduction

A *blocking set* B in a finite projective plane is a set of points intersecting every line. B is called *trivial* if it contains a line. Throughout this paper, we only consider *non-trivial* blocking sets. Two blocking sets are said to be *equivalent* if there is a collineation of the plane which maps one to the other one.

A blocking set is called *minimal* if no proper subset of it is a blocking set. If q is the order of the plane, and B has size $q + N$, then a line contains at most N points of B; if such a line exists, B is called of *Rédei type* and the line is said to be a *Rédei line*.

Minimal blocking sets of a desarguesian plane $PG(2, s)$, $s = p^n$, p prime, of size less than $\frac{3(s+1)}{2}$ are called *small*. They intersect every line in a number of points congruent to 1 modulo p (see [12]). Let B be a small minimal Rédei blocking set of $PG(2, s)$. Let e be the largest integer such that each secant of B meets B in $np^e + 1$ points, $n \in \mathbf{N}$. If $q = p^e > 2$, then $s = q^t$ (i.e., $GF(p^e)$ is a subfield of $GF(s)$), and $|B| = q^t + N$ with $q^{t-1} + 1 \leq N \leq q^{t-1} + \cdots + q + 1$ [1]. In particular, when $N = q^{t-1} + \cdots + q + 1$, $t > 2$, then there is exactly one Rédei line and all secants different from the Rédei line contain $q + 1$ points of B.

A $(t-1)$-*spread* $\mathcal{S}$ of $\Sigma = PG(2t-1, q)$ is a partition of the pointset of Σ in $(t-1)$-dimensional subspaces. Let $\mathcal{S}$ be a $(t-1)$-spread of $\Sigma = PG(2t-1, q)$. Embed Σ as a hyperplane in $\Sigma' = PG(2t, q)$ and define a point-line geometry $\pi = \pi(\Sigma', \Sigma, \mathcal{S})$ in the following way. The points of π are either the elements of $\mathcal{S}$ or the points of $\Sigma' \setminus \Sigma$. The lines of π are either Σ or the t-dimensional subspaces of Σ' which intersect Σ in an element of $\mathcal{S}$. The incidence is inherited by Σ'. Then, π is a translation plane with respect to the line represented by Σ, whose order is q^t (see [5] or [6]). If π is isomorphic to the desarguesian

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plane of order q^t, then $\mathcal{S}$ is a *desarguesian spread*. A subset $\mathcal{F}$ of a $(t-1)$-spread $\mathcal{S}$ of Σ is called *derivable partial spread* if there exists a partial spread $\mathcal{F}^*$ such that $\mathcal{S}^* = (\mathcal{S}\setminus\mathcal{F})\cup\mathcal{F}^*$ is a $(t-1)$-spread of Σ, and $\pi(\Sigma', \Sigma, \mathcal{S}^*)$ is called the *derived plane.*[1]

Let Γ be a t-dimensional subspace of $\Sigma' = PG(2t, q)$, with $\Gamma \not\subset \Sigma$, and let $\Delta = \Gamma \cap \Sigma$. If

$$\mathcal{D} = \{T \in \mathcal{S} \mid T \cap \Delta \neq \emptyset\}$$

then $B_\Gamma = (\Gamma\setminus\Delta) \cup \mathcal{D}$ is a minimal Rédei blocking set of π, and the translation line Σ is a Rédei line (see [2]). The order of B_Γ is $q^t + N$, where N is the order of the partial spread $\mathcal{D}$. In [4], Bruen showed that every derivable partial spread of $\mathcal{S}$ defines a Rédei blocking set of π. If $\mathcal{S}$ is desarguesian, then B_Γ belongs to the family of $GF(q)$-linear blocking sets constructed in [7], and every $GF(q)$-linear blocking set of Rédei type can be obtained in such a way ([7]).

Suppose that $\mathcal{S}$ is a desarguesian spread, and let B_Γ be a $GF(q)$-linear Rédei blocking set of $\pi(\Sigma', \Sigma, \mathcal{S})$. In this paper we prove that, if each element of $\mathcal{D}$ intersects Δ in a point (i.e., $N = q^{t-1}+\cdots+q+1$ and B_Γ has order $q^t+q^{t-1}+\cdots+q+1$), then $\mathcal{D}$ is a derivable partial spread. Finally, we construct a new example of $GF(q)$-linear Rédei blocking set of $PG(2, q^t)$ of size $q^t + q^{t-1} + \cdots + q + 1$.

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2. Derivable partial spreads

Let $PG(1, q^t) = PG(V, GF(q^t))$, where V is a 2-dimensional vector space over $GF(q^t)$. Regarding V as a $2t$-dimensional vector space over $GF(q)$, each point x of $PG(1, q^t)$ defines a $(t-1)$-dimensional subspace $P(x)$ of $\Sigma = PG(2t-1, q) = PG(V, GF(q))$ and $\mathcal{S} = \{P(x) \mid x \text{ is a point of } PG(1, q^t)\}$ is a spread of Σ. If $PG(2, q^t) = PG(\bar{V}, GF(q^t))$, where $\bar{V} = V \oplus \langle e\rangle$ is a 3-dimensional vector space over $GF(q^t)$, the map α defined by $\alpha : x \mapsto P(x)$ for all points x of $PG(1, q^t)$ and $\alpha : \langle v + e\rangle_{GF(q^t)} \mapsto \langle v + e\rangle_{GF(q)}$ for all points $\langle v + e\rangle_{GF(q^t)}$ of $PG(2, q^t)$ not in $PG(1, q^t)$, is an isomorphism from $PG(2, q^t)$ to $\pi(\Sigma', \Sigma, \mathcal{S})$, where $\Sigma' = PG(V', GF(q))$ and $V' = V \oplus \langle e\rangle_{GF(q)}$. Then $\mathcal{S}$ is a desarguesian spread of Σ.

For each λ in $GF(q^t)^*$, let τ_λ be the linear collineation of Σ defined by the linear map $v \mapsto \lambda v$. Then $\tau_\lambda = \tau_\mu$ if and only if $\lambda\mu^{-1} \in GF(q)$. Hence, $G = \{\tau_\lambda \mid \lambda \in GF(q^t)^*\}$ is a collineation group of Σ of order $q^{t-1} + \cdots + q + 1$ which fixes all the elements of $\mathcal{S}$ and acts sharply-transitively on the points of each $P(x)$.

Suppose that B_Γ is a $GF(q)$-linear Rédei blocking set of $PG(2, q^t)$ of maximum size and let $\Delta = \Sigma \cap \Gamma$. Then Δ is a $(t-1)$-subspace of Σ such that $P(x) \cap \Delta$ is either empty or a point. Put

$$\mathcal{D} = \{P(x) \in \mathcal{S} \mid P(x) \cap \Delta \neq \emptyset\},$$
$$\mathcal{D}^* = \{\Delta^g \mid g \in G\}.$$

Theorem 1 *$\mathcal{S}^* = (\mathcal{S} \setminus \mathcal{D}) \cup \mathcal{D}^*$ is a spread of Σ.*

Proof: As G fixes all the elements of $\mathcal{S}$, the subspace Δ^g intersects all the elements of $\mathcal{D}$ exactly at a point.

Let $P(x)$ be an element of $\mathcal{D}$, and let $z = P(x) \cap \Delta$. If y is a point of $P(x)$, then there is exactly one element g in G such that $y = z^g$. This implies that y belongs to Δ^g, and we have proved that

$$\bigcup_{P(x)\in\mathcal{D}} P(x) = \bigcup_{g\in G} \Delta^g.$$

To prove that $\mathcal{S}^*$ is a spread it is enough to prove that $\mathcal{D}^*$ is a partial spread. Suppose that y belongs to $\Delta^g \cap \Delta$. Then $y = z^g$ for some point z in Δ. As g fixes all the elements of $\mathcal{S}$, we have $z \in P(x)$ if and only if $y \in P(x)$. Therefore $z = y$, because $\Delta \cap P(x)$ is a point. This implies $g = 1$ because G is sharply transitive on the points of $P(x)$. Hence Δ^h and Δ^g are disjoint if and only if $h \neq g$. □

Let $\Lambda^* = PG(t-1, q^t)$ and let $\Lambda = PG(t-1, q)$ be a canonical subgeometry of Λ^*. Denote by Ω a $(t-3)$-dimensional subspace of Λ^* disjoint from all the lines of Λ. If $l = PG(1, q^t)$ is a line of Λ^* disjoint from Ω, then $D = \{\langle x, \Omega\rangle \cap l \mid x \in \Lambda\}$ is a set of $q^{t-1} + \cdots + q + 1$ points of l. Moreover, if $l = PG(1, q^t) = PG(V, GF(q^t))$, where V is a 2-dimensional vector space over $GF(q^t)$, then there is a subgroup W of the additive group of V such that W is a t-dimensional $GF(q)$-vector space and $D = \{\langle w\rangle \mid w \in W \setminus \{0\}\}$. Hence, $\mathcal{D} = \{P(x) \mid x \in D\}$ is a derivable partial spread of $PG(2t-1, q) = PG(V, GF(q))$ and $\mathcal{D}^* = \{W^\tau \mid \tau \in G\}$ (see [8]).

By [9], all derivable partial spreads defined by a Rédei blocking set of $PG(2, q^t)$ of size $q^t + q^{t-1} + \cdots + q + 1$ can be constructed in this way.

If $(X_0, X_1, \ldots, X_{t-1})$ are homogenous projective coordinates of Λ^*, then we can suppose $\Lambda = \{(a, a^q, \ldots, a^{q^{t-1}}) \mid a \in GF(q^t)^*\}$. The $(t-3)$-dimensional subspace Ω with equations $X_0 = X_1 = 0$ is disjoint from all lines of Λ, and the line l with equations $X_2 = X_3 = \cdots = X_{t-1} = 0$ is disjoint from Λ. Hence $D = \{(a, a^q, 0, \ldots, 0) \mid a \in GF(q^t)^*\}$. Denote by ξ a primitive element of $GF(q^t)$ and let μ be the collineation of Λ^* defined by

$$\mu : (X_0, X_1, \ldots, X_{t-1}) \mapsto \left(\xi X_0, \xi^q X_1, \ldots, \xi^{q^{t-1}} X_{t-1}\right).$$

Then μ has order $q^{t-1} + \cdots + q + 1$ and fixes Λ, Ω and the line l. Moreover, the group H generated by μ acts sharply transitively on Λ. Also, H fixes D and the two points $(1, 0, 0, \ldots, 0)$ and $(0, 1, 0, \ldots, 0)$ of l. We note that $\tau_\lambda\mu = \mu\tau_\lambda$. Hence the group GH is abelian. Let $V' = V \oplus \langle e\rangle_{GF(q)}$, where $l = PG(V, GF(q^t))$, and let $\Sigma' = PG(V', GF(q))$. If $\tau_\lambda\mu^i \in GH$, then $\tau_\lambda\mu^i$ induces a collineation of Σ' which maps the point $\langle(x, y) + \alpha e\rangle$ to the point $\langle(\lambda\xi^i x, \lambda\xi^{iq} y) + \alpha e\rangle$.

As GH maps elements of $\mathcal{D}$ to elements of $\mathcal{D}$ and elements of $\mathcal{D}^*$ to elements of $\mathcal{D}^*$, the derived plane $\pi(\Sigma', \Sigma, \mathcal{S}^*)$ has an abelian collineation group fixing the two lines $\langle P(1, 0), e\rangle$ and $\langle P(0, 1), e\rangle$. If $\langle P(1, x), e\rangle$, with $(1, x)$ not in D, the group G is the stabilizer of $P(1, x)$ in GH. Therefore, G defines a collineation group acting sharply transitively on the points of the line $\langle P((1, x)), e\rangle$ different from $\langle e\rangle$. If $X \in \mathcal{D}^*$, the stabilizer of X in GH coincides with H because G acts, by construction, sharply transitively on $\mathcal{D}^*$. Hence, H defines a collineation group of the plane acting sharply transitively on the points of $\langle X, e\rangle$ different from $\langle e\rangle$. By [6, Corollary 12.2] the plane $\pi(\Sigma', \Sigma, \mathcal{S}^*)$ is an André plane.

3. Some examples

Let $PG(2, q^t) = PG(V, GF(q^t))$. If e_0, e_1, e_2 is a fixed basis of V, denote by (x_0, x_1, x_2) the homogeneous projective coordinates of the point $\langle x_0e_0 + x_1e_1 + x_2e_2\rangle$ of $PG(2, q^t)$. Let $f : GF(q^t) \longrightarrow GF(q^t)$ be a $GF(q)$-linear map. The set

$$B = \{(x, f(x), a) : x \in GF(q^t), a \in GF(q)\}$$

is a $GF(q)$-linear Rédei blocking set of $PG(2, q^t)$ and the line $x_2 = 0$ is a Rédei line. Conversely, every small minimal Rédei blocking set of $PG(2, q^t)$ (with certain exception in characteristic two or three) can be obtained in such a way (see [1]). If

$$\mathcal{B} = \{(x, x^q, a) \mid x \in GF(q^t), a \in GF(q)\},$$

then $\mathcal{B}$ is a Rédei blocking set of size $q^t + q^{t-1} + \cdots + q + 1$ and, hence, the line $x_2 = 0$ is a Rédei line (see [3]). This is the only known Rédei blocking set of size $q^t + \cdots + q + 1$ and it is exactly the example constructed at the end of Section 2, where we have proved that the derived plane obtained from $\mathcal{B}$ is an André plane. See also [10] for a direct proof.

Let λ be a fixed element of $GF(q^t)$ different from 0, and denote by N the norm function of $GF(q^t)$ over $GF(q)$, i.e., $N(x) = x^{q^{t-1}+\cdots+q+1}$, for $x \in GF(q^t)$. Define

$$B_\lambda = \left\{\left(x, \lambda x^q + x^{q^{t-1}}, a\right) \mid x \in GF(q^t), a \in GF(q)\right\}.$$

Since $x \longrightarrow \lambda x^q + x^{q^{t-1}}$ is a $GF(q)$-linear map, B_λ is $GF(q)$-linear Rédei blocking set of $PG(2, q^t)$.

Theorem 2 *If $N(\lambda) \neq 1$, then B_λ is a blocking set of size $q^t + q^{t-1} + \cdots + q + 1$.*

Proof: By way of contradiction, suppose that $|B_\lambda| < q^t + q^{t-1} + \cdots + q + 1$. Then there exist $x, y \in GF(q^t)$, $a, b \in GF(q)$, and $\gamma \in GF(q^t)\backslash GF(q)$ such that

$$\left(x, \lambda x^q + x^{q^{t-1}}, a\right) = \gamma\left(y, \lambda y^q + y^{q^{t-1}}, b\right), \tag{1}$$

which implies $a = \gamma b$. As $\gamma \notin GF(q)$, we have $a = b = 0$. From (1), it follows

$$\begin{cases} x & = \gamma y \\ \lambda x^q + x^{q^{t-1}} & = \gamma\left(\lambda y^q + y^{q^{t-1}}\right), \end{cases}$$

which gives

$$\lambda = \frac{y^{q^{t-1}}}{y^q} \cdot \frac{\gamma - \gamma^{q^{t-1}}}{\left(\gamma - \gamma^{q^{t-1}}\right)^q} = \frac{y^{q^{t-1}-q}}{\left(\gamma - \gamma^{q^{t-1}}\right)^{q-1}}.$$

So, we obtain

$$N(\lambda) = N\left(y^{q^{t-1}-q}\right) \cdot N\left(\frac{1}{(\gamma - \gamma^{q^{t-1}})^{q-1}}\right) = 1.$$

Therefore, if $N(\lambda) \neq 1$, we have $|B_\lambda| = q^t + q^{t-1} + \cdots + q + 1$. □

Theorem 3 *If $N(\lambda) \neq 1$, $q > 3$ and $t \geq 4$, then B_λ and $\mathcal{B}$ are not isomorphic.*

Proof: Suppose there exists a linear collineation ω of $PG(2, q^t)$ which maps B_λ to $\mathcal{B}$. Denote by $A = (a_{ij})$, with $a_{ij} \in GF(q^t)$ and $i, j = 1, 2, 3$, the matrix associated with ω with respect to the basis e_0, e_1, e_2. As ω maps the Rédei line of B_λ to the Rédei line of $\mathcal{B}$, ω fixes the line $x_2 = 0$. Hence, $a_{13} = a_{23} = 0$, and $\det(A) = a_{33}(a_{11}a_{22} - a_{21}a_{12})$. Also, the points $(x, \lambda x^q + x^{q^{t-1}}, 0)$ of B_λ are mapped to the points $(y, y^q, 0)$ of $\mathcal{B}$, i.e.,

$$\left(x, \lambda x^q + x^{q^{t-1}}\right) \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \rho_x(y, y^q),$$

with $\rho_x \in GF(q^t)^*$. This implies

$$xa_{11} + \lambda a_{21} x^q + a_{21} x^{q^{t-1}} = \rho_x y \quad (2)$$

$$xa_{12} + \lambda a_{22} x^q + a_{22} x^{q^{t-1}} = \rho_x y^q. \quad (3)$$

From Eqs. (2) and (3), we have

$$y^{q-1} = \frac{xa_{12} + \lambda a_{22} x^q + a_{22} x^{q^{t-1}}}{xa_{11} + \lambda a_{21} x^q + a_{21} x^{q^{t-1}}},$$

which gives

$$N\left(xa_{11} + \lambda a_{21} x^q + a_{21} x^{q^{t-1}}\right) = N\left(xa_{12} + \lambda a_{22} x^q + a_{22} x^{q^{t-1}}\right),$$

i.e.

$$\prod_{i=0}^{t-1} \left(x^{q^i} a_{11}^{q^i} + \lambda^{q^i} a_{21}^{q^i} x^{q^{i+1}} + a_{21}^{q^i} x^{q^{t-1+i}}\right) = \prod_{i=0}^{t-1} \left(x^{q^i} a_{12}^{q^i} + \lambda^{q^i} a_{22}^{q^i} x^{q^{i+1}} + a_{22}^{q^i} x^{q^{t-1+i}}\right),$$

for all $x \in GF(q^t)$. As $x^{q^t} = x$, from the above equality we obtain two polynomials of degree at most $3q^{t-1} + q^{t-2} + \cdots + q^3 + q^2$. If $q > 3$, their degree is less than q^t, and hence they have the same coefficients. Comparing the coefficients of the terms of maximum degree $3q^{t-1}+q^{t-2}+\cdots+q^3+q^2$, for $t\geq 4$, we get

$$a_{21}\lambda^q a_{21}^q \lambda^{q^2} a_{21}^{q^2}\cdots\lambda^{q^{t-2}} a_{21}^{q^{t-2}} a_{11}^{q^{t-1}} = a_{22}\lambda^q a_{22}^q \lambda^{q^2} a_{22}^{q^2}\cdots\lambda^{q^{t-2}} a_{22}^{q^{t-2}} a_{12}^{q^{t-1}},$$

which implies

$$a_{21}^{q^{t-2}+\cdots+q+1} a_{11}^{q^{t-1}} = a_{22}^{q^{t-2}+\cdots+q+1} a_{12}^{q^{t-1}}. \tag{4}$$

On the other hand, comparing the coefficients of the terms of degree $3q^{t-1}+q^{t-2}+\cdots+q^3+q$, for $t\geq 4$, we have

$$a_{21}a_{11}^q \lambda^{q^2} a_{21}^{q^2}\cdots\lambda^{q^{t-2}} a_{21}^{q^{t-2}} a_{11}^{q^{t-1}} = a_{22}a_{12}^q \lambda^{q^2} a_{22}^{q^2}\cdots\lambda^{q^{t-2}} a_{22}^{q^{t-2}} a_{12}^{q^{t-1}},$$

which implies

$$a_{21}^{q^{t-2}+\cdots+q^2+1} a_{11}^{q^{t-1}+q} = a_{22}^{q^{t-2}+\cdots+q^2+1} a_{12}^{q^{t-1}+q}. \tag{5}$$

If $a_{21}a_{11}\neq 0$, dividing both sides of Eq. (4) by (5), we get

$$\frac{a_{21}^q}{a_{11}^q} = \frac{a_{22}^q}{a_{12}^q} \Longrightarrow a_{21}a_{12} = a_{22}a_{11},$$

i.e., $\det(A)=0$, a contradiction.

Now, suppose $a_{21}a_{11}=0$. From (5), it follows $a_{22}a_{12}=0$. As $\det(A)\neq 0$, the following cases may occur:

(a) $a_{12}=0$ and $a_{21}=0$
(b) $a_{22}=0$ and $a_{11}=0$.

In case (a), we have

$$N(xa_{11}) = N\big(\lambda a_{22}x^q + a_{22}x^{q^{t-1}}\big),$$

that is

$$N(a_{11})x^{q^{t-1}+\cdots+q+1} = N(a_{22})\big(\lambda x^q + x^{q^{t-1}}\big)\big(\lambda^q x^{q^2}+x\big)\cdots\big(\lambda^{q^{t-1}}x + x^{q^{t-2}}\big)$$

for all $x\in GF(q^t)$. Comparing the coefficents of the terms of degree $2q^{t-1}+q^{t-2}+\cdots+q^3+q^2$, we get $a_{22}=0$, which is impossible. The same way we can exclude case (b).

Finally, suppose there exists a collineation θ of $PG(2,q^t)$ which maps B_λ to $\mathcal{B}$. Let $A=(a_{ij})$, with $a_{ij}\in GF(q^t)$ and $i,j=1,2,3$, and σ denote respectively the matrix and the automorphism of $GF(q^t)$ associated with θ. The line $x_2 = 0$ is fixed by θ, hence $a_{13} = a_{23} = 0$ and $\det(A) = a_{33}(a_{11}a_{22} - a_{21}a_{12})$. Moreover,

$$\left(\sigma(x), \sigma\left(\lambda x^q + x^{q^{t-1}}\right)\right)\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \rho_x(y, y^q),$$

with $\rho_x \in GF(q^t)^*$. This implies

$$a_{11}\sigma(x) + a_{21}\sigma\left(\lambda x^q + x^{q^{t-1}}\right) = \rho_x y \tag{6}$$

$$a_{12}\sigma(x) + a_{22}\sigma\left(\lambda x^q + a_{22}x^{q^{t-1}}\right) = \rho_x y^q. \tag{7}$$

From Eqs. (6) and (7), we get

$$y^{q-1} = \frac{a_{12}\sigma(x) + a_{22}\sigma\left(\lambda x^q + x^{q^{t-1}}\right)}{a_{11}\sigma(x) + a_{21}\sigma\left(\lambda x^q + x^{q^{t-1}}\right)},$$

hence

$$N\left(a_{11}\sigma(x) + a_{21}\sigma\left(\lambda x^q + x^{q^{t-1}}\right)\right) = N\left(a_{12}\sigma(x) + a_{22}\sigma\left(\lambda x^q + x^{q^{t-1}}\right)\right).$$

If

$$\sigma(a'_{11}) = a_{11},\ \sigma(a'_{21}) = a_{21},\ \sigma(a'_{12}) = a_{12},\ \text{ and } \sigma(a'_{22}) = a_{22},$$

we can write

$$\sigma\left(N\left(a'_{11}x + \lambda a'_{21}x^q + a'_{21}x^{q^{t-1}}\right)\right) = \sigma\left(N\left(a'_{12}x + \lambda a'_{22}x^q + a'_{22}x^{q^{t-1}}\right)\right),$$

that is

$$N\left(xa'_{11} + \lambda a'_{21}x^q + a'_{21}x^{q^{t-1}}\right) = N\left(xa'_{12} + \lambda a'_{22}x^q + a'_{22}x^{q^{t-1}}\right),$$

for all $x \in GF(q^t)$. As before $a'_{11}a'_{22} - a'_{12}a'_{21} = 0$, which gives $\det(A) = a_{33}(a_{11}a_{22} - a_{12}a_{21}) = 0$, a contradiction. Then B_λ is not isomorphic to $\mathcal{B}$. □

Note

1. The incidence structure whose points are the points of $\Sigma' \setminus \Sigma$, and whose lines are the t-dimensional subspaces of Σ' containing an element of $\mathcal{F}$ is said a *derivable translation net* (see [11]).

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