RELATIONS AMONG LOW-DIMENSIONAL SIMPLE LIE GROUPS

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Abstract. The compact classical Lie groups can be regarded as groups of $n \times n$ matrices over the real, complex, and quaternion fields $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{Q}$ that satisfy metric- and volume-conserving conditions. These groups, $\mathrm{SO}(n,\mathbb{R})$, $\mathrm{SU}(n,\mathbb{C})$, and $\mathrm{Sp}(n,\mathbb{Q})$, are not all independent. Homomorphisms exist among some of these groups for small dimension. We review these relations by describing the Lie algebras of the compact forms and their complex extensions. Other noncompact real forms of these Lie algebras are constructed by systematic methods. The relations among all distinct real forms is presented.

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