



## SYED TWAREQUE ALI

### OBITUARY

*On the 25th of January, 2016, the world lost Syed Twareque Ali. He died while attending a conference in Malaysia. What follows are some remembrances of him written by some of his close colleagues and friends.*

#### S. Twareque Ali: The Early Years



I first met Twareque in Rochester when I was a graduate student there. Twareque was two or three years behind me, but we had the same thesis advisor, Gérard G. Emch. Emch's students at the time: Kalyan Sinha, Twareque, and I choose to work in the mathematics of the foundations of quantum mechanics (not Emch's field!) and Charley Radin worked in  $C^*$  algebras in solid state mechanics which was one of Emch's fields. This was quite funny if you did not know Professor Emch. He was a veritable fount of knowledge of the mathematics we would need. It did not matter whether he had worked in it or not.

Then I graduated, moved to Florida, and several years later discovered that Eduard Prugovečki (University of Toronto) was working in a parallel field to mine. We arranged to meet as follows: When phoning him to get some papers the first time, he suddenly shouted to his secretary "Are not we (Margaret Prugovečki and he) going to Florida next week? And where?" It was *in* Boca Raton where I was at the time. So, I took a lot of their vacation time discussing quantum mechanics with Eduard. As a result, we had two of us working in the field of "fuzzy quantum mechanics" or "stochastic quantum mechanics" or "quantum mechanics on phase

space” as it later was called. A short time after this, Twareque showed up and was a post doc under Eduard. The three of us founded the field together.

Twareque had written a paper “Fuzzy Observables in Quantum Mechanics” with Dr. Emch in 1974. Then came “On the Equivalence of Non-relativistic Quantum Mechanics Based upon Sharp and Fuzzy Measurements” with H.-D. Doebner in 1976. In this paper, Twareque had represented the “resolution vector” as a harmonic oscillator state. The “resolution vector” later turned out to be describing the instrument that one measures the unknown  $\psi$  against. It was not one vector but a collection of vectors (a coherent state) all based on the harmonic oscillator state. In 1977, he and Eduard published “Classical and Quantum Statistical Mechanics in a Common Liouville Space,” which has continued to have a big splash in the field of stochastic quantum mechanics. Also, these two published “Quantum Statistical Mechanics on Stochastic Phase Space.” By this time we were off and running.

There were many papers by Twareque, but I will mention just three important ones. In 1986, Ali and Prugovečki published “Extended Harmonic Analysis of Phase Space Representations of the Galilei Group” and “Harmonic Analysis and Systems of Covariance for Phase Space Representations of the Poincaré Group.” This set the stage for my doing everything in general eventually, but in a much more general setting with respect to the coherent state which one measures against. In 1988, Twareque, J. Brooke, P. Busch, R. Gagnion, and I published “Current Conservation as a Geometric Property of Space-Time.” This led the way for obtaining the current and the representations of the Poincaré group in a geometric setting which has now been completed to include spin. After that, he began to seriously work in coherent state theory, which is a subject for others.

While at a conference in Bulgaria, I was trying to explain to a group of foreign people in English, what the theory was all about. It did not work for me. Then Twareque said to me, “You should speak in *broken* English.” And he then proceeded to say the same thing I had said but in broken English! That worked, and everyone laughed. It was a typical Twarequeism.

Twareque had contributed a lot to the theory of quantum mechanics on phase space, had been a good person with which to work, and was a person that could be the life of the party at conferences. We will miss you, Twareque.

Franklin E. Schroeck, Jr.

## S. Twareque Ali: Some Recollections

I started to collaborate with Twareque in 1989, on the topic of coherent states. Actually he was referred to me, because he had trouble decomposing a phase space

representation of the Poincaré group. The solution was found quickly, just replacing a bounded Hilbert space operator (as in the non-relativistic case) by an unbounded one, whose dense domain leads to a triplet of Hilbert spaces. The next year, Jean-Pierre Gazeau joined the game and thus started our triangular collaboration (AAG), that continued up to the present day.

Since Twareque traveled a lot, we kept meeting each other all over the world, up to the present day. We often attended the same meetings, in addition to visiting each other at our home universities, including being respective invited professors. To name a few, Québec, Germany (Clausthal), Poland (Białowieża), Cuba, France, Italy, Bangladesh (Dhaka), China (Tianjin), India (Bangalore), and so on.

Actually, Twareque was also on the move professionally. After a MSc in Dhaka, Bangladesh, in 1966, and a PhD in Rochester, USA, in 1973, he occupied positions successfully at ICTP, Trieste (Italy), University of Toronto, University of Prince Edward Island, TU Clausthal (Germany), and finally Concordia University Montréal. In addition, he has been invited professor at uncountably many foreign universities throughout the world and invited speaker at as many conferences.

During his whole life, his scientific worldline was fully coherent. Indeed his entire career can be summarized in three words: phase space, quantization, coherent states. It is on the last topic that we collaborated for the first time in 1989 and continued up to the present day, together with J-P. Gazeau. Along the way, eleven joint papers were written, three books edited [17–19] (Białowieża proceedings), two more written as coauthors [5, 12] and the recent *Coherent States: A Contemporary Panorama*, a special issue of *J. Phys. A: Math. & Theor.* **45** (2012) co-edited with F. Bagarello.

Throughout these years, Twareque has relentlessly preached mathematical rigor in the field of coherent states, too often left to the rather sloppy treatment of quantum optics fans (in Wikipedia, for instance!). His favorite topics were positive operator valued measures, phase space formulation of quantum mechanics, vector and matrix-valued coherent states, coherent state quantization, and so on.

Let us be more specific. In the 1970s and 1980s, he devoted much time to measurement problems in (fuzzy) phase space and stochastic quantum mechanics, mostly with E. Prugovečki, in Toronto (which led to a memorable and lasting dispute with Gerald Kaiser), G. Emch in Rochester and H.-D. Doebner in Clausthal. Then he gradually focused on coherent states, relativistic or not. A notable result of the AAG collaboration was the extension of square integrability of group representations to homogeneous spaces [1] and the introduction of continuous frames in Hilbert spaces [2, 3] - the key to many applications, including wavelets. These papers are still often quoted today. At that point, the time was ripe for surveying the

whole topic of coherent states, which resulted in the overview [4], which in turn led to our first common textbook [5].

More recently, Twareque started to be interested in quantization, mostly Berezin or coherent states quantization. Notably, together with Miroslav Engliš, from Prague, he authored a useful survey of the various approaches to quantization [6]. He also got interested in the mathematics of signal processing, in particular their group-theoretical aspects, as can be seen, for instance in the textbook [12]. In the last years, he studied noncommutative quantum mechanics, quaternionic Hilbert spaces and complex orthogonal polynomials [11].

An important part of Twareque's life is the organization of meetings. Two series are notable. First, the Białowieża Workshops on Geometric Methods in Physics, famous for the wild forest, bison, vodka, Russian mathematicians of the highest caliber and, of course, their charismatic chief organizer, Anatol Odziejewicz. From 1992 on, the XI-th meeting, Twareque was instrumental in transforming a small local workshop into a full-fledged international event, still going strong - the last one was # XXXIV in June 2015.

Another remarkable achievement is the series of workshops in Havana, Cuba, organized jointly by Concordia University and the University of Havana. Here again, Twareque was one of the "chevilles ouvrières" of the meetings, who succeeded in attracting a number of recurrent distinguished participants to these beautiful surroundings and almost singlehandedly took care of the proceedings. More recently Twareque has also become a faithful participant in the school and workshops on mathematical physics (COPROMAPH) organized in Cotonou, Bénin, by Norbert Houkonnou.

Working with Twareque was a rewarding experience. His clean, elegant mathematical style is a pleasure to read, even if his papers are sometimes dense and compact. His nonscientific writings have the same quality, even poetry - I remember a memorable poem composed by him in Białowieża! His talks were always extremely clear and pedagogical. His ideas were often thought provoking. Altogether I feel fortunate to have been able to work with him for so long, and so do surely all his other collaborators. And everyone will remember his so characteristic laugh, compelling everyone to share his great and subtle sense of humor.

Jean-Pierre Antoine

## **S. Twareque Ali: Close Friend and Deep Thinker**

Twareque Ali and I met in Claustal, Germany during the 1980s, invited by Heinz-Dietrich Doebner to the Arnold Sommerfeld Institute. Each summer from 1992

to 2015, nearly without exception, we would see each other at the Workshop on Geometric Methods in Physics at Białowieża, Poland. With our spouses we toured Beijing and visited the Great Wall of China after the Group Theory Colloquium in Tianjin in 2012 (where he was honored on the occasion of his 70<sup>th</sup> birthday). We shared family weddings and other events. Twareque was not only a colleague and collaborator, but a close personal friend, an intimate confidant. He was someone with whom one could discuss the meaning of life's joys and disappointments without self-consciousness.

With his serious expression, his black hair and beard only lately streaked with grey, he looked thoughtful and wise - but his eyes twinkled, and he loved to laugh. He knew how to live life, to find humor in its seriousness. He would quote poetry extensively from memory, in Bengali, Italian, German, and of course English. He loved Omar Khayyam's *Rubaiyat*, in the Edward Fitzgerald translations. And his laughter always restored balance. He was especially fond of the novel *Small World* by David Lodge, which satirizes the sometimes pretentious academic scene we both knew so well. We imagined we could recognize Lodge's characters in people we knew, including (of course) ourselves.

And Twareque was a deep thinker. He believed deeply in peace, and gave generously of himself to the less-privileged in the world. For him, the truths of science were part of the ever-unattainable beauty for which he yearned, for which all of us yearn. He sought scientific truths through mathematics, especially an understanding of the mysteries of quantum mechanics. Influenced deeply by his teacher Gérard Emch, he in turn inspired numerous students and colleagues. Though Twareque has gone now, his inspiration, gentleness, and humanity will live on for many generations.

Twareque influenced me the most scientifically with his perspective on different methods of quantization, and in particular his work on coherent states quantization. However, I cannot actually rely on a particular paper or papers, because most of his influence on my thinking was through in-depth personal discussions. He taught me a lot, not only about quantization, but about ways of thinking scientifically and mathematically. But for a particular title, probably I would point to his paper Ali & Engliš [6], "Quantization Methods: A Guide for Physicists and Analysts".

Gerald Goldin

### **S. Twareque Ali: The Final Days**

During the last two to three years, I visited Twareque in Montreal proposing that he make a trip back to the theory of Positive Operator Valued Measures (POVMs).

The result on the characterization of POVMs as an integral over the projection valued measures he got in late 1982 played a crucial role in the theory of POVMs and in the applications to quantum mechanics. He was happy to be back to the early years of his research and was also happy to help a young guy interested in such questions. Later, in April of 2014, he came to visit me in Italy where we spent twenty wonderful days working on the connections between the theory of frames and some recent results on the characterization of joint measurability. I have a couple of works in progress of which he will be a coauthor.

It is very nice to recall that my children were really attracted by his unique laugh that he used as a magic quantum instrument to guide the sense and the direction of a conversation. He was a great man and his departure is a great loss for humanity.

Roberto Beneduci

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