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EXACT CURVATURE ELASTICA OF A THIN CANTILEVER UNDER TERMINAL LOADS

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Abstract. A thin, flexible, one side-built-in rod under a concentrated terminal force is studied in its elastic equilibrium configuration. In order to make the problem more tractable, a proper set of state variables is chosen, facing with a second order, nonlinear, but *autonomous* boundary value problem, in the rotation $\varphi(s)$ pertaining to each s-section. The search of the free end rotation φ_0, following the isoperimetric assumption, leads to a numerical sub-problem inside the main BVP. Furthermore, if $x(s)$ and $y(s)$ mean the elastica coordinates parametrized on the arclength s, one obtains $x'(s)$ and $y'(s)$ as elliptic functions of s. Finally, some minor changes have been shown in order to pass from a loading force to a more general free-end load combination, consisting of a force and a couple.

1. Introduction

The bending of an elastic beam under various loads provided the first problem of practical importance to theory of elasticity, and fed the initial development of beam theory, due to the names of Jakob and Johann Bernoulli, Euler and Lagrange. One of the simplest ones to consider is the loaded beam rigidly supported at one end only: a simple *cantilever*, whose *exact* shape is required when inflected under terminal loads. *Exact* means: taking into account its exact curvature. Then we wish to describe mathematically the shape assumed by a flexible rod built-in at one end, and affected at its free end by a general co-planar load consisting of a slender force and a bending couple. Such a cantilever is commonly treated assuming an approximate curvature expression and obtaining a third order deflection curve; but our viewpoint is different, and aims at giving an exact solution to the problem, assuming the exact curvature. As we told, such a problem is a very old one: its birth preceded the existence of calculus tools capable of solving it.