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SCATTERING OF VORTICES IN THE ABELIAN HIGGS MODEL

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Abstract. We study the scattering of vortices in the Abelian (2+1)-dimensional Higgs model. We show that in the case of the symmetric head-on collision of N vortices their trajectories are rotated by the angle π/N after the collision.

1. Introduction

In this paper we study the scattering of vortices in the Abelian (2+1)-dimensional Higgs model. The vortices, we are considering, are solutions of the vortex equations, arising in the superconductivity theory. They are given by smooth pairs (A, Φ), consisting of the (electromagnetic) gauge potential A and the (scalar) Higgs field Φ on $\mathbb{C}$. Such solutions are parameterized (up to gauge equivalence) by the zeros of the Higgs field Φ, so the moduli space of N vortices can be identified with $\mathbb{C}^N$. The dynamics of vortices in $\mathbb{C}$ is governed by the hyperbolic Ginzburg–Landau action functional. The dynamics of N vortices may be described approximately by geodesics of $\mathbb{C}^N$ in the metric, determined by the kinetic energy of the model. Unfortunately, this metric cannot be computed explicitly. But in a special case of the symmetric scattering of N vortices we can show, without using the explicit form of the metric, that after their head-on collision the configuration of vortices looks the same, only rotated by the angle π/N. In particular, in the case of two vortices, their trajectories are rotated by the angle $\pi/2$ after the head-on collision, so we have the right-angle scattering. This result was already obtained earlier in a number of papers (see [1, 4–6]).

2. Vortex Solutions in the Abelian Higgs Model

The *two-dimensional Abelian Higgs model* is determined by the following *action functional*

$$V(\alpha, \Phi) = \frac{1}{2} \int\limits_{\mathbb{R}^2} \left(|\mathrm{d}_\alpha \Phi|^2 + F_{12}^2 + \frac{\lambda}{4}(|\Phi|^2 - 1)^2 \right) \mathrm{d}x\mathrm{d}y \tag{1}$$