



TORIC KÄHLER METRICS: COHOMOGENEITY ONE EXAMPLES OF CONSTANT SCALAR CURVATURE IN ACTION-ANGLE COORDINATES

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Communicated by Izu Vaisman

Abstract. In these notes of the mini-course given by the author at the XI-th Conference on Geometry, Integrability and Quantization, Varna, Bulgaria, 2009 after the introduction of the toric Kähler geometry, we present Calabi's family of $U(n)$ -invariant extremal Kähler metrics in symplectic action-angle coordinates and show that it actually contains, as particular cases, many interesting cohomogeneity one examples of constant scalar curvature.

1. Introduction

In 1982 Calabi [7] constructed, using local complex coordinates, a general four-parameter family of $U(n)$ -invariant extremal Kähler metrics, which he used to put an extremal Kähler metric on

$$H_m^n := \mathbb{P}(\mathcal{O}(-m) \oplus \mathbb{C}) \rightarrow \mathbb{P}^{n-1}$$

for all $n, m \in \mathbb{N}$ and any possible Kähler cohomology class. In particular, when $n = 2$, on all Hirzebruch surfaces.

The main goal of these notes is to present Calabi's general family in local symplectic action-angle coordinates, using the set-up of [1, 2] for toric Kähler geometry, and show that it actually contains other interesting cohomogeneity one Kähler metrics as particular cases (see also [22]). These include:

- the Fubini-Study, flat and Bergman Kähler-Einstein metrics of constant holomorphic sectional curvature (positive, zero and negative, respectively).
- the complete Ricci flat Kähler metric on the total space of

$$\mathcal{O}(-n) \rightarrow \mathbb{P}^{n-1}$$

for all $n \in \mathbb{N}$ and any possible Kähler cohomology class, constructed by Calabi [6] in 1979.