JGSP **14** (2009) 35–49

JOURNAL OF
Geometry and Symmetry
in Physics

REMARK ON THE INTEGRALS OF MOTION ASSOCIATED WITH LEVEL k REALIZATION OF THE ELLIPTIC ALGEBRA $\mathrm{U}_{q,p}(\widehat{\mathfrak{sl}_2})$

TAKEO KOJIMA AND JUN'ICHI SHIRAISHI

Communicated by Gaetano Vilasi

Abstract. We give one parameter deformation of level k free field realization of the screening current of the elliptic algebra $\mathrm{U}_{q,p}(\widehat{\mathfrak{sl}_2})$. By means of these free field realizations, we construct infinitely many commutative operators, which are called the nonlocal integrals of motion associated with the elliptic algebra $\mathrm{U}_{q,p}(\widehat{\mathfrak{sl}_2})$ for level k. They are given as integrals involving a product of the screening current and elliptic theta functions. This paper give level k generalization of the nonlocal integrals of motion given in [1].

1. Introduction

One of the results in Bazhanov, Lukyanov and Zamolodchikov [4] is construction of field theoretical analogue of the commuting transfer matrix $\mathbf{T}(z)$, acting on the highest weight representation of the Virasoro algebra. Their commuting transfer matrix $\mathbf{T}(z)$ is the trace of the image of the universal R-matrix associated with the quantum affine symmetry $\mathrm{U}_q(\widehat{\mathfrak{sl}_2})$. This construction is very simple and the commutativity $[\mathbf{T}(z), \mathbf{T}(w)] = 0$ is direct consequence of the Yang-Baxter equation. They call the coefficients of the Taylor expansion of $\mathbf{T}(z)$ the nonlocal integrals of motion. The higher-rank generalization of [4] is considered in [5, 6]. The elliptic deformation of the nonlocal integrals of motion is considered in [1]. Bazhanov, Lukyanov and Zamolodchikov [4] constructed the continuous transfer matrix $\mathbf{T}(z)$ by taking the trace of the image of the universal R-matrix associated with $\mathrm{U}_q(\widehat{\mathfrak{sl}_2})$. However, it is not so easy to calculate the image of the elliptic version of the universal R-matrix, which is obtained by using the twister [10]. Hence the construction method of the elliptic version [1] should be completely different from those in [4]. Instead of considering the transfer matrix $\mathbf{T}(z)$, the authors [1] give the integral representation of the integrals of motion directly. The commutativity of the integrals of motion is not consequence of the Yang-Baxter equation. It is consequence of the commutative subalgebra of the Feigin-Odesskii