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MINIMAL SURFACE WITH A CAVITY OF GIVEN PERIMETER

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Abstract. We pose a variant of the classical minimum surface problem inspired by a simple experiment with soap films: to find the surface of least area containing a cavity of given perimeter. We show that the equilibrium surface is governed by a system of two equations one of which is the zero mean curvature condition. The other equation states that the curvature of the cavity's contour is constant and that its principal normal lies in the plane tangential to the surface. A gradient descent simulation confirms the analytical equilibrium conditions and yields configurations qualitatively consistent with experiment.

1. Introduction

Soap films have for centuries been a source of beautiful mathematical problems. The mathematical problem discussed here is a variant of the classical minimal surface problem: Given a curved contour Γ in three dimensions and a positive number L, find a surface of least area that spans Γ and contains a cavity of perimeter L.

This problem is inspired by the soap film experiment in which a curved wire loop is dipped in a soap solution. A closed loop thread is then placed inside the soap film and the film on the interior of the thread is punctured. The system then relaxes to the equilibrium configuration seen in Fig. 1.

Fluid film experiments, in which the film stays intact, can only expose the net *normal* effect of surface tension. In order to observe the influence of surface tension more directly, one must expose the edge of the fluid film and let it deform due to surface tension. The described experiment does just that and thereby gives greater insight into the pointwise nature of surface tension.

Thin films have long been at the center of attention for many theorists, experimentalists and engineers. In recent decades, the number of unexpected and unexplained phenomena has actually grown and this has been reflected in the volume of publications in leading scientific journals. There is little doubt that this trend will continue. Many new problems arise from cutting edge experiments enabled by modern tools such as high speed cameras. Remarkably, simple and inexpensive experiments

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