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RECURSION OPERATORS AND REDUCTIONS OF INTEGRABLE EQUATIONS ON SYMMETRIC SPACES

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Abstract. We study certain classes of integrable nonlinear differential equations related to the type symmetric spaces. Our main examples concern equations related to **A.III**-type symmetric spaces. We use the Cartan involution corresponding to this symmetric space as an element of the reduction group and restrict generic Lax operators to this symmetric space. Next we outline the spectral theory of the reduced Lax operator L and construct its fundamental analytic solutions. Analyzing the Wronskian relations we introduce the 'squared solutions' of L and derive the recursion operators by three different methods.

1. Introduction

One of classical models of statistical physics is provided by Heisenberg's equation

$$\mathbf{S}_t = \mathbf{S} \times \mathbf{S}_{xx}, \qquad \mathbf{S}^2 = 1 \tag{1}$$

which describes the behavior of an one-dimensional ferromagnet characterized by a spin vector $\mathbf{S}(x,t)$ in a closest neighbors approximation. By making use of the Lie algebras isomorphism

$$e_i \leftrightarrow \sigma_i, \qquad i = 1, 2, 3$$

where σ_i are Pauli's matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

one is able to set equation (1) into a matrix form

$$\mathrm{i}S_t = \frac{1}{2}[S, S_{xx}], \qquad S(x,t) = \sum_{k=1}^{3} S_k(x,t)\sigma_k, \qquad S^2 = \mathbb{1}. \tag{2}$$