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GEOMETRIC QUANTIZATION OF FINITE TODA SYSTEMS AND COHERENT STATES

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Abstract. Adler had showed that the Toda system can be given a coadjoint orbit description. We quantize the Toda system by viewing it as a single orbit of a multiplicative group of lower triangular matrices of determinant one with positive diagonal entries. We get a unitary representation of the group with square integrable polarized sections of the quantization as the module . We find the Rawnsley coherent states after completion of the above space of sections. We also find non-unitary finite dimensional quantum Hilbert spaces for the system. Finally we give an expression for the quantum Hamiltonian for the system.

MSC: 53D50, 81S10, 81R05
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1. Introduction

The connection between finite Toda system and coadjoint orbits was first explored by Adler [1]. We summarize the introduction to the Toda system as in [1]. The Hamiltonian considered is $H = \frac{1}{2}\sum_{i=1}^{n} y_i^2 + \sum_{i=1}^{n} \mathrm{e}^{x_i - x_{i+1}}$, $x_0 = x_{n+1}$. The Hamiltonian equations of motion are $\dot{x_i}= y_i$, $\dot{y_i}= \mathrm{e}^{x_{i-1}-x_i} - \mathrm{e}^{x_i - x_{i+1}}$, $i = 1, ..., n$. Define

$$a_i = \frac{1}{2}\mathrm{e}^{\frac{1}{2}(x_i - x_{i+1})}, \qquad i = 1, ..., n-1, \qquad b_i = \frac{1}{2} y_i, \qquad i = 1, ..., n.$$

Note that $a_i > 0$ for $i = 1, ..., n-1$.

Adler showed that the Hamiltonian equation of motion corresponds to a Lax equation and gave explicit expression for the integrals of motion which Poisson commute w.r.t. the following Poisson bracket

$$\{f, g\} = \dot{\sum} (a_{i-1} g_{a_{i-1}} - a_i g_{a_i}) f_{b_i} + \dot{\sum} a_i (g_{b_i} - g_{b_{i+1}}) f_{a_i}$$

where $\cdot$ means that terms with undefined elements, i.e., terms involving a_0, a_n, b_{n+1}.

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