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ALGEBRAS GENERATED BY ODD DERIVATIONS

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Abstract. Given an associative supercommutative algebra equipped with an odd derivation, one considers the space of vector fields it defines, and show, under suitable hypothesis, they form a Jordan superalgebra; in contrast with the Lie superalgebras of Virasoro type constructed from even derivations. Relations with Anti Lie algebras studied by Ovsienko and collaborators are then shown.

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Let $\mathcal{A}$ be an $\mathbb{Z}/2\mathbb{Z}$-graded, associative and graded commutative algebra.The degree of an element $a \in \mathcal{A}$ will be denoted by $|a|$, the same for the degree of an application; we shall consider derivations of $\mathcal{A}$. A map $\delta : \mathcal{A} \to \mathcal{A}$ is a derivation if for any $a, b \in \mathcal{A}$, one has

$$\delta(ab) = \delta(a)b + (-1)^{|a||\delta|}a\delta(b).$$

It is well-known that the space of derivations of a commutative associative algebra is a Lie algebra through commutator, a generic example being derivations of the algebra of smooth functions on a differentiable manifold, isomorphic to the Lie algebra of tangent vector fields through Lie derivative. This fact generalizes to the $\mathbb{Z}/2\mathbb{Z}$-graded case: graded bracket of derivations induces a Lie superalgebra structure, as can easily be deduced from the formula

$$[\delta_1, \delta_2] = \delta_1 \circ \delta_2 - (-1)^{|\delta_1||\delta_2|}\delta_2 \circ \delta_1.$$

For basic definitions and results about superalgebra, see [2, vol.1, Part 1].

1. About Virasorisation

We shall generalize here the construction of Virasoro algebra from the commutative and associative algebra of smooth functions on the unit circle with its natural derivative (cf. [4] for basic results about Virasoro algebra). Let δ be a derivation of $\mathcal{A}$, and $a \in \mathcal{A}$, then $a\delta$ defined as $[a\delta](b) = a\delta(b)$, is a derivation of degree

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