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# A CLASS OF LOCALIZED SOLUTIONS OF THE LINEAR AND NONLINEAR WAVE EQUATIONS

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**Abstract.** Following the tradition of the nano and picosecond optics, the basic theoretical studies continue to investigate the processes of propagation of femtosecond and attosecond laser pulses through the corresponding envelope equation for narrow-band laser pulses, working in paraxial approximation. We should point out here that this approximation is not valid for large band pulses. In air due to the small dispersion the wave equation as well as the $3D + 1$ amplitude equation describe more accurate the pulse dynamics. New exact localized solutions of the linear wave and amplitude equations are presented. The solutions discover non-paraxial semi-spherical diffraction of single-cycle and half-cycle laser pulses and a new class of spherically symmetric solutions of the wave equation. The propagation of large band optical pulses in nonlinear vacuum is investigated also in the frame of a system of nonlinear wave vector equations. We obtained exact vector solution with its own angular momentum in the form of a shock wave.

## 1. Introduction

With the progress of laser innovations it is very important to study the localized waves, especially pulses which admit only few cycles under the envelope and pulses in half-cycle regime. One important experimental result is that even in femtosecond region, the waist (transverse size) of an initially non modulated laser pulse continues to satisfy the Fresnel's law of diffraction. The parabolic diffraction equation governing Fresnel's evolution of a monochromatic wave in continuous regime (CW regime) is suggested for first time by Leontovich and Fock [5, 12, 13]

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + 4\mathrm{i}k_0 \frac{\partial w}{\partial z} = 0. \tag{1}$$

The solutions of equation (1) possess circular fundamental Gaussian mode [3] as well as higher-order modes, such as Laplace-Gauss [1,9,17], Helmholtz-Gauss and Bessel-Gauss [2, 6, 7, 9] beams. On the other hand the optics of laser pulses, especially in the femtosecond (fs) region operates with strongly polychromatic waves