



DRAWING THE FREE RIGID BODY DYNAMICS ACCORDING TO JACOBI

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Abstract. Guided by the Jacobi's work published one year before his death about the rotation of a rigid body, the behavior of the rotation matrix describing the dynamics of the free rigid body is studied. To illustrate this dynamics one draws on a unit sphere the trace of the three unit vectors, in the body system along the principal directions of inertia. A minimal set of properties of Jacobi's elliptic functions are used, those which allow to compute with the necessary precision the dynamics of the rigid body without torques, the so called Euler's top. Emphasis is on the paper published by Jacobi in 1850 on the explicit expression for the components of the rotation matrix. The tool used to compute the trajectories to be drawn are the Jacobi's Fourier series for *theta* and *eta* functions with extremely fast convergence. The Jacobi's *sn*, *cn* and *dn* functions, which are better known, are used also as ratios of *theta* functions which permit quick and accurate computation. Finally the main periodic part of the herpolhode curve was computed and graphically represented.

MSC: 70E15, 14K25

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