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CYLINDRICAL SHAPES OF HELFRICH SPONTANEOUS-CURVATURE MODEL

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Abstract. The governing equation of the Helfrich spontaneous-curvature model is the Helfrich equation. It is a coordinate free equation that describes the equilibrium shapes of biological (fluid) membranes. We make use of the conformal metric representation of the Helfrich equation and by applying the symmetry group reduction method we obtain a translationally invariant solution. Based on that solution, we derive analytic expressions for the position vector of special cylindrical equilibrium shapes. Plots of the graphs of some closed directrices of these shapes are presented.

1. Introduction

Biomembranes (membranes of living cells), or *fluid membranes* as they are generally called, consist of lipid compounds, mostly phospholipid molecules, having two parts – one or two hydrocarbon tails (the hydrophobic part) and a polar head group (the hydrophilic part). Placed in aqueous solution the phospholipid molecules spontaneously assemble to form closed *lipid bilayer* structure: two layers of molecules locating their hydrophilic heads to point outward in order to prevent the hydrophobic tails from direct contact with the water molecules. The lipid bilayers are the typical constituents of the semipermeable membranes of all the living cells (plant and animal) playing a dominant role for determining cell's shape.

The modern biomechanical models of the fluid membranes [2, 5, 20] dates back to the works of Canham [3] and Helfrich [7] in the early 70s of the previous century. Their basic concept, first suggested by Canham, is to think of the membrane as of a two-dimensional fluid in which the lipid molecules are moving freely in a way that no in-plane displacements can be developed, except bending. It follows from this idea that the shape of the living cells is controlled exclusively by the bending energy associated with the curvature elasticity of the lipid bilayer.

In the so-called Helfrich spontaneous-curvature model [7], the fluid membrane is described as a two-dimensional elastic surface $\mathcal{S}$, obtained as a solution of the

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