JGSP 20 (2010) 35–48

JOURNAL OF
Geometry and Symmetry
in Physics

IN-PLANE OSCILLATIONS OF A RING DRIVEN BY A SOAP FILM CATENOID

PAVEL GRINFELD

Communicated by Gregory Naber

Abstract. In this paper, we calculate the frequency of small oscillations of a ring confined to a plane that, along with a fixed ring of the same size, supports a soap film catenoid. The restoring force is provided by the film's surface tension. We assume that the soap film is massless and continuously assumes the shape of minimal area. Mathematically, the problem is equivalent to calculating the second derivative of the total area with respect to the displacement of the ring. The calculus of moving surfaces is extensively used in the presented calculation.

1. Introduction

In this paper we propose to study the harmonic oscillations of a ring confined to a plane under the influence of a massless soap film that continuously assumes the surface of minimum area. The presented calculation demonstrates the stability of the equilibrium catenoid with respect to an in-plane shift of the supporting rings. The calculation relies on the calculus of moving surfaces.

The catenoid has historically played an important role in the calculus of variations and the study of minimal surfaces. In 1744, Euler showed that the catenoid is a minimal surface of revolution in his celebrated work on the calculus of variations [2]. Since then, a number of embedded (that is, non-self-intersecting) surfaces have been discovered analytically: helicoid by Jean Meusnier in 1776, Scherk surfaces by Heinrich Scherk in 1834, Riemann surfaces in 1860 and the Schwarz quadrilateral in 1890. In recent decades, a number of minimal surfaces of genus greater than zero have been discovered, including surfaces by Enneper, Catalan, Henneberg and Costa. Minimal surfaces continue to be an active area of research (see the works of Meeks, for example [6] and references therein).

Minimal surfaces play an important role in modern physical applications including the study of fluid films [1], polymer networks [10], crystallography and protein structures [5] and smectic-A [7] and other liquid crystal phases.