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ELECTRIC FIELDS CREATED BY POINT CHARGES: SOME GEOMETRICAL AND TOPOLOGICAL RESULTS

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Abstract. We study some geometrical and topological properties of the electric fields created by point charges on Riemannian manifolds. Particularly, we characterize the spaces on which the electric lines emanating from a point charge are geodesics, and describe the topological properties of the basin boundary for N point charges. Several open problems will be posed.

1. Introduction

The discovery of the inverse-square law for Newtonian and Coulomb interactions is a milestone in the Physics of the XVII and XVIII centuries. The central claim [1, 14] is that, for both electric and gravitational interactions, the force per unit mass or charge experimented by a test particle situated at a point $\boldsymbol{x} \in \mathbb{R}^3$ is given by the field

$$\boldsymbol{E}(\boldsymbol{x}) = \frac{q}{4\pi} \frac{\boldsymbol{x} - \boldsymbol{p}}{|\boldsymbol{x} - \boldsymbol{p}|^3} .$$

Here $q \in \mathbb{R}$ is the charge (or minus the mass) of the point particle originating the interaction, $\boldsymbol{p} \in \mathbb{R}^3$ is its position and we have chosen Heaviside–Lorentz units.

Since then, the study of the electrostatic fields generated by N point charges q_i ($i = 1, \ldots, N$) in Euclidean space has become a classical problem in mathematical physics and potential theory [6]. When all the charges are negative, this is equivalent to studying the Newtonian field created by N point masses $-q_i$. In modern treatments, one usually defines the potential function $V : \mathbb{R}^3 \to \mathbb{R}$ of a unit point charge, which is a fundamental solution of the Poisson equation

$$-\Delta V = \delta_p$$

and obtains the electric field as $\boldsymbol{E} = -\nabla V$. Here δ_p stands for the Dirac distribution centered at p. The field of several charges can be calculated using the superposition principle.