



ON GENERALIZED FOURIER TRANSFORM FOR KAUP-KUPERSHMIDT TYPE EQUATIONS

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Abstract. We develop the Fourier transform interpretation of the inverse scattering method for nonlinear integrable evolution equations associated with a \mathbb{Z}_3 reduced Zakharov-Shabat system for the Lie algebra $\mathfrak{sl}(3, \mathbb{C})$. A simple representative of this integrable hierarchy is the well-known Kaup-Kupershmidt equation. Our results admit a natural extension for nonlinear equations connected to a deeply reduced Zakharov-Shabat system related to an arbitrary simple Lie algebra.

1. Introduction

The Kaup-Kupershmidt equation (KKE) is a $1 + 1$ nonlinear evolution equation

$$\partial_t f = \partial_{x^5} f + 10f \partial_{x^3} f + 25 \partial_x f \partial_{x^2} f + 20f^2 \partial_x f \tag{1}$$

where $f \in C^\infty(\mathbb{R}^2)$. It is integrable by means of the inverse scattering method: it is related to a third order spectral problem [11]

$$(\partial_{x^3} + 2f \partial_x + \partial_x f)y = \lambda^3 y$$

for some smooth function $y(x, t, \lambda)$. It can be easily transformed into a first order one [5] but related to the algebra $\mathfrak{sl}(3, \mathbb{C})$

$$(i\partial_x + q - \lambda J)\psi = 0, \quad q = \begin{pmatrix} u & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -u \end{pmatrix}, \quad J = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

with some additional symmetries imposed and for an appropriately chosen new dependent variable u . Thus a system of the Caudrey-Beals-Coifman (CBC) type occurs. This is a typical situation when transforming a scalar differential operator of order n to a $n \times n$ matrix one, see [4]. The spectral theory, the direct and inverse scattering transform for such systems have been worked out [1–3, 7] and more recently [9].