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COVERING MAPS AND IDEAL EMBEDDINGS OF COMPACT HOMOGENEOUS SPACES

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Abstract. The notion of ideal embeddings was introduced by the author at the Third Pacific Rim Geometry Conference held at Seoul in 1996. Roughly speaking, an ideal embedding is an isometrical embedding which receives the least possible amount of tension from the surrounding space at each point.

In this article, we study ideal embeddings of irreducible compact homogenous spaces in Euclidean spaces. Our main result states that if $\pi : M \to N$ is a covering map between two irreducible compact homogeneous spaces with $\lambda_1(M) \neq \lambda_1(N)$, then N does not admit an ideal embedding in a Euclidean space, although M could.

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1. Introduction

According to Nash's embedding theorem [15], every Riemannian manifold can be isometrically embedded in a Euclidean space with sufficiently large codimension. In other words, every Riemannian manifold can live in the Euclidean world if the codimension was sufficiently large.

Related to Nash's theorem, my main question raised in [4, 5] is the following.

Main Question. *Can a given Riemannian manifold live in a Euclidean world ideally?*

More precisely, can a given Riemannian manifold be isometrically embedded in a Euclidean space in such way that it receives the least possible tension from the surrounding space at each point?

It is well-known that the mean curvature vector field of a submanifold is exactly the tension field for an isometric immersion of a Riemannian manifold in another Riemannian manifold.

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