

# EXAMPLES OF GIBBS STATES OF MECHANICAL SYSTEMS WITH SYMMETRIES

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**Abstract.** In a previous paper, the notion of Gibbs state for the Hamiltonian action of a Lie group on a symplectic manifold was given, together with its applications in Statistical Mechanics, and the works in this field of the French mathematician and physicist Jean-Marie Souriau were presented. Using an adaptation of the cross product for pseudo-Euclidean three-dimensional vector spaces, we present several examples of such Gibbs states, together with the associated thermodynamic functions, for various two-dimensional symplectic manifolds, including the pseudo-spheres, the Poincaré disk and the Poincaré half-plane.

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