Journ@l Electronique d'Histoire des Probabilités et de la Statistique

Electronic Journ@l for History of Probability and Statistics

Vol 5, n°2; Décembre/December 2009

www.jehps.net

Probability and Statistics at the turn of 1900 : hopes and disappointments

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Résumé

Au tournant du 20ème siècle, avant qu'une nouvelle théorie des probabilités fondée sur la théorie de la mesure ne bouleverse les mathématiques de l'aléatoire, un certain nombre de textes prétendirent présenter l'état de l'art de la discipline. Dans cet article, nous suivons l'une de ces entreprises.

Abstract

At the turn of the 20th Century, before a renewal of the theory of probability by means of measure theory completely transformed the mathematics of randomness, several surveys, aimed at presenting the state of the art of the discipline, were published. In the present paper, we study one such undertaking.

Introduction

After three quarters of a century of what often seemed like wanderings in the desert, the beginning of the 20th century was marked by a renewed interest in Probability. This branch of mathematics sometimes considered unrewarding and barren, (and even at times unworthy of the interest of mathematicians) whose fundamental principles had hardly changed since the work of Laplace and Gauss cobbled together in 1825, was about to undergo some fundamental changes – Liapounov, Borel, Von Mises, Levy, Kolmogorov. The Statistics side has constantly been torn between administrative elements in the publishing of information (categories, nomenclature, description, investigations, means of publication) and the chosen structures of the logic/mathematics linked to the inferred inductive reasoning on which one can legitimately proceed from facts to generalities and from generalities to laws. Statistics has, to some extent, shaken free from its typical applications on account of the increasing autonomy in e.g. demography, economics or biology, which have their own proper organisations and magazines, in order to concentrate more fully

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on the one method of collating inferentially the many different values, and thus becoming in the process more mathematical[2].

The *Encyklopädie of mathematischen Wissenchaften, mit Einschluss ihrer Andwendugen*[3] was an ambitious project directed by the German mathematician Felix Klein, supported by the four Academies of Science in Gottingen, Leipzig, Munich and Vienna. The first volumes appeared in 1898 and the last in 1935 with a break during the First World War. The three main objectives present from the start were a well-referenced historical presentation, an up to date account of the sciences as they stood and, more importantly and innovatively, the application of these sciences.

The French edition[4] of the Encyclopaedia of the Pure Mathematical Sciences edited and published after the German edition is neither a straight translation nor a simple adaptation for the French readership. It was intended as an erudite and sometimes controversial re-reading of the German edition, by the group of scholars brought together by Jules Molk, professor of rational mechanics at the University of Nancy. He was familiar with the scientific literature in Germany having lived in Berlin between 1880 and 1884 and having written a thesis focused on an overview of the German breakthroughs in Mathematics. The first volumes of this French edition appeared in 1908, with the help of the two editors, Teubner and Gauthier Villars, and the last in 1916, two years after the death of Jules Molk. Only half of the volumes released from Teubner in 1908 were published, and that with some difficulty. Nevertheless, such a Franco-German scientific collaboration was remarkable and totally new.

The period covered by l'*Encyclopédie* gives us an excellent opportunity, in principle, to create an overview of the history of the mathematics of probability and statistics.

We refer to Anders Hald (1998)[5], Jan von Plato (2000)[6] and Krüger and al. (1987, 1989)[7], for general context, and to the work done by Hélène Gispert (1999)[8], for a close analysis of the circumstances and genesis of this scientific and publishing project. We concentrate on this sole attempt, limited yet fruitful, of evaluating the contribution of this publication in the realms of probability and statistics or as Borel put it, “The science of chance.” Our investigation is based on Molk’s French edition and shall only note where the French version differs from the German.

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[2] See our paper “Une discipline dans tous ses états: la Statistique à travers ses traités”, Revue de Synthèse, IV-2, 1991, and Les images de la Statistique à travers ses traités, *Journal électronique d’histoire des probabilités et de la statistique*, Vol.2/N°2, déc. 2006. www.jehps.net.or the study of the contents of the papers in the Journal de la Société de Statistique de Paris in Z.Kang’s thesis: *Lieu de Savoir social, la Société de Statistique de Paris au XIXème siècle*, Paris, 1989.

[3] Klein F., Weber H., Meyer F. (dir), *Encyklopädie der mathematischen Wissenschaften, mit Eischluss ihrer Anwebdungen*, Teubner, 1898-1935.

[4] Molk J. (dir), *Encyclopédie des Sciences Mathématiques pures et appliqués*, Gauthier-Villars, 1904-1916.

[5] Hald A., A history of probability and statistics from 1750 to 1930, New York : Wiley.

[6] Von Plato J., *Creating Modern Probability,* Cambridge university Press, 1994, 4 printing 2000.

[7] Kruger L., Daston L., Heidelberger M. (eds) , 1987, *The probabilistic revolution*, Vol 1: ideas in history, MIT Press, Cambridge, England. Kruger L., Gigerenzer G, Morgan M.S. (eds) , 1989 , *The probabilistic revolution*, Vol 2: Ideas in the sciences, MIT Press, Cambridge, England.

[8] H. Gispert, Les débuts de l'histoire des mathématiques sur les scènes internationales et le cas de l'entreprise encyclopédique de Felix Klein et Jules Molk, *Historica Mathematica,*26-4, p. 344-360.

Once we have digested the group of articles in the Encyclopédie that describe the field dealing with the mathematics of chance, our analysis concentrates on a few main topics.

The first deals with the breadth covered by the Encyclopédie: which theoretical questions are addressed in the articles and which are left in the dark. Going beyond a straightforward delineation of the areas covered by these disciplines and their representation, the more important second topic deals with the history of the mathematics of probability : how theoretical paradigms and applications have structured their history? The importance given to the history of this subject in the articles together with the wealth and detail of the bibliographical references offers us a rare synthetic vision of the subject. Released at the turn of the century, a key moment in the development of Mathematics and in particular the development of the Mathematics of Chance, Klein's Encyclopädie and its French counterpart edited by Jules Molk may obviously give us an overview of the changes in paradigm and methods adopted at the time. Reading these texts, we may benefit from their privileged analysis concerning the achievements and the failures of the Laplacian tradition, and of the theory of means, just after Bertrand's[9] famous critiques, and also concerning the new mathematics of statistics opened up by the German school of Lexis and Bortkiewicz and the English Biometric School of Galton and Pearson.

The initial intention of the German editors to orientate the Encyclopaedia towards application of mathematics, which represents more than 50% of the content of the books in terms of the number of pages, gives us, in addition, an informed insight into the meaning of the term "application;" all the more so that we are dealing with an avalanche of applications: from mathematics to probability, from probability to statistics and from statistics to numerous fields of knowledge, and often, from theory into the realm of action, combining evolutions within mathematics and the transformation of mathematical techniques into practical tools for analysis and decision making.

Third, the manner in which the French version was conceived, adopting a rather free translation from the German and with the particular use of asterisks to indicate where the commentary comes from additions made by French contributors, gives us the rare opportunity to observe the national differences in approach to the subject, and of the few schools covered: to see, in detail, the disagreement between these schools, and the importance of specific aspects in the field of the philosophy of knowledge; the way the research is organised and taught; and questions concerning the political agendas.

The articles concerning Probability and Statistics

Let us start by detailing a list of the articles in the French edition that we can ascribe to probability and statistics, thus allowing us to establish in the actual text the contribution of the Encyclopédie to the subject.
This is not so simple as it may seem because of the discouraging way they have been spread out through the volumes. Most of them are found in book I, devoted to arithmetic and algebra, and in volume 4 devoted to " Probability calculations, theory of error, abd diverse applications". All this is at odds with the logic of *L'Encyclopédie*, which places these "applications" in a volume "Pure Mathematics"

[9] See Joseph Bertrand : Calcul des Probabilités, 1889 (1st ed.)

and with the classification of the mathematics of probability in the "Jahrbuch"[10] and its subsequent inclusion in analysis.

Volume 4 is a mixture of applications operating at different levels. Two articles, on the" mathematics of probability" (I-20) and "the theory of error" (I-22), date from 1900 in the German edition, but whose French translation/adaptation have been delivered to the French editor between March 1906 and December 1908 , constitute the basic fundamentals of the "mathematics of chance", and the heart of the matter here, belonging nevertheless, in the eyes of pure mathematicians, to applied mathematics. The "calculation of differences and interpolation" (I-21) compiled by Henri Andoyer for Molk's edition is another branch of mathematics, important to probability computations, as are, for example, the "recurrent series" and "moment generating functions" used by de Moivre and Laplace. Furthermore we can consider this branch, and also combinatorics as a primary tool as well as an offshoot of probability computations. The very long article by R. Mehmke et Maurice d'Ocagne – inventor of nomography – "On numerical calculations" (I-23) can equally be applied to statistics by the remarkable work on tables and the abacus as with the calculating machines, the importance of which in treating statistical information, before mecanography and computers is well known. But here we are closer to the activity of Engineers rather than Mathematicians. Statistics, which was the specific subject of an article bearing the same name (I-24), occupied an ambiguous position at the time. After being named the "The science of the States" it was associated, even more so, with the science of demography and was closer to arithmetic than to the mathematics of probability. For others it had already been classed as the science of inference - using calculations to treat in a logical fashion any group of figures. In the same volume, the text "techniques of life insurance (I-26) represents an application of probability computations and statistics in improving the economic benefits inherent to the risks to human life, giving rise to an established professionally organized intermediary discipline, that of the actuary. The article *Economie mathématique* (I-26) by Vilfredo Pareto, which ends this book and Volume I constitutes the first systematic presentation (after his *Cours* of 1896) of those famous essays on ophemility, on the "surfaces of satisfaction", the demand curves and the resultant optimum equilibrium. Pareto immediately warns us of this totally hypothetical/deductive approach for excluding any other more statistical treatment of the values. Pareto's contribution to economic statistics, in particular his famous revenue curve, does not appear here, in this volume.

The last two articles[11] dealing with the mathematics of probability and their application to social questions are the only additions to the three articles of the basic fundamentals of probability and statistics. Articles pertaining to applications in physics and mechanics come, as we shall see later on in other volumes. Those articles dealing with other social issues…were not invited. This last observation means that we should give it a further thought. Laplace, in effect, in his *Essai philosophique sur les probabilités* had already envisaged applications of probability and statistics in "the

[10] The classification of probability calculation in the *Jahrbuch über die Fortschritte des Mathematik (1868-1942)* undergoes a change in 1914 from its original independent position existing as early as1868 to being a branch of analysis, and the percentage of pages devoted to it each year increases from ca. 2% at the beginning, to 3.6% in 1930 and to 7.6% in 1940.

[11] A third paper, on games, edited by C.A.Laisant after the German paper by W.Ahrens (Magdeburg) is anounced in the Teubner catalog of 1908, but is not in the volume published by Molk.

moral sciences" as well as in "natural philosophy". Certain areas which were important around 1900 are completely absent from Molk's *Encyclopädie*, such as medicine and health (Farr, Bertillon)[12], physical anthropology (Quetelet, Bertillon, Broca)[13], sociology (Quetelet, Durkheim, Halbwachs)[14], economic statistics (March, Bowley, Edgeworth, Simiand...)[15], biology and genetics (Galton, Pearson)[16], psychology (Thurstone, Spearman)[17].While these new fields are as much areas where statistical mathematics can be applied as they are fields where new concepts and analytical tools are emerging, as, for example, the theory of means and dispersion, as well as the theory of regression; correlation; factorial analysis; opinion polls…This gives rise to a body of cross referenced literature in the Jahrbuch which the editors of l'Encyclopédie were well aware of, and even more so those of the *Encyklopäedia*[18]. The lack of consideration given to the probability of judgements and testimony and a certain ignorance of the debates on social physics and the new science of eugenics on the part of mathematicians can perhaps explain why the Encyclopédie limited itself to subjects concerning the work of actuaries. However the revolution which occurred in these fields, starting in the 1880s, are largely due to the mix of mathematical structures imposed and the quantative statistics adopted by the profession, which, in turn, a little later, lead to the creation of intermediary disciplines and specialized magazines: biometry (with *Biometrika*) in 1901, econometrics (with *Econometrica*) in 1932, psychometrics (with *Psychometrika*) in 1935.

Now we come to the articles concerning the natural sciences, such as statistical mechanics (IV-2) and external ballistics (IV-21) which appear in the book about mechanics for two reasons: firstly, as a new theoretical branch of physics based on the principles of the science of chance; secondly, as an application of kinetics which has periodically made use of probabilistic results. Such is the case, also, with the following articles: "geodetic triangulation " (VI-1), "calibration of astronomical observations" (VII-3), and "how to determine longitude and latitude" (VII-4) which, respectively, appear in the following books Geodetics and Astronomy, matters in which the adjustement of an excess of eroneous data combined with the unknown character of some parameters implies using methods, such as least squares, where the mathematics of randomness are explicitely employed.

This first attempt at a synopsis shows the difficulty in uniting Mathematics and its application in the one publication. The editors had chosen to treat them separately ; an approach which does not favour a close understanding of their correlation. Probability and Statistics have been separated from the branch of mathematics that had largely

[12] See the works by Bernard Lécuyer, for instance "Probability in Vital and Social Statistics: Quetelet Farr and the Bertillons", in Krüger, Daston and Heidelberg eds., *The Probabilistic Revolutions*, Vol 1, MIT Press, 1987

[13] See for instance Claude Blanckaert, "Histoire de l'anthropologie", in *Bulletin et Mémoires de la Société d'Anthropologie de Paris,* 1989

[14] See Alain Desrosières, *La politique des grands nombres,* La Découverte, 2000, chap. 3 and chap.7.

[15] See our PhD, EHESS, 1995: *Histoire du modèle linéaire,* chapters 10-13.

[16] See Charles Lenay's thesis, 1989: *Enquête sur le hasard dans les théories biologiques,* and our PhD, 1995, op.cit., chapters 7-9.

[17] Olivier Martin's thesis, 1996: Eléments d'histoire de la mesure. Logique des outils de quantification dans les sciences psychologiques.

[18] In the E*lectronic Research Archive for Mathematics Jahrbuch Database*, one can find 95 references on correlation before 1907 among which 33 are relative to the British school (Edgeworth, Sheppard and Pearson). The latter totalizes 114 references with 82 anterior to 1908.

provided their fundamental tenets (analysis) and the applied areas, which provided in one go a testing ground for the concepts, the mathematical tools and methodology. It's the ridiculous division of the different disciplines that inhibits the proximity required to produce a one true encyclopaedia of Applied Mathematics. Other things that the editors failed to pick up on were the vertical links, sometimes straight sometimes indirect between pure and applied mathematics, as well as the direct transposition of mathematical concepts by analogy from one field of knowledge to another.

We shall now try to backup these first impressions with a closer reading of the more significant texts.

***I.20. Calcul des probabilités** (March 1906)*

The German article (August 1900), which was the model for the French edition, was written by Emmanuel Czuber (1851-1925). Born in Prague, professor at the Wiener Technische Hochschule from 1891 to 1921. He was one of the top experts in probability, having already published in 1900 a whole series of articles on the theory of errors, the St. Petersburg' problem, Geodetics, and questions about geometric probability. A work, dated 1884, bearing this title was translated into French[19] in 1902, and the first survey appeared in 1899[20]. But the treatise, in two very thick volumes, which he published after the article in the Encyklopädia, which was recognised all around Europe[21], testifies to his complete knowledge of the work of Laplace, English biometry, and, of course, the work of his fellow countrymen Gauss, Lexis and Bortkiewicz. Besides, he, himself, along with Bortkiewicz, are the most quoted German authors in Britain: second only to Laplace as the most frequently quoted reference in Keynes' treatise on probability (1921). The same cannot be said about the author chosen by the French editor Jules Molk to translate and amend his text. A dozen years later and he could have benefited from the blossoming of talent in the French school around Borel and Fréchet. An equivalent of a Czuber could have been found in, for example, Bertrand whose work dated 1889 became a reference, or, even, if not in Poincaré himself, the actuary Quiquet who edits his lecture notes from 1894, published in 1896, or also the actuary Laurent author of another *Traité du Calcul des Probabilités* in 1873. Jean Le Roux, professor of Applied Mathematics in Rennes and examiner at the Polytechnique, chosen by Molk to "rewrite" the article by Bortkiewicz was not a probabilist who marked his epoch. Nevertheless Molk did receive the Prix Leon Marie for his contributions in statistics and actuarial science in his Encyclopédie.

Like most articles, it begins with an historical introduction: Some preliminary comments retracing the philosophical conceptions of probability in the works of Laplace, Cournot, Bertrand and Poincaré, and raising the sensitive question about the foundations of the mathematics of probability. The definition of the calculation of probability given by Le Roux: the "study of relative frequency of uncertain events",

[19] Probabilités et moyennes géométriques", translation Herman Schuermans, Paris, Hermann, 1902; digitized on Gallica.

[20] *Die Entwicklung der Wahrscheinlichkeitstheorie und ihrer Anwendungen,* Deutsche Math. Ver., 1-279, 1899.

[21] Wahrcheinlichkeitsrechnung und ihre anwendung auf Fehlerausgleichung Statistik und lebensversicherung", Leipzig, Teubner, 2 tomes, 1st Ed. 1902-1903, 2nd Ed. 1908-1910, 3rd Ed.1914-1921

as well as the criticism of Laplace's subjective approach, and his circular definition of probability through "equal possibility," betrays the French editor's preference for the so-called frequentist theses of Cournot, placed to show the rift with "Philosophical probability." The influence of Bertrand's treatise, which is, in itself, a powerful attempt at undermining Laplacian theory, can be felt in a sentence as peremptory as: "it's just not possible to give a satisfactory definition of Probability." This leads on to two pages of introduction quite different to the original text by Czuber[22] whose presentation, finer and more focused on point of vocabulary, seems better balanced between the objective and subjective foundations of the calculation of probability[23]. The French modifications to Czuber's original text, in the rest of the article are really quite minor, apart from some bibliographical elements, some explanations of rather crude expositions by use of examples or formulae improving the didactic quality of the text and some details about vocabulary like the difference between, for example, the terms "*risque moyen*" et "*moyen risque* "[24]
stemming from a direct translation from the German rather than using a French terminology. So, really, we should be talking about Czuber's text.

In short, the structure of the text is very classical with no surprises. We can detect the same style found in Bertrand's chapters of the treatise, if less pronounced, but the same detachment with respect to the Laplacian tradition, more or less. The outline is the same Czuber maintains in the first book of his treatise. After a brief debate on the foundations, the straight calculation of probability by the rules of total and compound probability is shown without any anxiety over the unsound basis of the mathematics inherent to Laplacian reasoning. Then we broach the repeated proofs and theorems of Bernoulli and Poisson, thanks to the application of finite difference equations and the theory of moment generating functions of de Moivre and Laplace. But for Czuber, uncontradicted by Le Roux, this theory, "more or less forgotten today is of only historical interest" and it is necessary to substitute it with "the computation of operations" by G. Boole.

Following a presentation based on logic, consequently leading to a completely separate treatment of the a posteriori probability, that is to say what Laplace called "*probability of causes »* (adopting Bayes' rule), in addition twists the order of the historical presentation : Pascal's problem (problème des partis) is thus strangely separated from the question of mathematical expectation and from the St. Petersburg paradox (dealt with at the end of the text) which are, however, historically linked and which even pre-exist the notion of probability. And Geometric Probability precedes Bernouilli's theorem in the text, despite the latter having been released first.

What is also striking about this article is the existence of a type of presentation a longstanding formula used in probability and often used by actuaries such as H. Laurent: This takes the format of a series of puzzles or formal problems, each one solved separately and in any order as would be done in a collection of competitive examination problems: the de Moivre problem of the die with n faces (not connected

[22] Only a part of this introduction was accepted by the German editors, as there remain long sections between asteriks.
[23] Kries has replaced Cournot for the role of a supporter for objective probability with universal value.
[24] The first is a quadratic mean of discrepancies and the second a mean of absolute discrepancies.

to the finite differences), the problem of the gamblers' ruin, other games of chance (meetings, "jeu de la poule" ...). This hotchpotch of topics, in each long chapter shows that there is no ordering of arguments to replace the lack of chronological order, in sharp contrast to the works of Cournot and even Bertrand. Questions concerning the foundations and philosophical questions around the interpretation of probability[25] – are strangely absent from this article, apart from the initial comments on the work of Cournot in the French version, which misses out on all the work of the English Frequentists (Ellis and Venn). This, however, plays an important part in the logic of probability (Keynes), and in the transition from the rather subjective classical conception, to the more objective conceptions, which will subsequently support the inferential theory of Fisher and Neyman.

It seems we remain still rather reticent over this "shameful" part[26] of Mathematics, which refuses to confront directly the major controversies occuring in the history of probability calculus[27]. D'Alembert's doubts are described like "errors." The Bienaymé-Cauchy and Bienaymé-Poisson debates on the law of large numbers, on which depends the legacy of Laplacian Dogma, are mentioned without any analysis of their contents. The opposition to the principle of unsufficient reason sometimes used to set the a priori probabilities calculations is treated in the same allusive manner. The models of urns with changing contents of Poisson and Cournot are touched on but the questions concerning heterogeneity and the resultant instability, a German speciality (Lexis, Bortkiewicz), is implicitly relegated to "Mathematical Statistics." No mention is made of the British approach to dispersion (Edgeworth, Galton), although this question contains the beginnings of the radical break with probabilistic statistics of Laplace and, especially, Quetelet. The absence of the latter, who was the greatest populariser of probability calculations and the central reference for all the debates of the second half of the century concerning the founding of Social Sciences on a probabilistic basis, is the major gap of the paper. The controversies surrounding probability calculus are, perhaps, too stuffy for today's historians. One feels in the paper a touch of scientism with a clear emphasis on the shared gains of mathematical sciences.

In truth, the practical value of Probability, as Borel would have put it, or at least its use, is also missing from this article, despite the title "Mathematics Pure and Applied" adopted be the Encyclopédie. Neither the uses in the field of the physics of gases and thermodynamics introduced by Maxwell and Boltzmann, nor the work in biology by Galton and his followers, nor any of those concerning the newly founded social sciences are mentioned. To say nothing of the initial applications in finance by Bachelier, whose impact was clearly understood only much later. Now even if probability is simply considered as a measure, it is those applications which reveal the rifts or major developments in any possible interpretation. Thus is case with the notion of the mean as used by Quetelet with his invention so seminal yet controversial of the Average Man, or also the notion of chance as used by Maxwell and Darwin, in1869, with a conception more ontological than epistemological, to modelize the law of gases or that of heredity. On account of the choices made in the paper, the

[25] The debate over probability is at the same time very intense in the *Revue du Mois* and the *Revue de Métaphysique et de Morale*.

[26] Expression used by John Stuart Mill in his *Logic* (1843)

[27] But in the 1960s, William Feller was still supporting that the "philosophy of the foundations of probability "*be divorced*" *from mathematics and statistics*" and the *International Encyclopaedia of Social Sciences* (1968) deals with formal probability and its interpretation in two different papers.

introduction of randomness to provide new scientific explanations is unseen in the paper.

To sum up, the article *Calcul des probabilités*, much too short moreover (46 pages) compared to the huge articles on Geodetics or Insurance, is a very good historical introduction to the field, copiously referenced, but does not live up to its initial promise: the critical debates and the applications.

I-22. Theory of errors *(December 1908)*

Julius Bauschinger, professor of astronomy in Berlin (1896-1909) then in Straßburg (1909-1918) was the author of two articles in the German edition. The first on the "calculation of compensations"[28] (of errors) which was the source for the article " The Theory of Errors" in the Encyclopédie by Molk, and the second on "Interpolation" which the French author joined on to an article by Selivanov titled "Calculations of Differences." Henri Andoyer, the editor of the two articles was a member of the Academy of Sciences and the Office of Longitude, professor of astronomy at the Sorbonne. He was a close friend of Jules Molk[29] who gave him the task of editing the volume on Astronomy. In 1922 he went on to publish an honest if late popular version of "The scientific works of *Laplace," which* was blighted by the unfortunate preface decrying "the spirit of snobbism and adventure…obscure, ostentatious theories of the German mathematician, Einstein."

The article on the theory of errors veers little from the original German, and little separates the two versions, except a rather long list of extra references[30], written mainly to the memory of Laplace, and to Bertrand's objections to Gaussian Law. Even Gaussian notations were picked up on[31].

This article is one of the best introductions to the field of the Theory of Errors, which has an ambivalent status, associated with Mixed Mathematics during the 18th century, relevant, at times, to Physics with respect to the calculation of experimental error and, at times, with Mathematics treating error as hazard, once it had freed itself from its systematical components linked to identifiable physical causes and thus potentially susceptible to be corrected. The domain is rather dense and protean: Merriman, in 1877, had listed no fewer than 408 titles (works and articles) given to the subject, the majority of which were German, following the works of Gauss. For a pure mathematician the domain was all rather disturbing as the bases were rather unstable. As the writer of the article identified straight away, the resolution to the incompatibility between various discordant observations, corrupted with errors, is impossible unless you were adopting a method defining a priori either the kind of mean or of combination of observations to be considered (Encke's or Gauss's definition of mean), or the type of probability distribution followed by the errors (the Gauss-Laplace Law often justified, never "proven"), or the kind of minimum to be satisfied by a function of the gap between the observations and the true value (least squares, for example). However, such a principle would always be arbitrary and metaphysical (in the old sense of the word meaning coming in front of any physical analysis). Bauschinger and Andoyer are quite conscious of this, writing that one can

[28] Ausgleichungsrechnung
[29] Jules Molk was present at Henriette Andoyer's wedding (Letter of 11 March 1914)
[30] There are 65 references in the German version and, impressively, 105 in the French one.
[31] For instance, $[e^2] = e1?+ e2?+ \ldots + en^2$

easily choose any of the three principles in order to deduce the other two as a mathematical consequence. They consequently deal with the principle of means (Gauss's Law) and the method of least squares with a certain disenchantment leaving much room to the controversy over these principles. Otherwise the essay scrupulously follows the historical order in the synthesis by Laplace-Gauss: the first theory by Gauss deducing his law from the principle of the mean and the principle of maximum of the probability of observations, Laplace's theory combining a criterion of loss as an absolute value and asymptotic justification of normality; Gauss's second theory which built all the theory solely on the principle on minimisation of the quadratic mean error, finding with Laplace a vindication of the method of least squares which requires no hypothesis of normality, and which does not hide the quarrel of priority between Gauss et Legendre.
We can only just barely criticise this treatise for having started a bit late the historical outline without summoning the works of Lagrange, Simpson, Daniel Bernoulli and Laplace during the 1770s, and without giving due attention to other criteria as absolute-value norm (L1) by Boscovich and Laplace, principle of Minimax by the latter, and choice of the median by Fechner and Glaisher. We know that it was on Klein's recommendation that the references be limited to the 19th century, but here this might have been ignored, as was the case in other articles. More surprising is the lack of critical development devoted to Laplace's central limit theorem. The one line dedicated to the polemic between Cauchy and Bienaymé is not enough to give justice to the issues at stake. It would have been necessary to bring attention to the questioning of the convergence towards the normal distribution which Cauchy's counterexample emphasized, and to the restrictions discovered by Liapounov about its generality(1900)[32].
A second part of the article titled " practical problems" deals with, at the same time, the more complicated case of indirect observations (overdetermined linear model) and with conditional observations, i.e. determined by supplementary constraints such as the conditions allowing trangles to lock in Geodetics, or the aligorithms stemming from Gaussian elimination method (the notation of which is still used today) and its reformulations by determinants (Jacobi, Glaisher) or from the approximations of Seidel.

The Doolittle method (1878) though widely put to use, was not mentioned, and the thoroughness of the calculations suffer, in our opinion, from the confusion over unknown parameters with their estimators, but this is common up until the work done by Fisher on estimation (1922).
On the other hand, the beginning of the article gives an historical presentation of the tables of Laplace-Gauss distribution along with all the diversity of the measures of precision related to this distribution. Although Cauchy's method and the orthogonal polygons of Chebychev (1855) and Gram (1883) are notably missing, the text is complete and well balanced by references to the essential difficulties and controversies, including the various objections by Joseph Bertrand, albeit in a simplified form, which are the only significant French contribution mentioned.

The article on the Geodetic Triangulation by Noirel (VI-1), written after P.Pizzetti complements our previous article, containing as it does sections on the theory of error.

[32] The revision will continue in the works of Lindberg (1922) and Lévy (1937).

Noirel modified and completely restructured the long article on higher geodetics by Pizzetti based on four other German articles on Geodetics. The theory and practice of Gaussian methodology of the compensation of triangles and triangular networks adapted from the least squares method are described in both theoretical and practical aspects. It underlines the fact that "the method of the least square yields a unique solution to each individual case automatically and without any ambiguity or subjective appreciation. This could be an allusion to the decision of Polytechnique to take out the method from the syllabus in the middle of the 19th century as pointed out by Pierre Crépel[33], or again in Marie-Françoise Jozeau's thesis (1997) on the divergence between France and Germany in the practices of geodetics from 1830 to 1880. Despite the changes after 1870 marked by the French decision to sign up to the International Union of Geodesy, the rift between the two countries were, perhaps, never quite closed. The contribution to Geodetics of the work of the Belgian emphasized by M.F. Jozeau seems ignored here. However the innovation consisting of replacing the method of Borda's Circle by reiteration, or even more so by the so-called method of directions and of overviews are described in their principles on the subject and the methodology of compensation inferred.

Worthy of note, as another contribution to the two articles "Probabitity" and "Error" is the article titled "External ballistics" from the book on Mechanics (IV-21), written by C.Cranz for the German edition and adapted by E.Vallier, at least because, in considering the functional aspects of trajectories, it gives an understanding of some of the elements concerning dispersion of projectiles, reminiscent to some probability calculations whose main reference are "Mémorial de l'artillerie de la Marine" (Poisson 1830) and the works of Isidore Didion (Paris, 1858) and N. Zabudskij (Saint Petersbourg 1898). However the authors "did not go no further than pointing out what questions were raised".
A final adjunct to the theory of error can be gleaned from the articles "Reduction of astronomical observations" (VII-3) and "Astronomy and the calculation of longitude and latitude" (VII-4) in the book "Astronomy," where the methodology of least squares is demonstratively used .

I-24.Statitics *(october 1909)*
The article on statistics is undeniably the most informative to the historian of Mathematics of Chance because it is the most problematic. Ladislaus von Bortkiewicz (1868-1931), the author of the article delivered in april 1901 for the German edition, had already written several works on economic theory. He was born in St Petersbourg to a Polish family, associated with the Russians Chuprov and Markov, and had been professor in Berlin since 1901: he was, however, francophone. All of which clearly adds up to give him a solid grasp of the national cultures within the circle of statisticians in Europe. A pupil of Lexis, he was considered the main German master in what was called, after 1920, mathematical statistics. He opposed both the traditional administrative statistics as personified by Von Mayr, and the mechanical presuppositions in the statistical works of Laplace, which had been pushed by Quetlet towards an extreme cult of normal homogeneity. Bortkiewicz published in 1898 a famous text on "The law of small numbers" or, more precisely, the distribution of rare events today referred to as Poisson Distribution, and whose nomenclature and

[33] P. Crepel, "le calcul des probabilités de l'arithmétique sociale à l'art militaire", in *La formation polytechnicienne 1794-1994*, B. Belhoste, A. Dahan et A. Picon (dir), Dunod, 1994

originality, at the time, attracted no small amount of controversy. Bortkiewicz clearly represents a new school of statistics trying to found statistics on the basis of a renewed probability theory.

F. Oltramare, the author of the French version was an actuary, born to a family of Genevan Huguenots[34], a regular contributor to The Journal of Actuaries, but whose fame, once again, is not in the same rank as the author chosen by Klein. His "translation" of Bortkiewicz's article is very free, summing up in three lines what was developped over three pages in the original, and, inversely, dedicating a lot of print to clarifying his own personal ideas. Thus he starts the article with a rather long caveat :

Given the rather particular nature of the subject, it is necessary to point out the opinions determining the computation of statistical data by the calculations of probability distribution. The opinions concerning the very subject of statistics, which vary greatly from each other.

The subject covers a vast area: birth, death, susceptibility to illness, crime, etc.; the results are most often displayed in reports that may sometimes be considered to yield particular values of functions that are more or less determined. The principal task for the statistician is to quantify as best he can the actual value or the accuracy of the figures obtained, and, if need be, determine, as accurately as possible, the nature, form, and coefficients of the functions of which he has a certain number of precise values. Probability calculus supplies us with simple rules about this, but only in cases where its principles are reasonably applicable. The first thing to do is to check this (e.g. the sex ratio in births ..) If, as in this case, it is, then the theoretical and practical numbers will correspond to each other and make sense; and the task of the statistician would be more or less finished, requiring only to estimate the accuracy with which the frequency is in accordance with the examined probability. Generally, where no accordance is present, (...) the gaps that lie between these means and the set of data used to form them, even though they could be considered as accidental errors, often go beyond the limits allowed by the theory. Should we then, in this case reject outright the probability calculation, deny ourselves of its benefits or wait until the circumstances of the event are understood better? Some statisticians have failed to admit this; with a few tricks in the calculation, added to some considerations of variable probabilities they have managed to stretch out the area of application of these principles and bring together the outstanding/excess elements into a normal figure.

This time, as this long extract passage shows, the debate and controversy over the very nature of statistics, was not allowed to escape. On the contrary, it was portrayed as essential to put into pespective Bortkiewicz's contribution. Bortkiewicz, himself, in fact, did not reject this reproach, and linked his own text with the following observation: *"the opinions that have just been expressed differs slightly from the opinion of the majority of German authors engaged in broadening the principles of statistical mathematics.*

These authors maintain that the goals the statistician should set themselves are much more general. An explanation from nature must be found for the empirically obtained numbers rather than trying to fit them to some imagined construction expressed by one or several functions that have no bearings on the reality of the situation..(…) While they may acknowledge the usefulness probability calculations in statistics, they believe, from a practical standpoint, that its utility is extremely limited (...) In other

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[34] Among the members of this family, one finds a latinist, a leader of the extreme-right and a Gabriel Oltramare (1816-1906), a specialist of linear difference equations.

words, these authors deny any possibility of formulating general laws of statistics, and, instead, aim at coming up with formulae merely used as tools that can express, with reasonable accuracy, certain probabilities e.g. the risk of death as related to age or the rate of death related to age, but have none of the essential characteristics of the formulae found in analytical mechanics or mathematical physics.

His charge is not a light one and is an indication of how Bortkiewicz intended to distance himself from Laplacian dogma. We can, furthermore, observe the narrow view he has on Laplace work, reducing his contribution to three “theorems” : the convergence of a binomial distribution to a Laplace-Gauss distribution, the convergence of a difference of frequencies for two similar samples towards a common distribution, and the application of the analytical theory for the estimation of a population based on the sex ratio[35]. In these three cases, the emphasized defect is a systematic hypothesis of an independent repetition of identical Bernoulli trials, which allows Laplace to work with a model of constant urn, with a binomial distribution and its normal approximation. But in social and economic domains, well observed statistical data show that these conditions for the stability of the urn are irrealistic. Is it, therefore, necessary to throw out the baby with the bathwater and dismiss any probabilistic model when working with statistics? Bortkiewicz did not think so, and thus had also to contend with those who had lost all their faith in probability calculations, and declare straight out, like his first master Knapp, that statistics are too complex to entrust the specialists of probability with them. Leaning on Poisson and Bienaymé’s hypotheses on "causal variation " and "causal duration ", but especially on Lexis’ theory of dispersion[36], he used the tests of the latter to confirm that, excepting issues of demographic statistics (especially the famous sex ratio at birth) "*there is often a considerable discrepancy between the results obtained from the theory and the statistical facts (...) and the whole construct of P.S. Laplace is rendered effectively useless.*" Mercifully, "*it seems still possible to incorporate theory with experience in a large number of statistical situations, thanks to the use of variable probabilities,* and so maintain the usefulness of probability calculations.

However, Bortkiewicz did not really confirm this opinion in his paper, but mostly refers to Fechner and Pearson’s works. Moreover, prefering to follow Knapp and Zeuner, he develops, in the second part of the text, the "special problems " concerning the compilation and use of mortality tables, in which several "biometrical functions" (accidental death rate, mortality coefficient, average lifetime, probable lifetime...) are mathematically based on the same function V(x,t) continuous and differentiable from those still living on date x, born between 0 and t , and all this without any recourse to probability calculations, and without the slightest question of any statistical adjustment of the given formulae, two gaps, pointed out by Oltramare that seem rather paradoxical given the position taken by the author. Everything in the article was written as if the two ideas about the discipline, the old and the new, the former, focused on demography , and employing a simple adjustment of functions, the latter based on a probabilistic theory of inference, both expanded by Bortkiewicz,

[35] This third "theorem" well developed by Bortkiewicz was never taken up by Oltramare, who considered it only of historical interest.

[36] These hypotheses are much more developed in Bortkiewicz’s original text than Oltramare would let us believe (Cf the long note 13 on Poisson, Cournot and Gavarret, and hìs treatment of the unity of causes by Cournot).

coexisted happily and separately, without any real exchange. In truth this duality is very present during te second half of the 19th century amongst the economists, from Cournot via Edgeworth to Keynes.

Thus the article on statistics is perfectly placed at the centre of the arguments running at the time concerning the new principles underlying statistics, between administrative or descriptive uses and mathematical statistics whether based or not on probability, and dealing with a probability that lies between the Laplacian tradition of the major causes and the Lexisian tradition of "*Chancen systems.*" Statistics, a subject jostled about by the very sectors of its deployment, which have often constituted its "core usage": Practical Astronomy, Demography, the profession of the Actuary, Statistical Economics and Economic Mathematics. And it is this very position, at the crossroads in the central debate of its time that makes this article so interesting to present day historians.
An uninformed reader, trying to find a balanced overview of the whole domain from this article is liable to miss the distortion created by this point of view. Whole swathes have been left out, just as much from the period preceding 1900, as to what was happening at this moment in other places. The absence of any historical presentation of pre-Laplacian Statistics (the German *Staatkunde* and the French *Arithmétique politique*), the insufficient survey of the work of Laplace, the absence of any reference to Condorcet, or Quetelet[37] who was, however, the person most responsible for the misuse of the hypothesis of homogeneity opposed by the German School, the omission of the rejection of the British research concerning the theory of species as developed by Ellis and Venn, and of the theory of dispersion by Edgeworth and Galton, is disturbing. But we also wonder about the work concerning the theory of opinion poles by, amongst others, Kiaer and Bowley under the auspices of the International Statistical Institute, work concerning the adjustment of frequency curves by Pareto, March, Bowley and Pearson, or again the invention of correlation and of regression by Galton and Pearson, which find no place in the article, though they were both founders of the field of Mathematical Statistics. A close comparative study with contemporary treatises by Benini (1906) and Laurent (1908), or, more widely known, Bowley (1902) and Yule (1911) would show the scale of the missing domains within the discipline.

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I-25. Assurances sur la vie *(August 1911)*

Without doubt, one of the applications of Statistics, the application to insurances, is largely developed in a paper by H. Poterin du Motel based on G. Bohlmann : "Technique de l'assurance sur la vie" (I-25), which is, according to the author, the most successful model of all the types of insurance. Poterin du Motel, former pupil of the Ecole Polytechnique, was a member of the French Institute of Actuaries, founded in 1890 (on the ruins of the former Cercle des Actuaires of 1872). It is worth noting that the swing to the left in the politics of the time (Bloc des gauches) in France marked the apogee of the social state policy, very propitious for the trade organizations, the education of social economics, and insurance: the various related subjects taught at the CNAM were grouped together under the one chair of Insurance and Social Care handed to Léopold Mabilleau, and then to the polytechnicien and

[37] Quetelet is quoted just once, and that for a formula of which he has little to do with.

actuary René Risser[38] after 1927. Poterin du Motel published in 1899 a "*Theory of life insurance,*" which became the authoritative text[39]. The present article has adopted, roughly, the same structure as this text, the mathematical development, however, is richer at the expense of the description of the basics (probability, games...) and the everyday tables used by different companies. Contrary to other articles the *Encyclopédie,* does not emphasise the historical outlook. Starting from the idea of probability of death rate in eight premises giving the hypothesis of the calculations (of which Bohlmann is an expert), moving on to the different types of graphic, mechanical, and, especially, analytical adjustements: the Pearson method, Chebychev's series, Gompertz, Makeham and Quiquet's distribution. In the second part, the gross and net premiums investigated, that is, the amounts of annuity obtained, combining different form of mortality distributions, different formulae for determining premiums, and rules about updating. A third part deals with premium including or excluding expenses, which take administrative costs into account, cancellation, and dividends to obtain the concept of mathematical reserve. A final, seemingly original part touches on the theory of company risk, based on Laplace's central limit theorem, revised and extended by Liapounov (1900). This question, however, would be considerably developed further by, especially, the Swedish school (Lundberg, Cramer) not quoted in this article. To sum up, this article gives a good survey of the mathematical theory concerning insurance, as it has developed amongst the actuaries, especially in the Institut des Actuaires Français, whose Bulletin is useful to complete this article.

IV-2. Mécanique statistique *(March 1915)*

Finally, we shall say a few words about the very long article by Paul and Tatyana Ehrenfest (IV-2) on The conceptual foundations of the statistical approach in mechanics which deal with, in detail, studies," *which are far from the creation of a systematically developed discipline and which should rather be regarded as a collection of essays clarifying the former studies on the concepts of probability in the theory of gases* ".

Paul Ehrenfest[40], Professor of physics in Leiden, was a student of Boltzmann, and thus exposed to Boltzmann's fundamental theory concerning the movement of gases (1876) which was, according to the author, the point of departure of his own study in mechanical statistics even if he goes on to recall works by Krönig (1856), Clausius (1857), and Maxwell (1859), who were the first to postulate a probability distribution to form a model of molecular movement, and the formula of ideal gases, $pv = RT$. Maxwell was the first to make the hypothesis (inspired by Quetelet) of a three-

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[38] Another chair *Assurances sociales* was given to the jurist Etienne Antonelli in 1932, and Risser's chair, suppressed in 1937, was replaced by three courses as Dubourdieu's one on the *Théorie mathématique des assurances*. See Bénédicte Zimmermann et Bernard Bru's notices on Risser and Dubourdieu in the biographical dictionnary of professors at the Conservatoire National des Arts et Métiers.

[39] For instance, for P.J. Richard, author of a *Théorie mathématique des assurances* published in 1922 in the collection *Bibliothèque de mathématique appliquée* of M. d'Ocagne.

[40] Martin J. Klein devoted an intellectual biography to him: *Paul Ehrenfest, the Making of a Theoretical Physicist*, Amsterdam, North Holland, 1970. Ehrenfest had a strong political and philosophical influence over the young socialist mathematicians who had been his students, as the economist Jan Tinbergen, the sociologist of mathematics Dirk Struik, and the epistemologist Jan Burgers (see Gerard Alberts, "On connecting Socialism and Mathematics", *Historia Mathematica*, 21, 1994.)

dimensional gaussian distribution of the velocity distribution of particles in a gas at rest; the distribution was further generalised by Boltzmann (1868) for the case of particles placed in a gravitational field; he showed (1872) that this distribution is the only stationary one. Paul and Tatyana Ehrenfest insist at length on the paradox of explaining an irreversible occurrence by a model that is reversible, and so the advances made by Boltzmann's *H* theory for irreversible, non-stationary occurrences leading to a kinetic interpretation of thermodynamics, which maintains that entropy is constantly increasing.

The objections to this theory by Loschmidt followed by Zermelo lead to its reformulation through a model with a phase space, and to a discussion about ergodic systems in which the trajectory occurs in a period long enough to pass through all the energy phases (every distribution of position and speed are of equal probability). Adopting this hypothesis, Boltzmann shows that the "average velocity in the model of a gas during an indefinitely prolonged motion corresponds to the Maxwell-Boltzmann distribution". Thus do Paul and Tatyana Ehrenfest study in detail Boltzmann's results and the controversy surrounding the theorem. They explain how even the definition of ergodic systems contains contradictions The program set up by W. Gibbs (1901) is thus presented as an attempt to formulate an axiom of statistical mechanics, which was considered here as a semi-failure – *this author goes hardly beyond the work of J. C. Maxwell and L. Boltzmann*[41] *in keeping to his axiomatic* Neverthelesses the approaches adopted by Gibbs are analysed and compared to Boltzmann, on certain issues. Throughout this article the authors lean on an historical presentation to facilitate an understanding of the formalism of statistical mechanics emphasising the imprecision, the contradictions the paradoxes within the discipline, the difficulty in applying the discipline to anything other than the kinetics of gases, and finishing with the necessity "*for a wider development of the principles of statistical mechanics*." The article also points out the importance to the physical sciences according to Maxwell, with the concept (and terminology) of the "*model*," and the precise mathematical sense given by P and T. Ehrenfest. We can equally note the care taken by the authors in employing notions of "probability of an event", or "the most probable" distribution, ill-conceived in terms of frequency and sample space , and "employed loosely".
It would be an interesting path to compare the different meanings given to these expressions within the various uses of probability in mechanics. This time, the French translator, none other than Emile Borel, declined to make any commentary between asterisks, in favour of a 20 page supplement[42] devoted, in principle, "to works that appeared between October 1911 and January 1914", but which becomes the pretext for a long critique about the social status of the discipline. Borel continues straight on to present a hierarchy of the whole discipline of physical mathematics namely, Pure Mathematics, Physical Mathematics, researches into Physical Theory finishing with the "experimental work" not listed in this encyclopedia. Geometry is placed in the first category, *n* dimensional geometry, which is linked to Statistical Mechanics while n is close to 10^{24}, and the ergodic systems with regard to set theory and measure theory, two areas on which Borel published papers in 1912 and 1913. The works of Gibbs are placed in the second category. In writing about Gibbs, the author updates

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[41] Note 156, p. 235.
[42] It is in fact a second supplement, the first one, written by P. et T. Ehrenfest, being an update of their paper devoted to the studies dating between January1910 and September 1911.

the article by Paul and Tatyana Ehrenfest, by, for example, an analysis of a dissertation by Hertz on the concept of constellation or by his own studies on uncertainty in physical measurement revealing a different point of view from the authors of the German version – *The tendency towards the most probable state is not the attribute of one specific model, but is, in fact, a statistical property of the infinite number of models which correspond to a real gas after a fraction of a second* - or again with the reference to the work of Max Planck on quanta.

Physical theory, occupying the third category summons up the work on thermal radiation (Poincaré, Lorentz), Brownian motion (Haas Lorentz), radioactivity (Marie Curie) and the two general papers "*les Atomes*" by Jean Perrin and his own work "le Hasard"[43]. As we can see this article is of great interest, offering the opportunity to read the erudite observations of a science in full transformation, questioning itself constantly with the differing opinions on how to interpret the probability calculations. The practical value of probability, that is its special place in the collection of scientific theories, an idea developed from the very first issues of *la Revue du mois* (1906) by Borel, underlying the supplement, which thus goes beyond its place as part of the *Encyclopédie* to become an important work in its own right delivering such a large corpus on the work of Borel and his own personal exegeses[44].

By way of a conclusion we would say that the Encyclopédie of pure and applied mathematical sciences is an excellent tool for those who are involved in probability and statistics, taking into account especially the quality of the papers, the wealth of its references and its introduction to the history of the disciplines covered. As much of interest to present day historians, made aware of its lacunas and distortions, as for the complete beginner looking for an introduction to the field.. In the end, it does not live up to its promises. Clearly Jules Molk was not able to attract the top specialists in their field as Felix Klein had done in Germany, and the expected debate between the two scientific cultures, French and German, between the different competing paradigms, subsequently suffered somewhat because of this. By side-stepping the overly publicised controversies around the fundamental bases or interpretation of the subject (article "probabilité"), or conversely by being overly tied to a particular school (article "statistics"), the *Encyclopédie* does not faithfully mirror the dynamism present during this key period around the mathematics of chance when, it is true, the major tranformations would take place in the subsequent two decades.
Another disappointment is the way the applications are treated. The intention to lay emphasis on applied mathematics is only partially achieved: some are more than adequate in specific, often over-long articles, given the rather technical unproblematic nature of the domain (insurance, actuary..), while others are totally absent from the Encyclopédie, often because they lay outside the scope of, for example, engineering science: the very readership aimed for by the editors. And again, in the case of the biological and social sciences, excluded outright from the Encyclopédie for the stated reason that they were not yet considered as a science and even less a branch of applied mathematics Furthermore, when a field of application is not forgotten (astronomy,

[43] Emile Borel, *Le Hasard,* First Ed. 1914. Let us also recall two volumes of the monumental *Traité du Calcul des probabilités et de ses applications* (1925-40), written by Francis Perrin and Borel himself, are devoted to statistical mechanics.
[44] See for instance Maurice Fréchet, "Emile Borel", *Encyclopaedia Universalis*, or also Stéphane Callens, *Les maîtres de l'erreur*, PUF, 1996.

geodetics..) the pure and applied mathematics are dealt with in separate articles, with the result that the borrowings, the directions, the useful analogies, the feedback from applications into the mathematics itself are not analysed.

The sole exception to this of course is statistical mechanics where the theoretical questions and the mathematical concepts adopted or created thoroughly overlap.
The fundamental articles on probability and statistics are reduced to a rather undersized, simple, formal introduction and yield a neither honest nor complete overview of the domain, nor does it offer an introduction to the principal problems, which almost always springs from articulating the theory to its domain of application.