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DIRECT CONSTRUCTION OF A BI-HAMILTONIAN STRUCTURE FOR CUBIC HÉNON-HEILES SYSTEMS

NICOLA SOTTOCORNOLA

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Abstract. The problem of separating variables in integrable Hamiltonian systems has been extensively studied in the last decades. A recent approach is based on the so called Kowalewski's Conditions used to characterized a Control Matrix M whose eigenvalues give the desired coordinates. In this paper we calculate directly a second compatible Hamiltonian structure for the cubic Hénon-Heiles systems and in this way we obtain the separation variables as the eigenvalues of a recursion operator N. Finally we re-obtain the Control Matrix M from N.
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1. Introduction

The problem of the explicit integration in quadrature of Hamiltonian systems has received a lot of attention in the last decades. A well-known method consists in the determination of a new set of coordinates separating the Hamilton-Jacobi equation. Once separated, this equation can be integrated and the solutions can be obtained in the new coordinates. Inverting the change of coordinates, we can finally write the solutions in the original coordinates. In most cases the difficulty in the process consists in finding the *separation variables*. Even in low dimensional cases, this can be a challenging task. A class of systems that has been extensively studied is the family of the so called Hénon-Heiles (HH) systems whose Hamiltonian function is of the form

$$H(x,y,p,q) = \frac{1}{2}(p^2+q^2) + V(x,y)$$

in canonical coordinates (x,y,p,q) [3, 11]. It is known that seven non-trivial integrable cases exist with V a polynomial function: three are cubic and four quartic. Some additional inverse terms can be added to V in order to obtain the *generalized HH systems*. The separation variables have been found for all the cubic cases [1,9].

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