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NOTE ON REVERSION, ROTATION AND EXPONENTIATION IN DIMENSIONS FIVE AND SIX

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Abstract. The explicit matrix realizations of reversion and spin groups depend on the set of matrices chosen to represent a basis of one-vectors for a Clifford algebra. On the other hand, there are iterative procedures to obtain bases of one-vectors for higher dimensional Clifford algebras, starting from those for lower dimensional ones. For a basis of one-vectors for $\mathrm{Cl}(0,5)$, obtained by applying such procedures to the Pauli basis for $\mathrm{Cl}(3,0)$ the matrix form of reversion involves neither of the two standard matrices representing the symplectic form. However, by making use of the relation between 4×4 real matrices and the quaternion tensor product ($\mathbb{H}\otimes\mathbb{H}$), the matrix form of reversion for this basis of one-vectors is identified. The corresponding version of the Lie algebra of the spin group, $\mathfrak{spin}(5)$, has useful matrix properties which are explored. Next, the form of reversion for a basis of one-vectors for $\mathrm{Cl}(0,6)$ obtained iteratively from $\mathrm{Cl}(0,0)$ is obtained. This is then applied to computing exponentials of 5×5 and 6×6 real antisymmetric matrices in closed form, by reduction to the simpler task of computing exponentials of certain 4×4 matrices. For the latter purpose closed form expressions for the minimal polynomials of these 4×4 matrices are obtained, without availing of their eigenstructure.Among the byproducts of this work are natural interpretations for members of an orthogonal basis for $M(4,\mathbb{R})$ provided by the isomorphism with $\mathbb{H}\otimes\mathbb{H}$, and a first principles approach to the spin groups in dimensions five and six.

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