



A BETTER CALCULUS OF MOVING SURFACES

PAVEL GRINFELD

Communicated by Gregory L. Naber

Abstract. We introduce $\dot{\nabla}$, a new invariant time derivative with respect to a moving surface that is a modification of the classical δ/δ -derivative. The new operator offers significant advantages over its predecessor. In particular, it produces zero when applied to the surface metric tensors $S_{\alpha\beta}$ and $S^{\alpha\beta}$ and therefore permits free juggling of surface indices in the calculus of moving surfaces identities. As a result, the table of essential differential relationships is cut in half. To illustrate the utility of the operator, we present a calculus of moving surfaces proof of the Gauss-Bonnet theorem for smooth closed two dimensional hypersurfaces.

1. Introductory Remarks

The $\delta/\delta t$ -derivative has been the cornerstone of the calculus of moving surfaces, an extension of tensor calculus to deforming manifolds. Here, we introduce a new invariant time derivative $\dot{\nabla}$ which, because of the several conveniences that it offers over the $\delta/\delta t$ -derivative, will replace it in the future. As a demonstration of the new operator's utility, we will give a proof of a special case of the Gauss-Bonnet theorem by showing that the integral of Gaussian curvature over a closed surface remains unchanged under smooth surface evolutions that preserve its topology. Naturally, the Gauss-Bonnet theorem, which is a statement about the intrinsic geometry of a surface, is celebrated for its much deeper topological meaning. On the other hand, the calculus of moving surfaces (CMS) is extrinsic by its very nature. Nevertheless, the proof represents an effective application of the calculus of moving surfaces to a problem in which there are no moving surfaces.

In differential geometry on stationary manifolds, invariance is achieved by introducing the covariant derivative ∇_α that replaces the partial derivative $\partial/\partial S^\alpha$ [13], [8], [7], [9]. On moving surfaces, the calculus of moving surfaces plays a role analogous to that of tensor calculus on stationary surfaces. The central operator in the calculus of moving surfaces is the $\delta/\delta t$ -derivative, which is the analogue of the covariant derivative ∇_α . The calculus of moving surfaces has proven effective at analyzing problems in dynamics [2], [5], boundary perturbation [4], shape optimization [3] and, as seen in this paper, intrinsic problems on stationary surfaces.