



# CLIFFORD ALGEBRA IMPLEMENTATIONS IN MAXIMA

DIMITER PRODANOV

Communicated by Ivaïlo M. Mladenov

**Abstract.** This tutorial focuses on the packages `clifford` and `cliffordan` for the computer algebra system *Maxima*. *Maxima* is the open source descendant of the first computer algebra system and features a rich functionality from a large number of shared packages. The Maxima language is based on the ideas of functional programming, which is particularly well suited for transformations of formal mathematical expressions. While `clifford` implements Clifford algebras  $C\ell_{p,q,r}$  of arbitrary signatures and order based on the elementary construction of Macdonald, `cliffordan` features geometric calculus functionality. Using `clifford` expressions containing geometric, outer and inner products can be simplified. Applications of `clifford` and `cliffordan` in linear algebra and calculus are demonstrated.

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**Keywords:** Clifford product, computer algebra, Dirac operator, electromagnetism, geometric product, multilinear algebra, outer product, vector derivative

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