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THIRD-ORDER JACOBSTHAL GENERALIZED QUATERNIONS

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Abstract. In this paper, the third-order Jacobsthal generalized quaternions are introduced. We use the well-known identities related to the third-order Jacobsthal and third-order Jacobsthal-Lucas numbers to obtain the relations regarding these quaternions. Furthermore, the third-order Jacobsthal generalized quaternions are classified by considering the special cases of quaternionic units. We derive the relations between third-order Jacobsthal and third-order Jacobsthal-Lucas generalized quaternions.

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1. Introduction and Preliminaries

Recently, the topic of number sequences in real normed division algebras has attracted the attention of several researchers. It is worth noticing that there are exactly four real normed division algebras: real numbers ($\mathbb{R}$), complex numbers ($\mathbb{C}$), quaternions ($\mathbb{H}$) and octonions ($\mathbb{O}$). In [2] Baez gives a comprehensive discussion of these algebras.

The real quaternion algebra

$$\mathbb{H} = \{q = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k};\ \ q_s \in \mathbb{R},\ s = 0, 1, 2, 3\}$$

is a four-dimensional $\mathbb{R}$-vector space with basis $\{\mathbf{1} \simeq e_0, \mathbf{i} \simeq e_1, \mathbf{j} \simeq e_2, \mathbf{k} \simeq e_3\}$ satisfying multiplication rules $q_0\mathbf{1} = q_0$, $e_1e_2 = -e_2e_1 = e_3$, $e_2e_3 = -e_3e_2 = e_1$ and $e_3e_1 = -e_1e_3 = e_2$.

There has been an increasing interest on quaternions and octonions that play an important role in various areas such as computer sciences, physics, differential geometry, quantum physics, signal, color image processing and geostatics (for more details, see [1, 4, 10, 17]).

The origin of the topic of number sequences in division algebra can be traced back to the works by Horadam in [12] and by Iyer in [15]. In this sense, Horadam [12]

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