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BOOK REVIEW

Integrable Systems in Celestial Mechanics, by Diarmuid Ó Mathúna, Birkhäuser, Boston – Basel – Berlin 2008, x + 234 pp., ISBN 978-0-8176-4096-5

The two fixed centers problem is positioned somewhere in between the integrable two-body problem and the non-integrable restricted three-body problem and plays the role of the natural approximation for the latter one. In the last century it turns out that this old Euler problem can be used unexpectedly in a somewhat different direction: to approximate with a enough high precision the motion of the satellite of oblate planet. And this is the key issue of the whole book under consideration.

Trying to describe the book let us start with Chapter 0, which serves as a general introduction and presents detailed historical review of the problem and its development. The author builds so to speak a "ladder" by representing the three stage evolution of an integrable description of the motion dynamics in gravitational fields of different kinds. Grades of the ladder are as follows:

$$\text{the Kepler (two-body) problem} \longrightarrow \text{the Euler problem} \longrightarrow \text{the Vinti problem}$$

which actually were formulated by Darboux as mentioned by the author (see also the review paper [1]).

Regarding the Vinti's problem which concerns the satellite altitude dynamics and to appreciate the progress, one has to consult some more detailed descriptions of the intermediate steps. One excellent book within a scope under discussion and dealing mainly with the Hamilton–Jacobi approach, its application to the perturbation theories and summarizing, among others, the results of [3–8], is that by Demin [2]. For the oblate planet satellite dynamics and the intermediate orbits, one should confine the results presented in [9–11].

The real start of the book is Chapter 1 dealing with a descriptions of a few facts from Lagrangian mechanics and Liouville systems with separable configurational variables. Then in Chapter 2 an elegant vectorial treatment of the Kepler problem follows.

The major and central parts, Chapters 3 and 4, of the book cover the Euler problem description. The key point is that the latter one is considered as a natural generalization of the Kepler problem. So the Chapter 3 on the Kepler problem is a starting point to investigate the orbits of the Euler problem. All analytical transformations actually are steps of a certain unification process bringing the intermediate results to the final numeric procedures.

Concretely, the author makes use of the standard generic pattern

$$\left(\frac{\mathrm{d}y}{\mathrm{d}\tau}\right)^2 = \left(1-y^2\right)\left(1-k^2y^2\right) \tag{1}$$

and had applied it throughout the book to various dynamical cases. This equation has a well known solution

$$y(\tau) = \mathrm{sn}(\tau+\tau_0, k) \tag{2}$$

which evidently uniformize the corresponding elliptic curve (1).
One has to notice that the author demonstrates the power of analytical technique of high complexity in order to transform and reduce different cases of Euler problem integration to the generic case above.

Taking into account that solutions of the problem of two fixed centers and its generalization are expressed via Jacobian elliptic functions we note that the author is quite careful in keeping all obtained expressions of real type. In this setting one can easily apply the Landen transformation to compute the elliptic functions.
Alternatively, it is possible and enough efficient to apply the Weierstrass elliptic functions. In this case one can apply the uniform and fast technology for computing the elliptic functions by making use of the theta-functions as described in [12]. Moreover, in this way one can use the Weierstrass $\wp$-function invariants as new integrals of the problem, and finally construct the corresponding perturbations theory in a proper way.

Simultaneously one should understand that the book is aimed at enough experienced readers in celestial mechanics who know the background lying behind the whole complex analytical problems. The algebraic geometry tools like theta-functions and unimodular groups make it possible to treat the matter of the Euler problem from more general and surprisingly more simple viewpoint which is evidently due to its geometric nature. The geometric illustrations characterizing orbits of different kinds can be found in the Appendix to the book written by Vincent G. Hart and Seán Murray.

One can observe also a wide variety of different classes of solutions carefully categorized by the author into some virtual classification tree such that the book becomes somewhat like a directory reference for the problem, as frequently specifies the results up to some prescribed order of the small parameter (e.g., scaled distance between the fixed centers). The author manages all algebraic equations in an excellent manner, artistically so to speak. Thus one can surely confirm the book is written in a best traditions of the analytical celestial mechanics.

The final part of the book, Chapters 5 and 6, comprises a description of the satellite dynamics from the generalized two fixed centers problem (Vinti) viewpoint. Here the author still stands on the same position to apply the elliptic curve pattern mentioned above. And what is interesting is that at the very end of the book the author covers yet another known issue of the critical inclination arising in a satellite dynamics in the frame of perturbation theories.

Before to conclude this review let us note that the book as whole is prepared in the traditions of high publishing quality. Mistakes or even misprints are almost absent which can not be expected especially if one takes into account the numerous multilevel analytical constructions scattered throughout the book.

The final conclusion is that the book is highly recommended to all interested scientist and students and that probably very soon it will become a standard reference in the field.

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