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A NOTE ON THE CLASS OF SURFACES WITH CONSTANT SKEW CURVATURES

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Abstract. The goal of this paper is to analyze surfaces with constant skew curvature (CSkC), and show that the class of CSkC surfaces with non-constant principal curvatures does not contain any Bonnet surfaces.

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1. Introduction and Background

A CSkC (constant skew curvature) surface represents a smooth, immersed surface in a space form, whose difference of principal curvatures $k_1 - k_2 = a$ represents a positive constant. Note that this is equivalent with $H^2 - K = c^2 = \frac{a^2}{4}$ being constant, where H is the mean curvature, and K is the Gaussian curvature of the surface. This type of surface is a particular kind of *W-surface* (a surface that is characterized by a functional relationship between its principal curvatures, as stated by Chern in [2]). On the other hand, the characterization of a CSkC surface can be made in a more specific way. Surfaces in $\mathbb{R}^3$ whose principal curvatures satisfy a linear relation (i.e., $k_1 = pk_2 + q$, where p and q are real numbers) are called *linear Weingarten surfaces*. This class of surfaces has many relevant physical applications. For example, the Mylar balloon can be regarded as a specific example of a linear Weingarten surface (with $q = 0$) as it was done in [5], where a variational characterization was provided for linear Weingarten surfaces that generalize the Mylar balloon, in terms of beta functions.

Therefore, we may regard a CSkC surface as a linear Weingarten surface with $p = 1$ and q non-zero, by excluding umbilic points.

Separately, we will recall the notion of Bonnet surface. The notations used in this article are the standard ones from most books on surface theory, such as [6], for example. Let us consider an oriented surface $\mathbb{M}^2$ in $\mathbb{R}^3$ of Riemannian metric g, characterized by a smooth mean curvature H. One of the famous questions that