JGSP **12** (2008) 15–26

JOURNAL OF
Geometry and Symmetry in Physics

CONSTANT MEAN CURVATURE SURFACES IN EUCLIDEAN AND MINKOWSKI THREE-SPACES

DAVID BRANDER, WAYNE ROSSMAN AND NICHOLAS SCHMITT

Communicated by Ivaïlo M. Mladenov

Abstract. Spacelike constant mean curvature (CMC) surfaces in Minkowski 3-space $\mathbb{L}^3$ have an infinite dimensional generalized Weierstrass representation. This is analogous to that given by Dorfmeister, Pedit and Wu for CMC surfaces in Euclidean space, replacing the group $\mathrm{SU}(2)$ with $\mathrm{SU}(1,1)$. The non-compactness of the latter group, however, means that the Iwasawa decomposition of the loop group, used to construct the surfaces, is not global. The construction is described here, with an emphasis on the difference from the Euclidean case.

1. Introduction

This article expands on the content of a talk given at the X-th International Conference on *Geometry, Integrability and Quantization*, held in Varna 2008. It discusses the generalized Weierstrass representation for constant mean curvature surfaces in both the Euclidean and in the Minkowski three-space, with attention given to the difference between these cases. Detailed proofs and further results on the Minkowski case will appear in a forthcoming article by the authors [2].

2. Constant Mean Curvature Surfaces in Euclidean Three-space

2.1. Minimal Surfaces

Constant mean curvature surfaces are mathematical models for soap films and other fluid membranes. A special case is a *minimal surface*, where the mean curvature is zero. Mathematically, the study of minimal surfaces has been greatly assisted by the well-known *Weierstrass representation*, which allows one to construct all minimal surfaces from pairs of holomorphic functions via a simple formula. It is based on the fact that the Gauss map of a minimal surface is *holomorphic*, together with the fact that a CMC surface in general is determined by its