



HARMONIC SPHERES AND YANG–MILLS FIELDS

ARMEN SERGEEV

Communicated by Gregory L. Naber

Abstract. We study a relation between harmonic spheres in loop spaces of compact Lie groups and Yang–Mills fields on the Euclidean four-space \mathbb{R}^4 .

Contents

1	Harmonic Maps	2
1.1	Harmonic Self-maps of the Riemann Sphere	2
1.2	General Definition of Harmonic Maps	5
1.3	Harmonic Maps of Almost Complex Manifolds	7
2	Instantons and Yang–Mills Fields	8
2.1	Yang–Mills Equations on \mathbb{R}^4	8
2.2	Instantons	10
3	Twistor Interpretation of Instantons	12
3.1	Basic Twistor Bundle over \mathbb{S}^4	12
3.2	Atiyah–Hitchin–Singer Construction and Penrose Twistor Program	13
3.3	Atiyah–Ward and Donaldson Theorems	14
4	Twistor Interpretation of Harmonic Spheres	15
4.1	Eells–Salamon Theorem	15
4.2	Complex Grassmann Manifolds and Flag Bundles	16
4.3	Harmonic Spheres in Grassmann Manifolds: Burstall–Salamon Theorem	17
5	Atiyah Theorem and Harmonic Spheres Conjecture	18
5.1	Loop Spaces of Compact Lie Groups	18
5.2	Holomorphic Spheres in Loop Spaces: Theorem of Atiyah	19
5.3	Harmonic Spheres Conjecture	20
6	Twistor Bundle over the Loop Space	21
6.1	Hilbert–Schmidt Grassmannian	21
6.2	Virtual Flag Bundles and Harmonic Spheres in the Hilbert–Schmidt Grassmannian	22
6.3	Embedding of Loop Spaces into the Hilbert–Schmidt Grassmannian	22