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2+1-MOULTON CONFIGURATION

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Abstract. We pose a new problem of collinear central configuration in Newtonian n-body problem. For a given two-body, we ask whether we can add a new body in a way such that i) the configuration of the total three-body is also collinear central with the configuration of the initial two-body being fixed and further ii) the initial two-body keeps its motion without any change during the process. We find three solutions to the above problem. We also consider a similar problem such that while the condition i) is satisfied but by modifying the condition ii) the motion of the initial two-body is not necessarily equal to the original one. We also find explicit solutions to the second problem.

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1. Introduction

Leonard Euler had found the first solution of the three-body problem on a line, the collinear three-body problem [2]. In general, solutions of the n-body problem on a line, called a collinear n-body problem, become *collinear central configuration*, that is, the ratios of the distances of the bodies from the center of mass are constants. Moulton [5] proved that for a fixed mass vector $\mathbf{m} = (m_1, \ldots, m_n)$ and a fixed ordering of the bodies along the line, there exists a unique collinear central configuration $\mathbf{q} = (q_1, \ldots, q_n)$ with mass $\mathbf{m} = (m_1, \ldots, m_n)$ (up to translation and scaling), where q_i denotes the position of the i th-body on a line $i = 1, \ldots, n$. The configuration is called a *Moulton Configuration*, which will be abbreviated as MC.

In this paper, we consider the following problem. We assume we are given a MC $\mathbf{q}_A = (q_{A_1}, q_{A_2})$ of two bodies A_1, A_2 such that $q_{A_1} < q_{A_2}$ with mass $\mathbf{m}_A = (m_{A_1}, m_{A_2})$. We consider to add a body B of position q_B with mass m_B, to A_1, A_2 on the same line containing A_1, A_2 so that i) the configuration of A_1, A_2 and B is MC with the configuration of the initial two-body being fixed and ii) the motion of A_1, A_2 are kept invariant during the process. More precisely, let q_i denote one