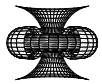
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THE PROPER-TIME LORENTZ GROUP DEMYSTIFIED

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Abstract. Proper velocities are measured by proper time as opposed to coordinate velocities, which are measured by coordinate time. The standard Lorentz transformation group, in which each transformation is expressed by a coordinate velocity and an orientation between two inertial frames, is well known. In contrast, the equivalent proper-time Lorentz transformation group, in which each transformation is expressed by a proper velocity and an orientation between two inertial frames is unknown. The dignity of special relativity theory requires that every possible means be explored for the solution of a problem so elegant and so celebrated. Fortunately, a so called *gyro-formalism* approach to special relativity enables the elusive proper-time Lorentz transformation group to be uncovered.

1. Introduction

Einstein velocity addition is a century-old idea whose time has come back. Following the discovery of the *gyrovector space structure* to which Einstein velocity addition gives rise [26], and unleashing the power of its hyperbolic geometry [27], hitherto important unsolved problems of special relativity theory can be solved. The problem in case is the determination of the proper-time Lorentz transformation group, a problem that eluded all its previous explorers as we demonstrate in this article.

Coordinate time, or observer's time, is the time t of a moving object measured by an observer at rest. Accordingly, special relativity theory is formulated in terms of Coordinate time. Contrasting coordinate time, proper time, or traveller's time, is the time τ of a moving object measured by a co-moving observer. Proper time is useful, for instance, in the understanding of the twin paradox [11], and the mean life time of unstable moving particles, like muons.

The mean lifetime of muons between creation, in the upper atmosphere, and disintegration is 2.2μs (proper lifetime) measured by their proper time. This proper time of the moving muon, measured by the muon own clock, is several orders