NONLINEAR 1-D OSCILLATIONS OF A CHARGE PARTICLE UNDER COULOMB FORCES AND DRY FRICTION

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Abstract. We study the 1-D motions of a charge under Coulomb force, within the electrostatic approximation. When the electrostatic force is attractive, no oscillating motion takes place. When repulsive, nonlinear oscillations will arise. In both cases dry friction has been taken into account and time equations have been solved providing time as elliptic integrals of first and second kind. A short phase plane analysis has been included. The oscillation period has been exactly computed and found to increase versus the initial speed of the mobile.

1. Introduction

Most physical, biological, economic systems are inherently not linear so that they lead to nonlinear ordinary differential equations. As a consequence a branch of applied research is looking for exact solutions, by means of either special functions of the Mathematical Physics, or iterative approaches or perturbations, and so on. For instance, A. More [3] highlights that a closed form solution makes the analysis by far more elaborate and easier as its behavior becomes at once clear, when expressed in closed form in terms of known functions.

Our Problem

A mobile M-point mass, always constrained on a straight trajectory, undergoes dry friction, say $\mu > 0$ its dynamic coefficient, having as propelling cause an electrostatic force. The motion time law is required.

In fact, two invariable electric charges of *opposite signs* are placed: q_0, at a certain fixed point; the latter, say q, on the moving particle of mass $M > 0$. Let be L the initial distance between them: we put the origin O of the reference at the start of the mobile: so that the resting charge stays at $x = -L$ and the Coulomb force will be directed as the negative sense of x with a intensity $F(x) = kqq_0/(L + x)^2$, being $x = x(t) \geq 0$ the (unknown) particle's position at time t, and $k = 9 \times 10^9 N \times$