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BOOK REVIEW

Total Mean Curvature and Submanifolds of Finite Type, by Bang-Yen Chen, World Scientific, Singapore 2015, xviii + 467 pp., ISBN: 978-981-4616-68-3 (hardcover), ISBN: 978-981-4616-69-0 (softcover).

The book *Total Mean Curvature and Submanifolds of Finite Type* is the new edition of the book published under the same title in 1984 by one of the leading experts on the geometry of submanifolds. The book presents the most important topics in the theory of submanifolds. The readers will find this new edition both a good introduction to the area of total mean curvature and finite type theory, and a very useful reference to recent results on both subjects, which have great value both within mathematics as well as in other natural sciences. Reading this book the readers will deepen their understanding and improve their appreciation of the concepts and theories under discussion.

The study of submanifolds of finite type has been a very active research subject in the recent years. In particular, biharmonic submanifolds have been receiving a growing attention and have become a popular subject of research with many important progresses. The first edition of the book collected the major results, up to the early 1980s, on total mean curvature and submanifolds of finite type. In the second edition the author presents a great number of important new results on both subjects, including recent developments and results on biharmonic submanifolds and biharmonic conjectures obtained after the publishing of the first edition.

The book consists of ten chapters. Chapters 1, 2, 3, and 4 on *Differentiable Manifolds*, *Riemannian and Pseudo-Riemannian Manifolds*, *Hodge Theory and Spectral Geometry*, and *Submanifolds* of the new edition are reworked versions of the first four chapters of the first edition. Chapters 5 and 6 on *Total Mean Curvature* and on *Submanifolds of Finite Type* of the first edition are modified and expanded and new extra chapters 7, 8, 9 and 10 on *Biharmonic Submanifolds and Biharmonic Conjectures*, *λ-biharmonic and Null 2-Type Submanifolds*, *Applications of Finite Type Theory*, and *Additional Topics in Finite Type Theory* are added in the new edition. In all chapters the author has included many historical notes and observations on the presented material for a better and comprehensive readability.

Chapters 1 and 2 present the basic notions and results on differentiable manifolds, and especially on Riemannian and pseudo-Riemannian manifolds, providing the necessary background on this subject. In Chapter 3 the readers are provided with basic facts about Hodge theory, elliptic differential operators and Jacobi's elliptic functions. The Hodge-de Rham decomposition theorem is given and some of its applications are presented. In a special section the author gives the spectra of some important Riemannian manifolds. Chapter 4 presents basic formulas and theorems in the theory of submanifolds of Riemannian or pseudo-Riemannian manifolds. Here the author has collected the main theorems regarding some special submanifolds such as totally geodesic submanifolds, parallel submanifolds, totally umbilical submanifolds, pseudo-umbilical submanifolds, and minimal Lorentzian surfaces.

Chapter 5 deals with total mean curvature of the submanifolds. The author presents results on total mean curvature of surfaces in real space forms in relation with conformal invariants. A special section is devoted to a variational problem on total mean curvature. In Chapter 6 the author presents the theory of finite type submanifolds. The notion of finite type submanifolds provides a natural way to combine the spectral geometry with the theory of submanifolds. The study of finite type submanifolds began in the late 1970s with the author's attempts to find the best possible estimates of the total mean curvature of a closed submanifold of Euclidean space and to find a notion of "degree" for submanifolds of Euclidean space. The first results on finite type theory were collected in the first edition of the book. Since then many geometers have contributed to this theory. In this second edition the author presents numerous important results on finite type theory obtained after the publication of the first edition. Many new references on this subject are included.

In Chapter 7 the author provides a comprehensive account of the latest updates and the new results on biharmonic submanifolds and biharmonic conjectures. The longstanding biharmonic conjecture of the author is presented in details. The most recent results which provide supports of this conjecture are included in this chapter. The author presents also many results on biharmonic submanifolds in hyperbolic space and provides a list of recent results concerning the generalized biharmonic conjecture. A list of studies on biharmonic submanifolds in various model spaces other than real space forms is also given.

Chapter 8 deals with λ-biharmonic submanifolds and null 2-type submanifolds. Many classification results on null 2-type submanifolds with parallel mean curvature vector field as well as with parallel normalized mean curvature vector field are presented in this chapter. The classification of null 2-type surfaces with constant mean curvature in the Euclidean four-space and the classification of null 2-type

spacelike or Lorentzian surfaces with constant mean curvature in Minkowski four-space are given. Null 2-type marginally trapped surfaces in the Minkowski four-space and null 2-type quasi-minimal surfaces in the pseudo-Euclidean four-space with neutral metric are also classified. The chapter ends with results concerning the question to classify λ-biharmonic submanifolds in Euclidean and pseudo-Euclidean spaces.

Chapter 9 provides numerous applications of finite type theory to several important subjects in differential geometry for Riemannian submanifolds and maps via their spectral decompositions. The author gives some sharp relationships between total mean curvature and the order of Euclidean submanifolds, as well as sharp relationships between total mean curvature and the first and second nonzero eigenvalues of the Laplacian for submanifolds of Euclidean space, the unit hypersphere, and the projective space. Total mean curvature is also estimated in terms of the order of immersions and circumscribed radii. Some applications of finite type theory to smooth maps are presented. In particular, relations between finite type theory and topology of Gauss map of Euclidean submanifolds are given. The author provides also relations between linear algebras and differential geometry via finite type theory.

In the last Chapter 10 of the book the author presents some additional topics in finite type theory. Numerous results on submanifolds with finite type Gauss map, submanifolds with pointwise 1-type Gauss map, and submanifolds with finite type spherical Gauss map are provided. Results concerning finite type submanifolds in Sasakian manifolds and finite type submanifolds of real and complex hyperbolic spaces are also given.

I strongly recommend this second edition of the book to the mathematical community for its great value both in presenting the developments and the most important steps in the evolution of geometry of submanifolds in the last years, as well as for providing a great number of references for researchers interested in the theory of submanifolds of finite type and total mean curvature. This book will be of great use for graduate students and researchers in the field of modern differential geometry.

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