JGSP **41** (2016) 1–16

JOURNAL OF
Geometry and Symmetry in Physics
ISSN 1312-5192

INVOLUTIONS IN SEMI-QUATERNIONS

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Communicated by Abraham Ungar

Abstract. Involutions are self-inverse and homomorphic linear mappings. Rotations, reflections and rigid-body (screw) motions in three-dimensional Euclidean space $\mathbb{R}^3$ can be represented by involution mappings obtained by quaternions. For example, a reflection of a vector in a plane can be represented by an involution mapping obtained by real-quaternions, while a reflection of a line about a line can be represented by an involution mapping obtained by dual-quaternions. In this paper, we will consider two involution mappings obtained by semi-quternions, and a geometric interpretation of each as a planar-motion in $\mathbb{R}^3$.

MSC: 11R52, 53A25, 53A35, 70B10
Keywords: Dual-quaternions, involutions, planar-motion, semi-quaternions

1. Introduction

The adventure of quaternions started in the mid-19th century as a geometric and algebraic interest. Soon after they were found to have applications in mechanics, physics, computer graphic technology, mixed and augmented systems, etc. The main difficulty in the development of quaternions occured while defining the multiplication rule. Rumor says that the Irish mathematician Sir William Rowan Hamilton was looking for a way to formalize points in three-space in the same way that points in the plane can be defined in the complex field. For many years, he knew how to add and subtract points in three-space. However, he had failed by the problem of multiplication for over ten years. Finally, on 16 October 1843 in Dublin, Hamilton solved the multiplication problem and his intuition was that the algebra of quaternions would require three imaginary parts satisfying

$$\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = \boldsymbol{ijk} = -1.$$

Quaternions are useful tools for representing rotations, reflections and rigid-body (screw) motions in three-dimensional spaces. Ell and Sangwine [5] represented an involution and an anti-involution mapping of real-quaternions with their geometrical meanings as reflections or rotations in three-dimensional Euclidean space $\mathbb{R}^3$.

doi: 10.7546/jgsp-41-2016-1-16