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BALLOONS, DOMES AND GEOMETRY

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Abstract. In this survey, we present physical and mathematical ideas surrounding the geometry of balloons and of pneumatic domes.

1. Introduction

The fact that geometric principles and tools are fundamental to structural engineering is well-appreciated in the engineering world, but little-known in Mathematics. The point of the present work is to survey the types of mathematical and physical arguments that arise in the study of inflatable membranes in an attempt to make the subject accessible to mathematicians. In previous works (see [19, 20]), we have considered the shape of a Mylar balloon in the context of surface theory. While there we presented some physical arguments that we hoped would place the balloon in a broader context, here we want to carry this further to show the beautiful connections between structure and geometry. Of course, we are not the first to do this. The bibliography contains references (such as [2–5, 9, 13, 15, 17, 23, 24]) that will at least start the reader on the path to investigating an area where differential geometry plays a leading role in our understanding of practical engineering problems.

Membranes are a special class of structures; they are made up of thin "films" which carry load (almost) exclusively in tension (due to a lack of bending stiffness). Examples of membranes include soap bubbles and balloons (especially Mylar balloons *and* large scientific balloons). With the advent of new technologies, membranes have found a place in structural engineering as designs for pneumatic domes (enclosing arenas for instance). Inflated membranes are structures where the membrane remains in place due to some internal pressure. Thus, the forces acting on an inflated membrane are the pressure difference across the membrane and the tension in the membrane (as in a soap film or bubble).

For many years, NASA has been interested in inflatable membranes because they have small mass and can be transported while deflated — thus occupying a small