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POISSON-LIE STRUCTURE ON THE TANGENT BUNDLE OF A POISSON-LIE GROUP, AND POISSON ACTION LIFTING

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Abstract. We show in this paper that the tangent bundle TG, of a Poisson-Lie group G has a Poisson-Lie group structure given by the canonical lifting of that of G. We determine the dual group of TG, its Lie bialgebra and its double Lie algebra.

We also show that any Poisson action of G on a Poisson manifold P is lifted on a Poisson action of TG on the tanget bundle TP.

1. Introduction

Poisson-Lie group theory was first introduced by Drinfel'd [1] [2] and Semenov-Tian-Shansky [11]. Semenov and Kosmann-Schwarzbach [4] used Poisson-Lie groups to understand the Hamiltonian structure of the group of dressing transformations of certain integrable systems. These Poisson-Lie groups play the role of symmetry groups. Theory of Poisson-Lie groups was remarkably developed by Weinstein [9] [13], Drinfel'd [3] and Jiang-Hua Lu [6] [7].

Let (G, ω) be a Poisson-Lie group with Lie algebra $\mathcal{G}$ and multiplication $m : G \times G \longrightarrow G$.

We assume that the tangent bundle TG is equipped with the Poisson structure Ω_{TG} introduced by Sanchez de Alvarez in [10]. In this case, TG has a Poisson-Lie group structure with dual Poisson-Lie group (TG^*, Ω_{TG^*}) and Lie bialgebra $(\mathcal{G} \dashv \mathcal{G}, \mathcal{G}^* \vdash \mathcal{G}^*)$, where G^* is the dual of G, $\mathcal{G} \dashv \mathcal{G}$ is the semi-direct product Lie algebra with bracket

$$[(x, y), (x', y')] = ([x, x'], [x, y'] + [y, x']), \text{ where } (x, y),\ (x', y') \in \mathcal{G} \times \mathcal{G}$$

and $\mathcal{G}^* \vdash \mathcal{G}^*$ is the semi-direct product Lie algebra with bracket

$$[(\alpha, \beta), (\alpha', \beta')] = ([\alpha, \beta'] + [\beta, \alpha'], [\beta, \beta']), \text{ where } (\alpha, \beta),\ (\alpha', \beta') \in \mathcal{G}^* \times \mathcal{G}^*.$$