



VISUALIZING LIGHT CONES IN SCHWARZSCHILD SPACE

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Abstract. We present a numerical approach to the visualization of the light cones, and hence the causal structure, of exterior Schwarzschild space, taking advantage of the symmetries of Schwarzschild space and the conformal invariance of null geodesics.

1. Introduction

The purpose of this paper is to describe a numerical tool for the investigation of the causal structure of exterior Schwarzschild space by visualization of the light cones. The tool should also be of use to an instructor teaching a course including material on the causal structure of space-time, or to a student on such a course. Much of the material should be accessible to a student of general relativity who has previously studied a course at the level of, for example, d’Inverno [3] or Hughston and Tod [2], and the tool should help to bridge the gap between such texts and the more advanced treatments of Wald [8] or Hawking and Ellis [1]. We will use the Einstein summation convention throughout.

In Section 2, we provide a brief review of the causal structure of Minkowski space, as is generally discussed in introductory courses on special relativity, and how this generalizes to general space-times, with particular reference to the importance of null geodesics. In Section 3, we consider static space-times, and demonstrate how conformal transformations can be used to reduce the problem of finding null geodesics in space-time to that of finding geodesics in a Riemannian manifold of lower dimension. Section 4 carries this procedure out in the case of exterior Schwarzschild space-time. In Section 5 we provide a description of the numerical approach used, with some illustrative figures. Section 6 briefly discusses how the visualization tool might be used as an educational tool. The code itself is freely available to download.