



## CHERN-SIMONS AND STRING THEORY

KISHORE MARATHE

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**Abstract.** We discuss a special case of the gauge theory to string theory correspondence where exact results are available. We show how the Witten-Reshetikhin-Turaev invariant in  $SU(n)$  Chern-Simons theory on  $S^3$  is related via conifold transition to the all genus generating function of the topological string amplitudes on a Calabi-Yau manifold.

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### 1. Introduction

Last year several of us (lucky mathematical physicists) attended the Bayrischzell workshop 2004 on noncommutativity and physics. One of the speakers told us that gauge theory, open string theory, closed string theory and noncommutative QFT are the same. Perhaps the heady alpine air had something to do with this euphoric statement. Nevertheless several bits and pieces of evidence supporting the existence of links between these theories has accumulated over the last few years. However, stealing a line from Cliff Taubes, I would like to say that at least from a mathematical point of view we would be lucky if in a few years we know what are the right questions to ask. This year we are celebrating the 25th anniversary of the marriage between Gauge Theory and the Geometry of Fiber Bundles from the sometime warring tribes of Physics and Mathematics. Marriage brokers were none other than Chern and Simons. The 1978 paper by Wu and Yang can be regarded as the announcement of this union. It has led to many wonderful offspring. The theories of Donaldson, Chern-Simons, Floer-Fukaya, Seiberg-Witten, and TQFT are just some of the more famous members of their extended family. Quantum Groups, CFT, Supersymmetry, String Theory and Gravity also have close ties with this family. In this talk we will discuss one particular relationship that has recently come to light. The qualitative aspects of Chern-Simons theory as string theory were investigated by Witten [6] almost ten years ago. Before recounting the main idea of this work we review the Feynman path integral method of quantization which is particularly suited for studying topological quantum field